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FEATURES OF THE MATHEMATICAL MODELING OF FOUNDATIONS INTERACTION WITH COMPACTING SOILS, WITH ANISOTROPIC PROPERTIES

Features of the mathematical modeling of foundations interaction with compaction soils with anisotropic properties, by ultimate elements method in the physically and geometrically non-linear presentation are confirmed. To prove this fact it was obtained henomenological soil model that describes its state during building, work of compacted subsoil. Famous physical correlations of orthotropic medium were used in model. Modeling of cast work -in-situ pile with leading borehole and enlarged base for isotropic basic and for transversely-isotropic medium were compared. Reality of decisions obtained by modeling are provided by properties of ultimate elements, by sizes of calculated field, by choice of design schemes of soil compaction, by conformity of model state soil parameters.

Keywords: *soil natural and directed anisotropy, foundation, soil compaction zone, orthotropic medium, method of ultimate elements, axis-symmetrical problem.*

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ОСОБЛИВОСТІ МАТЕМАТИЧНОГО МОДЕЛЮВАННЯ ВЗАЄМОДІЇ ФУНДАМЕНТІВ З УЩІЛЬНЕНИМИ ҐРУНТАМИ, ЩО МАЮТЬ АНІЗОТРОПНІ ВЛАСТИВОСТІ

Викладено особливості моделювання методом скінченних елементів у фізично й геометрично нелінійній постановці взаємодії фундаментів з ущільненими ґрунтами, що мають анізотропні властивості. Для цього розроблено феноменологічну модель ґрунту, що описує його стан при влаштуванні та роботі ущільнених основ. У моделі використано відомі фізичні співвідношення ортотропного середовища. Наведено порівняння результатів моделювання роботи набивної палі з лідируючою свердловиною й розширенням для ізотропної основи та для трансверсально-ізотропного середовища. Достовірність рішень, отриманих моделюванням, забезпечено властивостями скінченних елементів, розмірами розрахункової області, вибором розрахункових схем ущільнення ґрунту, відповідністю параметрів моделі стану ґрунту.

Ключові слова: *природна та наведена анізотропія ґрунту, фундамент, ущільнена зона, ортотропне середовище, метод скінченних елементів, вісесиметрична задача.*

Introduction. Classic calculation methods are tested for common foundations with compacted subsoil. They are not universal because of soil conditions difference and new structural and technologic solutions, which during designing often leads to necessity of additional natural tests. In the same time current level of soft allows to use methods of bases and foundations stress-strain state (SSS) modeling on solution of practical geotechnical problems. Current practice [1] recommends to use tested program complex of finite elements method (FEM), especially in the physically and geometrically non-linear presentation.

Analysis of recent sources of research and publications. O. Bugrov, F. Gabibov, N. Zotsenko, V. Lushnikov, Yu. Osipov, E. Sergeev, L. Timofeeva, G. Cherny, O. Shkola, B. Amadei, A. Bishop, I. Duncan, G. Gazetas, H. Kulatilake, K. Lo, J. Magnan, M. Oda, H. Seed and other [1 – 7] investigated anisotropy of mechanical soil properties. I. Boyko, G. Geniyv, M. Goldshteyn, S. Klovanych, O. Кorobova, V. Кovtun, O. Petrakov, S. Thimbal, P. Allahverdizadeh and other [8 – 11] developed the advantages of anisotropic model over isotropic.

Because of specificity for clay deposits with aqueous origin, loess, strip clays primary (natural) mechanical (deformation, strength) anisotropy are caused by their natural structure (ordered structure with priority parallel orientation of particles or pores in certain direction), its origin, formation condition, (including the process of sedimentation), etc., and secondary anisotropy, the nature and the laws depending on the natural structure of the soil and on the particular technologies of foundations (the direction of displacement of soil working body piles, blocks of sizes between Base Area) [2 – 7 11] – it allows to use physical models in their relations anisotropic, and firstly orthotropic environment.

But presented environmental variables in the formation of its foundations and bases of determining soil compaction simulation of the transient processes include soil compaction by different technologies, with FEM physically and geometrically nonlinear statement [10 – 15].

Identification of general problem parts unsolved before. Today, even with help of modern program complex of FEM in the physically and geometrically non-linear presentation fair evaluation of SSS of soil foundations arranged with soil compaction with anisotropic properties, were not investigated.

