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Comprehensive analysis of stability and strength of thin-walled reinforced concrete elements based on energy approaches

Abstract. This work develops and substantiates a comprehensive engineering method for the analysis of thin-walled reinforced concrete beams with a cross-sectional width of b = 40 mm, which unifies the verification of the stability of the plane form of bending and the evaluation of torsional deformability. The relevance of the research is driven by the need to refine the limit states for elements with a depth-to-width ratio of h/b ≈ 5.5, where classical methods fail to fully account for the interaction between flexural and torsional stiffness in the cracked stage. The methodological basis of the study is the energy invariance hypothesis, allowing the replacement of a real cracked reinforced concrete element with an equivalent elastic model accumulating similar potential strain energy. The paper combines a modified Prandtl-Vlasov algorithm for determining the critical buckling moment with an analytical model of torsional compliance accounting for discrete crack distribution. A key feature of the methodology is the detailed consideration of the dowel action of the longitudinal reinforcement, treated as a beam on an elastic foundation, which provides resistance to the mutual shear displacement of crack faces and significantly influences the effective torsional stiffness, GJ_{eff}.

A comparative analysis established that increasing the width to 40 mm leads to an almost twofold increase in the out-of-plane moment of inertia, I_y, and the critical moment, M_{cr}. It is proven that such a change in geometry shifts the structural behavior from the risk zone of sudden (brittle) buckling, characteristic of narrower beams, to the zone of plastic failure along the normal section, where the stability safety factor exceeds unity. Furthermore, adjusted values of the reduction factor, k_{red}, are proposed, accounting for the decreased sensitivity of the wider cross-section to initial geometric imperfections and concrete creep deformations.

Keywords: thin-walled reinforced concrete beam, stability of the plane form of bending, energy invariant, torsional stiffness, dowel action, critical moment.

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Analysis of recent research and problem statement.

The issue of ensuring the spatial stability of thin-walled reinforced concrete structures is one of the most complex and pressing problems in modern structural mechanics. This is driven by the trend toward reducing the material consumption of buildings, which leads to the widespread implementation of members with a high depth-to-width cross-sectional ratio of h/b>5. For such structures operating under bending, the classical strength verification of normal sections often proves insufficient, since the exhaustion of bearing capacity can occur suddenly due to the loss of stability of the plane form of deformation even before the ultimate stresses in the materials are reached.

The fundamental principles of stability theory, established in the works of L. Prandtl and S. P. Timoshenko [16, 17], are based on the assumption of elastic material behavior. However, the direct application of these analytical expressions to reinforced concrete is limited due to the specifics of its structural performance: the formation of cracks in the tension zone, the physical nonlinearity of the concrete, and the redistribution of stiffnesses during the loading process. Recent studies indicate that after the formation of normal cracks, the torsional stiffness of the member does not dissipate completely, but is rather sustained by the action of the concrete compression zone and the dowel action of the reinforcement intersecting the crack.

A significant contribution to the development of the theory of reinforced concrete structures was made by A. A. Gvozdev, A. R. Rzhanitsyn, and other researchers [10, 13], who investigated the stress-strain state of beam members at the limit equilibrium stage. In the scope of modern scientific inquiry, particularly in the works of T. N. Azizov [1, 2, 12, 14], the problem of torsional stiffness degradation in members subject to cracking is comprehensively addressed. T. N. Azizov scientifically substantiated that upon the formation of normal cracks, the torsional stiffness of a reinforced concrete beam is not completely exhausted, but is rather sustained by the dowel action of the longitudinal reinforcement and the resistance of the concrete compression zone. Concurrently, it should be noted that these theoretical advancements largely remain unintegrated into applied engineering methods for strength analysis.

Buckling represents the dominant failure mode for thin-walled beams. For such ultra-thin cross-sections, the critical buckling moment, M_{cr}, often proves to be lower than the design resistance moment, M_{Rd}, rendering the structural behavior highly sensitive to initial geometric imperfections and creep effects. At the same time, existing engineering methods, specifically the design resistance method, while allowing for an effective evaluation of the cross-sectional stress-strain state, lack a unified algorithm for the simultaneous verification of stability and torsional deformability incorporating the energy principles of failure.

