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TRANSVERSAL LOADED PILES DEFORMATION TASK DECISION METHOD

The deformation method allows to describe the stress-strain state of foundation structures by means of dependency that binds settling of foundations to the parameters of stiffness in the system «base – piles foundation» at different stiffness coefficients of the basis, along length or depth of foundation. The proposed methodology allows improving the calculations of the stress-strain state of laterally loaded piles, which can significantly improve the performance of buildings and structures. Deformations of a foundation structure are described by approximate dependency that includes the sinking of ends of foundation and the stiffness parameter of the system "basis-foundation". The calculation embraces various (linear and nonlinear) distributions patterns of the of the stiffness coefficient of the basis along the length of the structure as well as distribution properties of the ground basis.

Keywords: *pile, Winkler basis, lateral load, horizontal deformations, soil stiffness coefficient.*

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ДЕФОРМАЦІЙНИЙ СПОСІБ РОЗВ'ЯЗАННЯ ЗАДАЧ ПОПЕРЕЧНО НАВАНТАЖЕНИХ ПАЛЬ

Розв'язання задач деформаційним способом дозволяє описувати напружено-деформований стан фундаментних конструкцій за допомогою залежності, яка пов'язує осідання фундаментів з параметрами жорсткості в системі «основа – пальовий фундамент» при різних коефіцієнтах жорсткості основи по довжині або глибині фундаментів. Запропонований спосіб дозволяє вдосконалити розрахунки напружено-деформованого стану поперечно навантажених паль, що дає змогу значно підвищити експлуатаційні якості будівель і споруд. Описано деформації фундаментної конструкції наближеною залежністю, що включає в себе осідання кінців фундаменту та параметр жорсткості системи «основа – фундамент». Розрахунок охоплює різноманітні (лінійні й нелінійні) закономірності розподілу коефіцієнта жорсткості основи по довжині конструкції та розподільні властивості ґрунтової основи.

Ключові слова: *паль, основа Вінклера, поперечне навантаження, горизонтальні деформації, коефіцієнт жорсткості ґрунту.*

Introduction. In view of construction and reconstruction expansion [5] of buildings and facilities in town cramped conditions, as well as in complicated engineering and investigation conditions, there is a significant growth of piles usage for foundations and enclosure structures. New experimental and technical research [1 – 4, 6] has been accumulated within last years, which allows to make significant improvements in the regulations of existing standards, and besides, in many cases it allows to produce more cost-effective or more reliable buildings. The deformation method allows to improve the calculation of stress-strained state of piles and other foundation types through the expression of foundation structures' deformation as a dependency of foundation settling on the “basis-foundation” system stiffness and the basis stiffness coefficient that varies along the length or the depth of foundations, which allows to improve calculations quality for various types of foundations.

Analysis of the latest sources of research and publications. V.A. Florin [8] in his research came very close to the «deformation» method in solving of contact problems for foundation calculations, on the basis of the bed coefficient linear distribution. He examined only absolutely stiff structures. However, V.A. Florin demonstrated how, having used the laws of bed coefficient variation and foundation deformation, it would be possible to obtain the balance equations from where angle of structure twist can be obtained, and subsequently, the response of the basis can be obtained as well.

The general problem parts selection unsolved earlier. This approximation method relates to the piles calculation implementing various laws of stiffness variation in the Winkler's basis [4 – 5], where solving of differential equations for bending of structures, based on such a basis, causes mathematical challenges, and sometimes would be impossible altogether, for example, if it is tried to implement the basis stiffness coefficient with nonlinear distribution along the length, into a bending pile axis equation, it is obtained a non-converging function that neither allows to give a specific solution nor allows to generate the calculation tables.

Problem definition. The task is to define the calculation method for laterally loaded piles foundation that would more accurately describe its interaction with the Winkler's basis, as well as developing the calculation methodology of «pile-ground» system.

To achieve this task, the following problems were being solved:

- the analysis of the current calculation models of the «laterally loaded pile-soil» system as well as experimental laboratory and field studies;
- the formulation the of analytical and numerical calculation method;
- the comparison of the theoretical results obtained with the current solutions and results that were produced in the software package.

The primary material and results. The essence of the deformation method is the replacement of the equation of a bending pile axis or foundation by an approximation equation that includes a correction coefficient for stiff structures – ξ .