That is why the **goal** of this article is to improve the methodic of modeling in the condition of axis-symmetrical problem of FEM in the physically and geometrically non-linear presentation of foundations (or piles) interaction with compaction soils, which have anisotropic properties.

Basic material and results. The authors investigated the mechanical anisotropic soil properties by taking soil samples by cutting rings, oriented in different angles (α = 0; 45; 90^o) to horizontal plane (it's taken as isotropic plane), with tests in the odometers, direct shear apparatuses, penetrometers. Penetration was performed directly within the array by penetrometers PD-2M and MV-2 perpendicular to the sites cleaned in different directions to the plane of isotropy. In array of points for all research areas at the plane isotropy has coefficient of variation values of soil mechanical properties submitted in the form of quadrants or travel time, which is graphical representation of the behavior of the soil mechanical characteristics from the angle α [5 – 7].

The coefficients of mechanical anisotropic properties were defined:

$$n_{E,\alpha} = E_\alpha / E_- \; ; \tag{1}$$

$$n_{c,\alpha} = c_\alpha / c_- \; ; \tag{2}$$

$$n_{\varphi,\alpha} = tg\varphi_\alpha / tg\varphi_- \quad ; \tag{3}$$

$$n_{R,\alpha} = R_\alpha / R_- \quad , \tag{4}$$

where E_- – modulus of soil deformation in isotropic plane from impact of stresses in the same plane (orientation of rings under angle $\alpha = 0^o$ relatively to horizontal plane);

E_α – the same, respectively for plane, inclined to isotropic plane on angle α, c_-, c_α, φ_-;

φ_α – unit cohesion and angle of internal soil friction in plane of shift respectively is parallel ($\alpha = 0^o$) and inclined to isotropic plane on angle α;

R_- and R_α – unit penetration resistance according to $\alpha = 0^o$ and $\alpha \neq 0^o$ to isotropic plane.

In massif of loessial loam (W_L = 0,29 – 0,33; W_P = 0,18 – 0,20; e = 0,83 – 0,99; w = 0,13 – 0,23) established that the highest values of mechanical properties are character for the samples, which are taken for angle $\alpha = 0^o$ to horizontal plane, the smallest – for angle $\alpha = 45^o$ (n_α = 0,6 – 0,9). With depths soil anisotropy increased: in depth 1 m from surface $n_{\alpha=90}$ = 0,85 – 0,9 and in depth 4,0 – 4,5 m – $n_{\alpha=90}$ = 0,7 – 0,8. Under water saturation and silicatization loess has isotropic properties ($n_\alpha \to 1{,}0$).

Bulk loams for 10 – 40 years of compaction under own weight gained anisotropic properties ($n_{\alpha=90}$ = 0,65 – 0,95). Its options of anisotropy depend on time of own weight compaction. After soil compaction priority directions of locus answered to displacement directions of the soil by pile working body or prefabricated elements (n_α = 0,5 – 2,0) [6, 7].

For conditions, when coefficients of soil anisotropy differ significantly from n_α = 1,0, calculation accuracy of the soil base SSS could be increased by its using in the model of physical correlations of orthotropic or transversely-isotropic medium. Parameters describing these bodies (in the cylindrical coordinate system – which scheme is presented in Figure 1)are: modulus of deformation in isotropic plane E_r and E_Θ, and also transverse direction E_z; respectively Poisson's ratios $\nu_{r\Theta}$, ν_{rz}, $\nu_{\Theta z}$. In the case of model of transversely-isotropic medium using in calculation shows that $E_\Theta = E_r$.

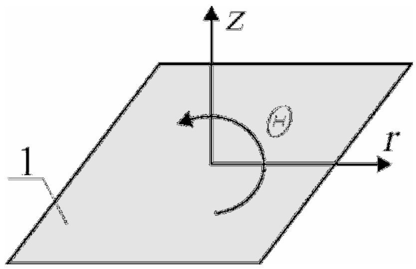

Figure 1 – The cylindrical coordinate system: 1 – isotropic plane

According to classification of compaction methods and phenomenological elastoplastic soil model were created program complex «PRIZ-Pile». It is realized in form of axis-symmetrical problem solution by FEM in physically and geometrically non-linear form [10, 11]. Geotechnic modelling:

1) different by geometry, scheme of soil displacement, character and speed of pressure transition processes of foundations arrangement with compacted subsoils, which result is SSS of massif and presented values of soil properties;