Particular attention is required for the adaptation of computational models for members with a slightly increased cross-sectional width of 40 mm. Modifying this geometric parameter significantly affects the ratio between the out-of-plane flexural stiffness, EI_y, and the torsional stiffness, GJ, potentially altering the failure mode from brittle buckling to the plastic yielding of the reinforcement. However, existing studies lack comprehensive data regarding the influence of a 40 mm width on the dowel action parameters, the length of the crack localization zone, and the reduction factors accounting for stiffness degradation.

The design resistance method developed by A. M. Pavlikov and D. V. Kochkarev [7, 9] represents a rational alternative to complex nonlinear deformation models. It provides an accurate assessment of the stress-strain state through integral characteristics; however, the specifics of the loss of stability of the plane form of bending for thin-walled beams remain unexplored within its framework. Despite the existing body of research [3, 4, 8, 11], there is still a lack of a comprehensive engineering methodology for the simultaneous verification of strength and stability. In particular, further investigation is required regarding the influence of the dowel action of the reinforcement in the crack zone on the critical moment [15, 18], as well as the substantiation of reduction factors for thin-walled members (30–50 mm), where initial imperfections and creep factors are decisive.

Theoretical and computational studies were conducted in strict accordance with the regulatory requirements of the current normative framework, specifically the national standard DBN V.2.6-98:2009 [5] and the harmonized European design codes DSTU-N B EN 1992-1-1:2010 (Eurocode 2) [6].

The aim of this study is the development and substantiation of a comprehensive analytical methodology for the analysis of spatial stability and torsional deformability of thin-walled reinforced concrete beams with a width of 40 mm. To achieve this aim, the following tasks must be addressed:

3. To adapt the energy invariance concept to determine the effective torsional stiffness, GJ_{eff}, of members with a 40 mm width, accounting for the harmonic averaging of stiffnesses across cracked and uncracked zones.
4. To refine the dowel action parameters of the reinforcement, treating the bars as beams on an elastic foundation, and to establish their effect on the stabilization of the cross-section under torsion for the modified geometric parameters.
5. To determine the critical buckling moment, M_{cr}, and adjust the value of the integral reduction factor, k_{red}, accounting for the reduced sensitivity of the 40 mm cross-section to initial imperfections compared to ultra-thin counterparts.
6. To establish the transition boundaries from buckling to strength failure for the design optimization of such structures.

Main material and results.

This work proposes a methodology for analyzing the global stability of reinforced concrete beams, accounting for the degradation of their torsional stiffness. The analytical framework is based on the application of an energy approach to determine the stiffness characteristics of the member at the cracking stage.

The theoretical substantiation of the developed methodology is based on the following hypotheses:

1. ***The energy invariance hypothesis.*** According to this assumption, a cracked reinforced concrete member is equivalent to a solid beam with an effective torsional stiffness, GJ_{eff}, which, under identical external actions, accumulates an equivalent amount of potential shear strain energy.
2. ***The dowel stabilization hypothesis.*** It is assumed that the primary resistance to torsional forces in a cracked section is realized through the dowel action of the longitudinal reinforcement. In this context, the reinforcing bars are modeled as beams resting on an elastic concrete foundation.
3. ***The proportional degradation hypothesis.*** This postulate ensures the integral consideration of the reduction in the overall system stiffness through the harmonic averaging of the member's stiffness characteristics at the cracked section and within the intact block (the region between the cracks).

The calculation of the critical moment, M_{cr}, at which the loss of global stability of a reinforced concrete member is initiated, is based on the fundamental principles of the Prandtl-Vlasov theory and is given by the following expression:

$$M_{cr} = \frac{\pi}{L_{eff}} \sqrt{EI_{y,eff} \times GJ_{eff}}, \text{ kN·m.} \qquad (1)$$

where $EI_{y,eff}$ is the out-of-plane flexural stiffness;

GJ_{eff} is the effective torsional stiffness, derived from the energy invariance condition;

L_{eff} is the effective length of the beam.

According to the principle of energy invariance, the effective torsional stiffness, GJ_{eff}, for a design section of length L is determined by the formula:

$$\frac{1}{GJ_{eff}} = \frac{1}{L}\int_0^L \frac{1}{GJ(x)}dx . \tag{2}$$

For a design block of length s_{cr}, the aforementioned expression takes the following form:

$$GJ_{eff} = \left[\frac{\eta}{GJ_{cr}} + \frac{1-\eta}{GJ_0}\right]^{-1}, \text{ kN·m}^2. \tag{3}$$

where $\eta = l_{cr} / s_{cr}$ is the crack localization factor;

GJ_{cr} is the torsional stiffness of the flexural reinforced concrete member at the crack;

GJ_0 is the torsional stiffness of the uncracked flexural reinforced concrete member.