If analyse from the simple to the complex, it can be considered this device to be used the simplest single-span beams (Fig. 1, a) and consoles (Fig. 1, b, c).

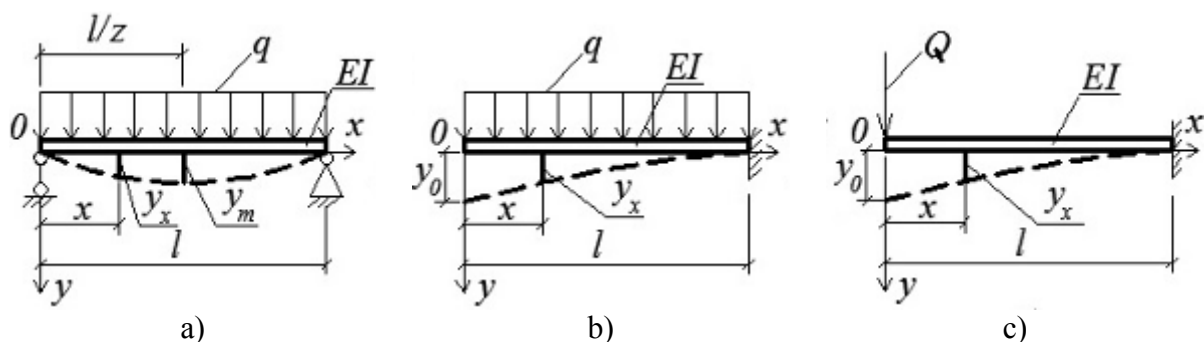


Figure 1 – The simplest bending structures

The solution of differential equations for bending of these structures is rendered in the course of Materials Resistance like this:

$$a) y_x = \frac{q \cdot l^4}{24 \cdot EI} \cdot \left(\bar{x}^4 - 2 \cdot \bar{x}^3 + \bar{x} \right), \quad (1-a)$$

$$b) y_x = \frac{q \cdot l^4}{24 \cdot EI} \cdot \left(\bar{x}^4 - 4 \cdot \bar{x} + 3 \right), \quad (1-c)$$

$$c) y_x = \frac{Q \cdot l^3}{6 \cdot EI} \cdot \left(\bar{x}^3 - 3 \cdot \bar{x} + 2 \right). \quad (1-c)$$

Various dependencies were applied to describe the deformation of these beam structures. For example, it is suggested for a single-span beam that is symmetrically loaded

$$y_1 \approx 4 \cdot y_{max} \cdot \bar{x} \cdot (1 - \bar{x}), \quad (2-a)$$

or

$$y_1 \approx y_{max} \cdot \sin(\pi \bar{x}). \quad (2-b)$$

Where in this case of the load

$$y_{max} = \frac{5}{384} \cdot \frac{q \cdot l^4}{EI}.$$

For console beams it was recommended:

$$y_1 = y_0 \cdot \left\{ 1 - \bar{x} \cdot \left[1 + \xi \cdot (1 - \bar{x}) \right] \right\}. \quad (3)$$

Wherein in the cases $\delta)$ and $\epsilon)$ it is got

$$\left. \begin{aligned} y_0 &= \frac{q \cdot l^4}{8 \cdot EI}; \\ y_0 &= \frac{Q \cdot l^3}{3 \cdot EI}. \end{aligned} \right\} \quad (4)$$

As demonstrated by the results of calculations found in the Table 1, the deformations in strict solution can be rendered accurately enough when applying recommended dependencies. In this case, however, the structures of uniform bending stiffness were considered.

However, this dependency cannot be applied for beam and banded structures having contact with the basis, as both ends of these structures get two types of movement such as y_0 and y_l (Fig. 2) under the load, and besides, these types of movement may have different signs.

The deformation of piles (Fig. 2, a) is represented by the formula

$$y_z = y_0 - \bar{z} \cdot (y_0 - y_l) \cdot \left[1 + \xi \cdot (1 - \bar{z}) \right], \quad (5)$$

which can be rendered in relevant units as

$$\bar{y}_z = 1 - (1 - \bar{y}_l) \cdot \left[1 + \xi \cdot (1 - \bar{z}) \right] \cdot \bar{z}, \quad (6)$$