2) further work of such soil basis and foundations under load. Eight-nodal isoparametric FE can change its shape and volume, which allows using rectangular and curved mesh. In presenting of the soil isotropic medium physical correlations of SS in matrix form with form:

$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \sigma_z\\ \tau_{rz}\end{Bmatrix}=\frac{E}{\Omega}\begin{bmatrix}1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0\\ \frac{\nu}{1-\nu} & 1 & \frac{\nu}{1-\nu} & 0\\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & 0\\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)}\end{bmatrix}\begin{Bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \varepsilon_z\\ \gamma_{rz}\end{Bmatrix}; \qquad (5)$$

$$\Omega=[(1+\nu)(1-2\nu)]/(1-\nu), \qquad (6)$$

where σ_r, σ_Θ, σ_z, τ_{rz} – normal and tangential stresses in UE y cylindrical coordinates;
E – modulus of soil deformation those FE;
ν – Poisson's soil ratio of FE;
ε_r, ε_Θ, ε_z, γ_{rz} – axial and angle components of strains in FE.

In presenting soil by anisotropic (orthotropic) medium physical equations of SS in matrix form with form:

$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \sigma_z\\ \tau_{rz}\end{Bmatrix}=\frac{1}{\Omega}\begin{bmatrix}E_r(1-\nu_{\theta z}\nu_{z\theta}) & E_r(\nu_{r\theta}+\nu_{rz}\nu_{z\theta}) & E_r(\nu_{rz}+\nu_{r\theta}\nu_{\theta z}) & 0\\ E_\theta(\nu_{\theta r}+\nu_{zr}\nu_{\theta z}) & E_\theta(1-\nu_{rz}\nu_{zr}) & E_\theta(\nu_{\theta z}+\nu_{rz}\nu_{\theta r}) & 0\\ E_z(\nu_{zr}+\nu_{\theta z}\nu_{z\theta}) & E_z(\nu_{z\theta}+\nu_{r\theta}\nu_{zr}) & E_z(1-\nu_{rz}\nu_{\theta r}) & 0\\ 0 & 0 & 0 & \Omega G_{rz}\end{bmatrix}\begin{Bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \varepsilon_z\\ \gamma_{rz}\end{Bmatrix}; \qquad (7)$$

$$\Omega=1-2\nu_{\theta r}\nu_{rz}\nu_{z\theta}-\nu_{r\theta}\nu_{\theta r}-\nu_{\theta z}\nu_{z\theta}-\nu_{rz}\nu_{zr}, \qquad (8)$$

where E_r, E_Θ, E_z – modulus of soil deformation for relevant directions;
$\nu_{r\Theta}$, ν_{rz} $\nu_{\Theta z}$ – respective Poisson's ratios, which are calculated as follow:

$$\nu_{\theta r}=\frac{E_r}{E_\theta}\cdot\nu_{r\theta}; \qquad \nu_{zr}=\frac{E_r}{E_z}\cdot\nu_{rz}; \qquad \nu_{z\theta}=\frac{E_\theta}{E_z}\cdot\nu_{\theta z}. \qquad (9)$$

For transversely-isotropic medium $E_\Theta = E_r$.
G_{rz} – shear modulus, according to formula of S. Lekhnitcky:

$$G_{rz}=\frac{E_rE_z}{E_z+E_r(1+2\nu_{rz})}. \qquad (10)$$

For each layer of the soil (stiffness) is defined:

– initial stiffness – depending on the characteristics consideration of anisotropy. In the case of soil presentation by isotropic medium the stiffness characteristics presented in the form of entry modulus of deformation and Poisson's ratio ν. In presenting of the soil by orthotropic medium as stiffness characteristics modulus of deformation E_r, E_Θ, E_z and respective Poisson's ratios $\nu_{r\Theta}$, ν_{rz} $\nu_{\Theta z}$ are considered Acceptance of the hypothesis of transversely-isotropic medium [2] is possible. Then: $E_\Theta = E_r$; $\nu_{\Theta z} = \nu_{rz}$;

– correlations of deformation modulus from volume (or porosity) of soil $E_i/E_\circ = f(V_i/V_\circ)$ in the form of analytical expression [11] or table. For the first phase of this dependence, it is considered the rate of applied load, corresponding to foundation arrangement technology, and the second - static load;

– correlations of soil resistance shift from normal stress $\tau = f(\sigma)$;

– soil unit weight γ.