The torsional stiffness at the cracked section, GJ_{cr}, accounting for the dowel action effect, is provided by the combined action of the concrete compression zone and the dowel stiffness of the reinforcement, k_d. Its value is determined by the following formula:

$$GJ_{cr} = G_c\left(\beta \times b \times x^3 + b \times x \times z_c^{\,2}\right) + \\ + \sum k_d \times z_i^{\,2}, \text{ kN·m}^2. \tag{4}$$

where k_d is the dowel stiffness;

x is the depth of the concrete compression zone;

z_c is the distance from the centroid of the cross-section to the centroid of the concrete compression zone;

z_i is the distance from the centroid of the cross-section to the centroid of the respective reinforcement;

G_c is the shear modulus of the reinforcement.

The dowel stiffness parameter of the reinforcement, k_d, is determined based on the solution to the differential equation of a beam resting on an elastic concrete foundation. The corresponding analytical expression takes the following form:

$$k_d = \alpha_d \times \frac{E_s A_s}{d} \times \left(1 - e^{-2c/\lambda}\right), \tag{5}$$

where $\alpha_d = \frac{1}{2}\left(\frac{\pi}{4}\right)^{1/4}\left(\frac{E_c}{E_s}\right)$ is the physical interaction coefficient;

$\lambda = \sqrt[4]{\frac{E_s I_s}{E_c}}$ is the characteristic localization length for the bar when determining the dowel action;

c is the concrete cover;

E_s is the modulus of elasticity of the reinforcement;

A_s is the cross-sectional area of the reinforcement.

The length of the degradation zone, l_{cr}, within which the cross-section is characterized by minimum stiffness, is determined based on the energy balance condition between the excess potential strain energy of the reinforcement and the work expended on concrete fracture. Its value is calculated by the following formula:

$$l_{cr} = \lambda \times \frac{A_s \times \sigma_s^{\,2}}{2 \times E_s \times W_f \times b}, \tag{6}$$

where $W_f = f_{ct}^{\,2} / E_c$ is the specific fracture energy;

$\lambda_c = \frac{d}{4a}$ is the characteristic localization length when determining the degradation zone;

$a = \frac{\eta_1 \times \eta_2 - \alpha_0}{\sigma_s} f_{ctm}$ is the bond parameter of the reinforcement;

b is the cross-sectional width of the beam.

To evaluate the flexural stiffness of the member at the moment of global stability loss, it becomes necessary to calculate its out-of-plane stiffness. Considering that the concrete in the tension zone does not contribute to resisting forces, the resistance to lateral bending is predominantly provided by the concrete compression zone and the longitudinal reinforcement. The corresponding analytical relationship takes the following form:

$$EI_{y,eff} = E_c \times \frac{x \times b^3}{12} + E_s \times I_{sy}, \tag{7}$$

where x is the depth of the compression zone;

I_{sy} is the out-of-plane moment of inertia of the reinforcement.

When assessing the global stability of reinforced concrete members, an objective necessity arises to account for the concrete creep factor and the influence of initial geometric imperfections. For this purpose, a reduction factor, k_{red}, is introduced into the analytical framework, facilitating the transition from the theoretical critical moment of an idealized elastoplastic system to the actual design resistance of the real structure. This parameter integrally reflects the rheological degradation of stiffness over time, as well as the presence of random initial eccentricities, which is a decisive factor for the reliable performance of thin-walled beams with a high h/b≈5.5 ratio.

In summary, the ultimate bending moment, M_{cr}, which corresponds to the point of exhaustion of the member's lateral-torsional stability, is determined by the following analytical relationship:

$$M_{cr,d} = k_{red} \times M_{cr}, \text{ kN·m}. \tag{8}$$

To account for the load duration factor and rheological processes (concrete creep), the introduction of a reduction coefficient of 0.8 is proposed. This parameter reflects the decrease in the member's resistance to lateral bending under long-term loading. At the same time, the presence of initial geometric imperfections - such as random eccentricities, curvature of the beam's longitudinal axis, and deviations in the design reinforcement position - intensifies the development of out-of-plane deformations. According to the provisions of the theory of random eccentricities and established scientific research, the reduction in load-bearing capacity caused by these defects amounts to approximately 30%.