Table 1 – The results of the simplest bending structures calculation

Structure	$\frac{\bar{x}}{\bar{y}}$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
	Beam a)	\bar{y}	0.0	0.300	0.594	0.813	0.952	1.0	0.952	0.813	0.594	0.300
$\bar{y}_1 - a$		0.0	0.360	0.640	0.840	0.960	1.0	0.960	0.840	0.640	0.360	0.0
$\bar{y}_1 - b$		0.0	0.309	0.588	0.809	0.951	1.0	0.951	0.809	0.588	0.309	0.0
Console b)	\bar{y}	1.0	0.867	0.734	0.603	0.475	0.354	0.243	0.147	0.070	0.019	0.0
	\bar{y}_1 if $\xi=0.6$	1.0	0.846	0.704	0.574	0.456	0.350	0.256	0.174	0.104	0.046	0.0
Console c)	\bar{y}	1.0	0.850	0.704	0.564	0.432	0.313	0.208	0.122	0.056	0.015	0.0
	\bar{y}_1 if $\xi=0.6$	1.0	0.846	0.704	0.574	0.456	0.350	0.256	0.174	0.104	0.0	0.0

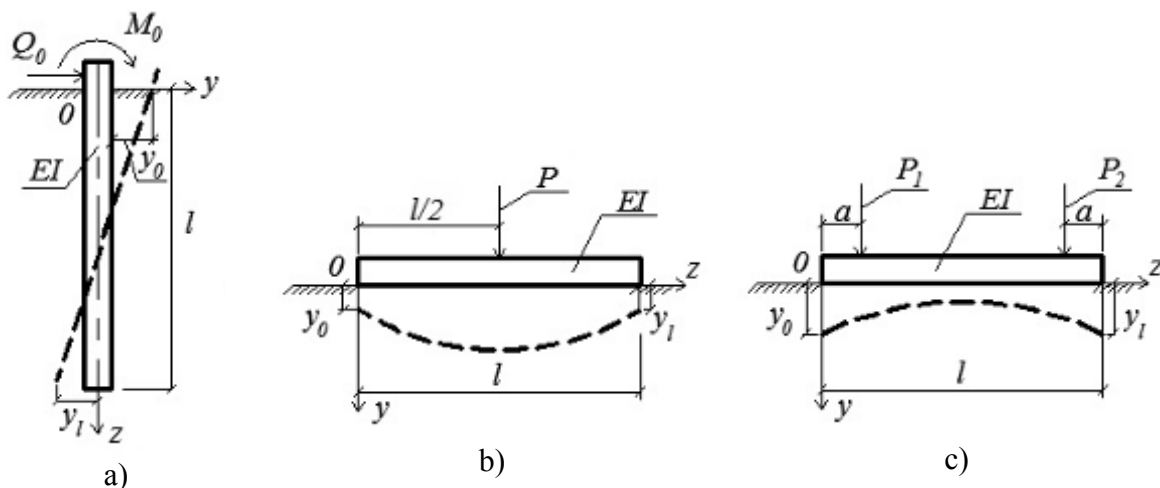


Figure 2 – Piles, beam and banded foundations

If it is calculated a band foundation that is symmetrically loaded at the center (Fig. 2, b), it would be convenient to describe the deformation as the following dependencies

$$y_z = y_0 \cdot \left[1 + 4 \cdot \xi \cdot \bar{z} \cdot (1 - \bar{z}) \right], \quad (7-a)$$

or

$$y_z = y_0 \cdot \left[1 + \xi \cdot \sin(\pi \cdot \bar{z}) \right]. \quad (7-b)$$

In case of symmetrically loaded band ends (Fig. 2, c) it is used the following formulas

$$y_z = y_0 \cdot \left[1 - 4 \cdot \xi \cdot \bar{z} \cdot (1 - \bar{z}) \right], \quad (8-a)$$

or

$$y_z = y_0 \cdot \left[1 - \xi \cdot \sin(\pi \cdot \bar{z}) \right] \quad (8-b)$$

The calculation of laterally loaded piles with linearly increasing law of bed coefficient distribution.

If it is considered three types of piles loadings from an external load to be: Q_0, M_0 – at free end of a pile and q – along the length of a pile trunk (Fig. 3).