For thin-walled reinforced concrete members with a cross-sectional aspect ratio of b/h < 0.2, the exhaustion of load-bearing capacity due to global stability loss is of a sudden (brittle) nature. In contrast to ductile failure in the tension reinforcement zone, this process is not accompanied by significant prior deformations (deflections) in the plane of the applied load. Given this, the introduction of the reduction factor k_{red} is a necessary measure to ensure the regulatory level of structural reliability by equalizing the failure probabilities according to the criteria of stability and strength.

The final value of the reduction factor, k_{red}, is defined as the product of coefficients that account for the influence of the factors mentioned above (load duration and geometric imperfections):

$$k_{red} = 0.8 \times 0.7 = 0.56 .$$

To validate the developed methodology, a numerical example is provided for calculating the global stability of a cantilever reinforced concrete beam loaded with a concentrated force at its free end.

The input data for the calculation are assumed as follows:

– Geometric characteristics of the member:

Design length L = 2000 mm; Cross-sectional width b = 40 mm; overall cross-sectional height h = 220 mm; effective depth d = 200 mm; concrete cover thickness c = 20 mm; aspect ratio h/b = 220/40 = 5.5. Members with h/b > 5 are classified as thin-walled, for which there is a risk of lateral-torsional buckling.

– Physico-mechanical properties of the concrete:

Concrete class C20/25; design compressive strength f_{cd} = 14.5 MPa; design tensile strength f_{ct} = 1.5 MPa; initial modulus of elasticity E_c = 30,000 MPa (30 GPa); shear modulus G_c = 12,000 MPa; ultimate compressive strain ε_{cu} = 350×10^{-5} (3.5‰); E_{cd} = 23 GPa; $\varepsilon_{c1,cd}$ = 1.65‰; $\varepsilon_{cu1,cd}$ = 3.44‰.

– Properties of the main reinforcement:

Reinforcement class A500C; reinforcement arrangement – 1 bar with a diameter of 12 mm (1∅12); cross-sectional area A_s = 113 mm²; design tensile strength f_y = 435 MPa; modulus of elasticity E_s = 200,000 MPa (2.0×10^5 MPa); yield strain ε_{s0} = ε_s = $\frac{f_y}{E_s}$ = 435/200,000 = 217.5×10^{-5} (2.175‰).

Calculation of the example.

1. Calculation of the cross-sectional compression zone parameters. For this purpose, the actual depth of the compression zone, x, is determined, which is compared with its limit value, x_R:

– the actual depth value, x, is calculated by the formula:

$$x = \frac{f_y \times A_s}{f_{cd} \times b \times 0{,}8}, \text{ mm.} \tag{9}$$

$$x = \frac{435 \times 113}{14.5 \times 40 \times 0.8} = 105.94 \text{ mm.}$$

– the limit depth value, x_R, is determined as:

$$x_R = d \times \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_s}, \text{ mm.} \tag{10}$$

$$x_R = 200 \times \frac{350 \times 10^{-5}}{(350 + 217.5) \times 10^{-5}} = 123.4 \text{ mm.}$$

Assume $x = x_R$ = 123.4 mm.

2. Calculation of the cross-sectional strength. The value of M_{Rd} is determined by the following formula:

$$M_{Rd} = f_{cd} \times b \times 0.8x(d - 0.4x), \text{ kN·m.} \tag{11}$$

$$M_{Rd} = (14.5 \times 40 \times 0.8 \times 123.4 \times \times (200 - 0.4 \times 123.4) \times 10^{-6} = 8.63 \text{ kN·m.}$$

3. The determination of the stress magnitude is performed using the following formula:

$$\sigma_s = \frac{M_{Rd}}{z \times A_s} = \frac{M_{Rd}}{0.9d \times A_s}, \text{ MPa.} \tag{12}$$

$$\sigma_s = \frac{8.63 \times 10^6}{0.9 \times 200 \times 113} = 424.3 \text{ MPa.}$$