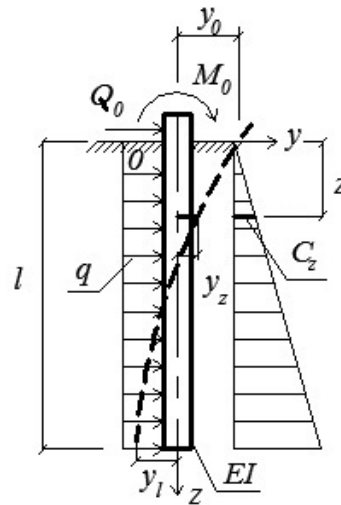


Figure 3 – The calculation diagram of a laterally loaded pile

It is represented the equation of a bending pile axis as

$$y_z = y_0 - (y_0 - y_l) \cdot \bar{z} \cdot [1 + \xi \cdot (1 - \bar{z})] \quad (9)$$

The law of the bed coefficient variation

$$C_z = K \cdot z \quad (10)$$

Therefore, the law of contact strain variation σ_z along the depth of a pile will be expressed as

$$\sigma_z = K \cdot l \cdot \bar{z} \cdot \{y_0 - (y_0 - y_l) \cdot \bar{z} \cdot [1 + \xi \cdot (1 - \bar{z})]\} \quad (11)$$

Next, it is got the total resistance of a basis

$$\begin{aligned} \int_0^l \sigma_z dz &= K \cdot l^2 \cdot \left\{ \frac{y_0}{2} - (y_0 - y_l) \cdot \left[\frac{1}{3} + \xi \cdot \left(\frac{1}{3} - \frac{1}{4} \right) \right] \right\} = \\ &= \frac{K \cdot l^2}{12} \cdot [6 \cdot y_0 - (y_0 - y_l) \cdot (4 + \xi)] \end{aligned} \quad (12)$$

The equation (11) will be represented as

$$\frac{\sigma_z}{K \cdot l} = y_0 \cdot F_1^z + y_l \cdot F_2^z \quad (13)$$

where

$$\left. \begin{aligned} F_1^z &= \bar{z} - \bar{z}^2 \cdot [1 + \xi \cdot (1 - \bar{z})]; \\ F_2^z &= \bar{z}^2 \cdot [1 + \xi \cdot (1 - \bar{z})]. \end{aligned} \right\} \quad (14)$$

Next, it is discussed various cases of the load on a pile.

1. Evenly distributed load q

It is put together the conditions of pile balance.

From the condition $\Sigma N = 0$, applying the equation (12), it is got

$$y_0 \cdot (2 - \xi) + y_l \cdot (4 + \xi) = \frac{12 \cdot q}{K \cdot l} \quad (15)$$

From the condition $\Sigma M = 0$ and the equation (13) it can get

$$\left(y_0 \cdot f_1 \cdot \bar{z}_1^0 + y_l \cdot f_2 \cdot \bar{z}_2^0 \right) \cdot K \cdot l^3 = \frac{q \cdot l^2}{2} \quad (16)$$

Wherein f_i and \bar{z}_i^0 are the areas of functions F_1^z , F_2^z and their distance to the axle F_i , are calculated with the help of integration of these functions.

Therefore, the system of two equations (15) and (16) was derived to determine the two unknown variables y_0 and y_l ,

For example, if a pile is «long», with the reduced length $\bar{l} = 4$, according to the proposed methodology, it is assumed $\xi = 1$. Next, it is calculated the values $f_1 = 0,0833$; $f_2 = 0,41667$; $\bar{z}_1^0 = 0,404$; $\bar{z}_2^0 = 0,720$ (Fig. 4), which allows to put together the system of equations (15) and (16):

$$\left. \begin{aligned} y_0 + 5 \cdot y_l &= \frac{12 \cdot q}{K \cdot l} \\ y_0 \cdot 0,0833 \cdot 0,404 + y_l \cdot 0,41667 \cdot 0,720 &= \frac{1}{2} \cdot \frac{q}{K \cdot l} \end{aligned} \right\}$$

The solving of this problem gives $y_l = 0,729 \cdot \frac{q}{K \cdot l}$; $y_0 = 8,355 \cdot \frac{q}{K \cdot l}$.

When it is put it into (9), it is obtained the equation for pile deflections, and when it is put it into (11), it is obtained the distribution of contact strains, which allows to draw diagram of bending moments.

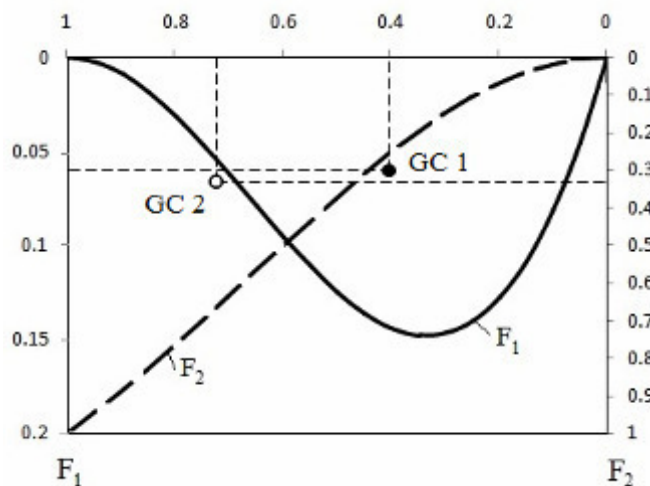


Figure 4 – Functions F_1 , F_2 and positions of their gravity centers if $\xi = 1$

The equation for deflections and resistances of the soil can be represented as

$$\left. \begin{aligned} y_z &= L_{1q}^z \cdot \frac{q}{K \cdot l} \\ \sigma_z &= L_{2q}^z \cdot q \end{aligned} \right\} \quad (17)$$

where L_1^z and L_2^z are functions of deflections and resistances,

$$\left. \begin{aligned} L_{1q}^z &= 0.729 + 7.626 \cdot (1 - \bar{z})^2 \\ L_{2q}^z &= \bar{z} \cdot [0.729 + 7.626 \cdot (1 - \bar{z})^2] \end{aligned} \right\} \quad (18)$$

If it is replaced, for example, the resistance by ten concentrated forces

$$-P_z = 0.1 \cdot L_2^z \cdot q \cdot l,$$

And also, if it is replaced the distributed load q by ten forces

$$Q_z = 0.1 \cdot q \cdot l,$$

Then it becomes possible to figure out the bending moments along the axis of a pile (a beam)

$$M_z = L_3^z \cdot q \cdot l^2, \quad (19)$$

where L_3^z is a function of bending moments.

The values of new functions L_1^z , L_2^z and L_3^z are represented in the Table 2.

Table 2 – Values of functions L_1^z , L_2^z and L_3^z for various relative depths \bar{z}

\bar{z}	0.00	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95	1.00
L_1^z	8.355	7.6115	6.239	5.019	3.951	3.036	2.273	1.663	1.2056	0.9006	0.748	0.729
L_2^z	0.000	0.3806	0.936	1.2547	1.383	1.366	1.2503	1.081	0.9042	0.7655	0.7107	0.729
L_3^z	0.000	0.0048	0.0113	0.0185	0.0231	0.0238	0.0210	0.0156	0.0094	0.0042	0.0004	0.000

Calculation example 1

To draw diagrams for y_z , σ_z and M_z for a flexible pile ($\alpha_d \cdot l = 4$), evenly loaded along its buried part, the load being $q = 10 \text{ t/m}^2$.

Initial data: pile length $l = 5 \text{ m}$, design width $b_p = 1 \text{ m}$, deformation coefficient

$$\alpha_o = \sqrt[5]{\frac{K \cdot b_p}{EI}} = 0.8 \text{ m}^{-1}.$$

The calculation was completed according to the Table 2 and represented at the illustration 5. Concurrently, pile calculations were carried out with the software package «SCAD» and according to the methodology by A.S. Maliev [7].

The suggested deformation methodology gave some intermediate values for deflections, resistances and bending moments.

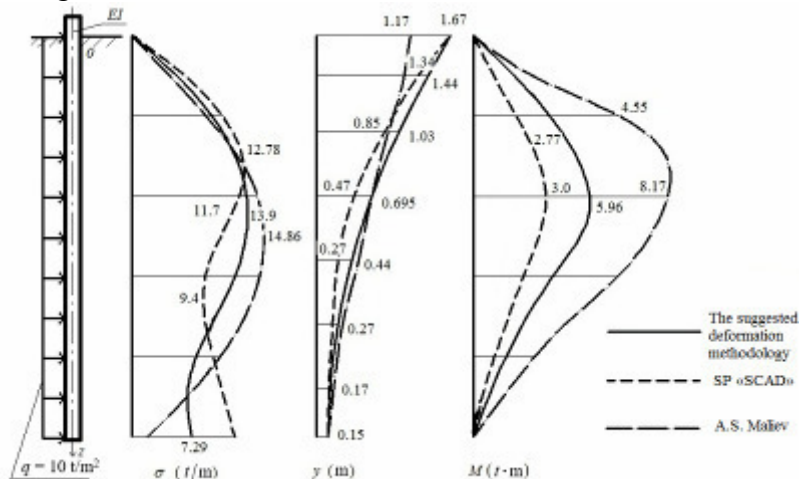


Figure 5 – An example of calculation for a flexible pile with distributed load

2. Concentrated load Q_0

To solve this problem it is meant to make up balance equations $\Sigma N = 0$ and $\Sigma M = 0$, which in this case is:

$$\left. \begin{aligned} y_0 \cdot (2 - \xi) + y_l \cdot (4 + \xi) &= \frac{12 \cdot Q_0}{K \cdot l^2} \\ y_0 \cdot f_1 \cdot \bar{z}_1^0 + y_l \cdot f_2 \cdot \bar{z}_2^0 &= 0 \end{aligned} \right\} . \quad (20)$$

If note that the functions F_1^z and F_2^z are dependent only on what model is accepted for basis, and in this case they are described by formulas (14). Therefore, when it is determined f_i and \bar{z}_i^0 their values depend on what stiffness coefficient ξ has been assumed. With this type of the load on a long pile ($\bar{l} = 4$) it is recommended to assume $\xi = 1.4$.

In this case the numerical methodology would give

$$f_1 = 0.05 ; f_2 = 0.450 ; \bar{z}_1^0 = 0.3603 ; \bar{z}_2^0 = 0.7111 .$$

When applying these values to solve the system (20), it is got (if $\xi = 1.4$)

$$y_l = -2.2824 \cdot \frac{Q_0}{K \cdot l^2} ; y_0 = 40.54 \cdot \frac{Q_0}{K \cdot l^2} .$$

Wherein the functions of deflections and resistances are:

$$\left. \begin{aligned} L_{1Q}^z &= 40.54 + 42.822 \cdot (1 - \bar{z})^2 \\ L_{2Q}^z &= \bar{z} \cdot \left[40.54 + 42.822 \cdot (1 - \bar{z})^2 \right] \end{aligned} \right\} . \quad (21)$$

3. Pile loaded by the moment M_0

In this case of the load, the calculation gets somewhat simplified, because from the condition $\Sigma N = 0$ it is obtained

$$y_0 \cdot (2 - \xi) + y_l \cdot (4 + \xi) = 0,$$

where the ratio of movements of the ends:

$$\frac{y_0}{y_l} = -\frac{(4 + \xi)}{(2 - \xi)} . \quad (22)$$

Next, from the condition $\Sigma M = 0$ it is obtained

$$\left(y_0 \cdot f_1 \cdot \bar{z}_1^0 + y_l \cdot f_2 \cdot \bar{z}_2^0 \right) = -\frac{M_0}{K \cdot l^3} . \quad (23)$$

When it is considered a long pile ($\bar{l} = 4$), it should be assumed $\xi = 1.6$.

In this case, it is obtained (provided $\xi = 1.6$ is assumed)

$$f_1 = 0.0333 ; f_2 = 0.4667 ; \bar{z}_1^0 = 0.3933 ; \bar{z}_2^0 = 0.7071 .$$

Using the conditions (22), (23), it is obtained

$$y_l = -7.069 \cdot \frac{M_0}{K \cdot l^3} ; y_0 = 98.968 \cdot \frac{M_0}{K \cdot l^3} .$$

Wherein the functions of deflections and resistances are:

$$\left. \begin{aligned} L_{1M}^z &= 98.968 + 106.037 \cdot (1 - \bar{z})^2 \\ L_{2M}^z &= \bar{z} \cdot \left[98.968 + 106.037 \cdot (1 - \bar{z})^2 \right] \end{aligned} \right\} \quad (24)$$

Conclusions. The above mentioned calculation approaches can be united into the unified «deformation» methodology, which allows:

– to express deformation of foundation structure as approximate (or exact) dependency that includes the settling of foundation ends and unknown (or preset) parameter ζ ; of the «basis-foundation» system stiffness

– to utilize random law of basis stiffness coefficient variation (proportionality), including the one imitating distribution properties of the current models that have distribution properties (half-plane, half-space, finite thickness basis, etc.);

– to establish the law for basis resistance variation along the structure;

– to find out the unknown movements, by integration, using balance conditions.

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