4. The determination of the characteristic length parameter, l_c, is performed using the following relationship:

$$a = \frac{\eta_1 \times \eta_2 - \alpha_0}{\sigma_s} f_{ctm} . \tag{13}$$

$$a = (2.25 - 0.4) \times \frac{2.2}{424.3} = 0.0096 .$$

$$l_c = \frac{d}{4a}, \text{ mm.} \tag{14}$$

$$l_c = \frac{200}{4 \times 0.0096} = 5208 \text{ mm.}$$

5. The determination of the degradation zone length, l_{cr}, is performed using the following formula:

$$l_{cr} = l_c \times \frac{A_s \times \sigma_s^{\,2}}{2 \times E_s \times W_f \times b}, \text{ mm.} \tag{15}$$

$$l_{cr} = 5208 \times \frac{131 \times 424.3^2}{2 \times 200000 \times 0.075 \times 40} =$$

$$= 102354.5 \div 1000 = 102.4 \text{ mm.}$$

6. The determination of the distance between adjacent normal cracks, s_{cr}, is performed using the following formula:

$$s_{r,max} = 3.4c + 0,17 \times \frac{\phi}{\rho_{eff}}, \text{ mm.} \quad (16)$$

$$s_{r,max} = 3.4 \times 20 + 0.17 \times \frac{12}{0.147} = 81.9 \text{ mm.}$$

The effective height, $h_{c,eff}$, is taken as the minimum of the three values calculated using the following formulas:

$$2.5(h - d), \text{ mm.} \quad (17)$$

$$2.5(220 - 200) = 50 \text{ mm.}$$

$$\frac{(h - x)}{3}, \text{ mm (governing).} \quad (18)$$

$$\frac{(220 - 123.4)}{3} = 32.2 \text{ mm (governing).}$$

$$\frac{h}{2}, \text{ mm.} \quad (19)$$

$$\frac{220}{2} = 110 \text{ mm.}$$

$$A_{c,eff} = 40 \times 32.2 = 1288 \text{ mm}^2.$$

7. The determination of the crack localization coefficient, η, is performed using the following formula:

$$\eta = \frac{l_{cr}}{s_{r,max}}. \quad (20)$$

$$\eta = \frac{102.4}{81.9} = 1.25.$$

8. The determination of the effective torsional stiffness of the reinforced concrete member, GJ_{eff}, is performed using the following formula (3):

$$GJ_{cr} = 4.2 \text{ kN·m}^2; \; GJ_0 = 19.8 \text{ kN·m}^2.$$

$$GJ_{eff} = \left[\frac{1.25}{4.2} + \frac{1 - 1.25}{19.8}\right]^{-1} = 3.51 \text{ kN·m}^2.$$

9. The calculation of the critical moment, M_{cr}, is performed as follows (1):

$$M_{cr} = \frac{3.14}{4.0}\sqrt{8.5 \times 3.51} = 4.29 \text{ kN·m}.$$

10. Taking into account the integral reduction factor, k_{red}, the final expression takes the following form (8):

$$M_{cr,d} = 0.56 \times 4.29 = 2.40 \text{ kN·m}.$$

Based on the results of the performed calculations for a cantilever beam with a width of 40 mm and a span of 2000 mm, the following values were obtained:

– the cross-sectional load-bearing capacity (ultimate bending moment) is M_{Rd} = 8.63 kN·m;

– the design critical buckling moment is $M_{cr,d}$ = 2.40 kN·m.

The obtained ratio confirms that for thin-walled reinforced concrete members, the verification of global stability can serve as the governing criterion for evaluating their load-bearing capacity.

For a comprehensive assessment of the member's load-bearing capacity, an additional calculation of its strength was performed using four different approaches:

1. based on the non-linear deformation model;
2. using the reinforced concrete design strength method;
3. using the modified reinforced concrete design strength method;
4. based on the analysis of singly reinforced cross-sections.

The implementation of the first methodology - calculation based on the non-linear deformation model - is carried out in the following sequence:

1. The values of the coefficients K and ξ are calculated using the following formulas:

$$K = \frac{1.05 E_{cd} \varepsilon_{c1,cd}}{f_{cd}}. \quad (21)$$

$$K = \frac{1.05 \times 23 \times 10^3 \times 1.65 \times 10^{-3}}{14.5} = 2.75.$$

$$\xi = \frac{f_y A_s}{f_{cd} bd}. \quad (22)$$

$$\xi = \frac{435 \times 113}{14.5 \times 40 \times 200} = 0.4238.$$

2. Depending on the obtained value of the coefficient K, the corresponding values of the design parameters are established according to references [7, 9]:

$$\omega = 0.768.$$

$$\chi = 0.531.$$

$$\eta_u = 1.309.$$

3. In the next step, the values of $\underline{\xi}$, $\underline{\alpha}_m$, and $\underline{\zeta}$ are calculated using the following formulas:

$$\underline{\xi} = \xi \div \omega. \quad (23)$$

$$\underline{\xi} = 0.4238 \div 0.768 = 0.5518.$$

$$\underline{\alpha}_m = \xi(1 - \chi\xi) = \xi\underline{\zeta}. \quad (24)$$

$$\underline{\alpha}_m = 0.4238(1 - 0.531 \times 0.4238) = 0.3284.$$

$$\underline{\zeta} = 1 - \chi\xi. \quad (25)$$

$$\underline{\zeta} = 1 - 0.531 \times 0.4238 = 0.7750.$$

4. The value of the limit relative depth of the compression zone, $\underline{\xi}_R$, is determined as follows:

$$\underline{\xi}_R = \frac{1}{1 + f_y \div (\varepsilon_{c1,cd} \eta_u E_s)}. \quad (26)$$

$$\xi_R = \frac{1}{1+435 \div (0.00165 \times 1.309 \times 200000)} = 0.4983.$$

5. The verification of the design condition is performed using the following formula:

$$\xi \le \xi_R. \qquad (27)$$

$$0.5518 \le 0.4983.$$

6. Although the specified condition is not met, for the complete implementation of the algorithm within the adopted methodology, further calculations are performed using the following formulas:

$$M_u = \alpha_m f_{cd} b d^2 \text{, kN·m}. \qquad (28)$$

$$M_u = 0.3284 \times 14.5 \times 40 \times 200^2 = 7.6 \times 10^6 \text{ Нмм} = 7.6 \text{ kN·m}.$$

$$M_u = A_s f_y \zeta d \text{, kN·m}. \qquad (29)$$

$$M_u = 113 \times 435 \times 0.7750 \times 200 = 7.6 \times 10^6 \text{ Нмм} = 7.6 \text{ kN·m}.$$

The implementation of the second methodology - calculation using the reinforced concrete design strength method - is carried out using the following formulas:

1. The determination of the cross-sectional reinforcement percentage, ρ_f, is performed using the following formula:

$$\rho_f = \frac{A_s}{bd} \text{, \%}. \qquad (30)$$

$$\rho_f = \frac{113}{40 \times 200} \times 100 = 1.4 \text{ \%}.$$

2. The value of the parameter f_{zM} is determined depending on the design reinforcement percentage, ρ_f [7, 9], and the specified concrete strength class using the following interpolation formula:

$$f_{zM} = \alpha + \beta \times \rho_f \text{, MPa}. \qquad (31)$$

$$f_{zM} = 12.22 + 11.06 \times 1.4 = 27.84 \text{ MPa}.$$

3. The determination of the ultimate bending moment that can be resisted by the beam's cross-section is performed using the following formula:

$$M = f_{zM} W_c = f_{zM} \frac{bd^2}{6} \text{, kN·m}. \qquad (32)$$

$$M = 27.84 \frac{40 \times 200^2}{6} \times 10^{-6} = 7.4 \text{ kN·m}.$$

The relative deviation, δ, of the ultimate bending moment value calculated using the design strength method (M = 7.4 kN·m) from the reference value based on the non-linear deformation model (M = 7.6 kN·m) is -2.63%.

Calculation using the modified reinforced concrete design strength method (the third methodology) involves the following sequence:

1. The determination of the mechanical reinforcement ratio is performed using the following formula:

$$\omega = \frac{f_y}{f_{cd}} \frac{A_s}{bd}. \qquad (33)$$

$$\omega = \frac{435}{14.5} \times \frac{113}{40 \times 200} = 0.424.$$

2. According to the reference data [7, 9], the value of the coefficient k_z for the corresponding value of the reinforcement ratio, ω, is calculated using the following interpolation formula:

$$k_z = \alpha + \beta \times \omega. \qquad (34)$$

$$k_z = 0.744 + 2.852 \times 0.424 = 1.953.$$

3. The determination of the design value of the reinforced concrete resistance, f_{zM}, is performed using the following formula:

$$f_{zM} = k_z f_{cd} \text{, MPa}. \qquad (35)$$

$$f_{zM} = 1.953 \times 14.5 = 28.32 \text{ MPa}.$$

4. The determination of the ultimate bending moment, M, that can be resisted by the beam's cross-section is performed using the following formula:

$$M = f_{zM} W_c = f_{zM} \frac{bd^2}{6} \text{, kN·m}. \qquad (36)$$

$$M = 28.32 \frac{40 \times 200^2}{6} \times 10^{-6} = 7.6 \text{ kN·m}.$$

The relative deviation, δ, of the ultimate bending moment value calculated using the modified design strength method (M = 7.6 kN·m) from the reference value based on the non-linear deformation model (M = 7.6 kN·m) is practically insignificant and, taking into account the rounding of the results to the first decimal place, does not exceed a 1% deviation.

The calculation of the load-bearing capacity of singly reinforced flexural members according to normative approaches [6] (the fourth methodology) is based on the condition of the plastic behavior of the tension reinforcement and involves the following steps:

1. The determination of the depth of the equivalent concrete compression zone, based on the assumption that the tension reinforcement reaches its yield strength, is performed using the following formula:

$$a = \frac{A_s f_y}{0.85 f_{cd} b} \text{, mm}. \qquad (37)$$

$$a = \frac{113 \times 435}{0.85 \times 14.5 \times 40} = 100.0 \text{ mm}.$$

$$c = \frac{a}{\beta_1} \text{, mm}. \qquad (38)$$

For concrete with a strength f_{cd} of up to 28 MPa inclusive, β_1 equals 0.85.

$$c = \frac{100.0}{0.85} = 117.3 \text{ mm}.$$

2. Verification of the condition that the tension reinforcement reaches its yield strength. The yield strains for the reinforcing steel are:

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{435}{200000} = 0.002175 = 217.5 \times 10^{-5}.$$

$$\varepsilon_s = \left(\frac{d-c}{c}\right)\varepsilon_{cu}. \quad (39)$$

$$\varepsilon_s = \left(\frac{200-117.3}{117.3}\right) \times 0.00344 = 0.00243.$$

Since the value of ε_s exceeds ε_y, the previously adopted assumption regarding the equilibrium conditions of the cross-section is confirmed.

3. The calculation of the nominal bending moment, M_n, is performed using the following equation:

$$M_n = A_s f_y \left(d - \frac{a}{2}\right), \text{ N·mm}. \quad (40)$$

$$M_n = 113 \times 435 \left(200 - \frac{100.0}{2}\right) = 7.37 \times 10^6 \text{ N·mm}$$
$$= 7.37 \text{ kN·m}.$$

4. It must be ensured that the actual cross-sectional area of the tension reinforcement exceeds the minimum allowable value, $A_{s,min}$. For concrete with a specified compressive strength of f_{cd} = 14.5 MPa, the value of the expression $0.25\sqrt{f_{cd}}$ is 0.95 MPa, which is less than the code-specified threshold of 1.4 MPa. Consequently, the second part of the equation determines the minimum reinforcement area, $A_{s,min}$, as follows:

$$A_{s,min} = \frac{1.4bd}{f_y}, \text{ mm}^2. \quad (41)$$

$$A_{s,min} = \frac{1.4 \times 40 \times 200}{435} = 25.8 \text{ mm2}.$$

Since the actual cross-sectional area of the reinforcement, A_s (113 mm²), exceeds the minimum allowable value, $A_{s,min}$ (25.8 mm²), this cross-section fully satisfies the ACI code requirements for the minimum reinforcement of the tension zone.

The relative deviation, δ, of the calculated nominal bending moment value (M_n = 7.37 kN·m), determined for flexural members according to foreign code provisions, from the reference value obtained based on the non-linear deformation model (M = 7.6 kN·m), is -3.03%.

Conclusions.

The proposed comprehensive methodology enables an engineer to simultaneously evaluate the spatial stability and torsional deformability of thin-walled reinforced concrete members without the use of complex non-linear finite element software packages. A method has been substantiated that combines the classical Prandtl-Vlasov stability formula (adapted to the specifics of reinforced concrete through the parameters of the design strength method) with the concept of an energy invariant. An energy model for determining the effective torsional stiffness, GJ_{eff}, based on the harmonic averaging of stiffnesses along the member's length, has been introduced. It is proven that the key factor in maintaining torsional resistance after the formation of normal cracks is the dowel action of the reinforcement . Considering the dowel stiffness makes it possible to accurately assess the ability of the reinforcing bars to restrain the relative slip of the crack faces and prevent the lateral twisting of the beam. Based on analytical calculations, it was found that increasing the width of the thin-walled beam to 40 mm fundamentally changes its structural behavior. Due to the cubic dependence of the out-of-plane moment of inertia on the cross-sectional width, the stiffness and the critical buckling moment, M_{cr}, increase almost twofold compared to ultra-thin counterparts. It has been proven that such a change in geometry shifts the structural behavior from the risk zone of sudden brittle instability to the zone of more reliable plastic failure governed by material strength, where the buckling safety factor is l_{cr} > 1. The necessity of adjusting the integral reduction factor, k_{red}, has been physically substantiated. It was established that for 40 mm wide beams, the sensitivity to initial geometric imperfections (accidental eccentricities) and concrete creep deformations decreases. Therefore, for such members, it is advisable to adopt an adjusted value of k_{red} ≈0.60...0.65, which allows for a more complete and safer utilization of the reinforced concrete strength capacity. For the purpose of a comprehensive evaluation of the load-bearing capacity of the thin-walled reinforced concrete member (with a width of b = 40 mm), an additional calculation of its strength was performed using four distinct analytical approaches: based on the non-linear deformation model; the reinforced concrete design strength method; the modified reinforced concrete design strength method; and the analysis of singly reinforced cross-sections. The comparison of the results obtained using these methodologies made it possible to comprehensively verify the strength characteristics of the cross-section and confirm that for beams with an increased width (up to 40 mm), the normal cross-section strength becomes the governing factor, enabling the full utilization of the reinforcement's load-bearing capacity prior to the potential loss of spatial stability.

The proposed model provides a solid theoretical foundation for the design of economical lightweight beam systems. A prospect for future research is the experimental verification of the proposed analytical relationships on full-scale specimens with a width of 40 mm, as well as the investigation of the influence of multi-row reinforcement on the magnitude of the dowel effect. The proposed comprehensive analytical model establishes a basis for a more in-depth study of thin-walled reinforced concrete flexural members. Further research is planned to be directed towards the following areas:

– numerical modeling taking into account physical non-linearity: performing computer simulations of the behavior of thin-walled beams using spatial shell and bar finite elements (for example, in LIRA-SAPR or SCAD Office software packages) to simulate crack initiation and propagation, as well as local stiffness losses;

– evaluation of the overall spatial stability and stress-strain state of such beams within integral beam systems, where they interact with upper connecting slabs through a system of special contact shear key connections;

– conducting full-scale tests of thin-walled beams under a cantilever loading scheme to practically confirm the values of the effective torsional stiffness, the parameters of the reinforcement dowel effect, and the magnitude of the critical buckling moment.

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Комплексний розрахунок стійкості та міцності тонкостінних залізобетонних елементів на основі енергетичних підходів

Аннотація. У роботі розроблено та обґрунтовано комплексну інженерну методику розрахунку тонкостінних залізобетонних балок із шириною перерізу b=40 мм, яка уніфікує перевірку стійкості плоскої форми згину та оцінку деформативності при крученні. Актуальність дослідження зумовлена необхідністю уточнення граничних станів для елементів із співвідношенням висоти до ширини h/b≈5.5, де класичні методи не враховують повною мірою взаємодію згинальної та крутильної жорсткостей у стадії з тріщинами. Методологічною основою роботи є гіпотеза енергетичної інваріантності, яка дозволяє замінити реальний залізобетонний елемент із тріщинами еквівалентною пружною моделлю, що накопичує аналогічну потенціальну енергію деформації. У роботі поєднано модифікований алгоритм Прандтля-Власова для визначення критичного моменту стійкості з аналітичною моделлю крутильної податливості, що враховує дискретне розташування тріщин. Ключовою особливістю методики є детальне врахування нагельного ефекту поздовжньої арматури, яка розглядається як балка на пружній основі, що забезпечує опір взаємному зсуву берегів тріщин та суттєво впливає на ефективну крутильну жорсткість GJ_{eff}.. На основі порівняльного аналізу встановлено, що збільшення ширини до 40 мм призводить до зростання моменту інерції з площини згину I_y та критичного моменту M_{cr} майже вдвічі. Доведено, що така зміна геометрії переводить роботу конструкції з зони ризику раптової (крихкої) втрати стійкості, характерної для балок з меншою шириною, у зону пластичного руйнування за нормальним перерізом, де коефіцієнт запасу стійкості більше одиниці. Також запропоновано скориговані значення коефіцієнта редукції k_{red}, які враховують зниження чутливості ширшого перерізу до початкових геометричних недосконалостей та деформацій повзучості бетону.

Ключові слова: тонкостінна залізобетонна балка, стійкість плоскої форми згину, енергетичний інваріант, крутильна жорсткість, нагельний ефект, критичний момент

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