Except geometric dimensions, which are result of the first stage, as initial parameters of foundations on the second stage physical and mechanical properties of materials (unit weight, angle of internal soil friction, unit cohesion, modulus of deformation) and lateral earth pressure coefficients λ and the impact of sliding side surface foundation on modulus of soil deformation are considered (from 0 to 1). When the foundation has several components, these parameters are determined for each foundation component.

Calculation in complex is performed in two stages. On the first stage is simulating the formation of cavity under foundation (pile, artificial base). The cavity axis coincides with a symmetry axis of the designed field. External influence is given in a form of forced vertical and horizontal displacements of unit of FE net, which lay on axis of rotation on the highest limit of designed field or occupy in it another position, which simulates the process of pressing out the soil by pile (rammer). These displacements lead to decreasing FE value and thus to reduction of soil porosity and the increase of its deformation modulus and strength though the process of soil boiling is possible. Because the forced displacements coincide with FE sizes of calculated scheme it is corrected on each stage by correction unit coordinates accounting displacements of previous step. With the change of coordinates, the FE values change and it gives the possibility to correct sol modulus of deformation in each FE for speed of loading, which corresponds to technology of the foundation arrangement. Soil void ratio in each FE is

$$e_i = e_\circ - (1 + e_\circ)(1 - V_i / V_\circ). \qquad (11)$$

The result of the first stage (and its steps) are new coordinates of FE units, the given soil characteristics, displacements of units of FE net, strength in the massif which are given in a form of tables, charts, isolines. Considering the fact that calculation on first stage are connected with step by step solution of the problem of defined moving and are deformed by the scheme at every step, then, as a rule, here is significant change in the shape of FE, which can lead to degeneration of the FE. Therefore, it should be considered moving the nodes less than the magnitude of product size element and soil void in this FE or appropriately select the size of FE.

Calculated soil characteristics and SSS massif enable to pass to the second stage – simulation of behavior of pile (foundation) under load. The hollow received by pressing out the soil is filled in by construction material, its characteristics are designed and new FE simulating foundation is introduced. So in contrast to models with fixed value of deformation modulus, this model describes its changes in volumetric strains especially in compaction, according to the change of soil porosity and transmission speed of pressure on it.

Peculiarity of the model on second stage is that in complicated SSS (compression with shear) general strains include linear (elastic) and plastic parts, thus plastic part of strain appears after SSS reach of strength limit according to Mises-Shleikher-Botkin. The compaction of soil is considered, its transfer into plastic stage with the reach of strength limit according to condition of strength, the possibility of sliding of side surface of foundation relatively the soil. The latter is realized by controlling tangential stresses in FE soil which are in contact «pile – soil». The task is checked by

$$\tau_{rz} \le (\sigma_r + \gamma h \lambda) tg\varphi + c \ , \qquad (12)$$

where σ_r – radial normal stresses;
h – distance from surface;
λ – lateral earth pressure coefficient;
φ – angle of internal friction;
c – unit cohesion.

With further loading on the surface of the FE, adjacent to soil, it was applied uniformly distributed load by friction forces on the foundation by soil $p = \gamma h \lambda \ tg\varphi$. Results of stage are: dependence of foundation settlement on load; displacements of each FE node; stresses in massif; soil transition in the fluid state in some FE; there soil properties are presented.

Examples of initial design scheme area division of the FE after the formation of leading borehole with diameter 0.5 m and depth 5.0 m for modeling SSS of cast-in-situ pile with enlarged base with crushed stone are presented in Figure 2 (borehole limited by nodes 865, 867 and 1186). The scheme has 369 FE with dimensions from 0,25×0,25 up to 0,8×1,0 m and 1204 nodes (150 fixed). Calculated region – cylinder with diameter 9,1 m and height 15 m. To depth 1.5 m is laying loam heavy silty, stiff (ρ_d = 1,41 g/cm^3), the range 1,5 – 3,5 m – loam light silty, stiff (ρ_d = 1,49 g/cm^3; E = 5.8 MPa), lower – clay light silty, stiff (ρ_d = 1,54 g/cm^3; E = 14 MPa). The enlarged base is created by soil compaction with crushed stone V_{cr} = 1,5 m^3 ($V_{cr.1}$ = 0,25 m^3) by the rammer with diameter 430 mm. The enlarged base has form of ellipsoid with half-axes: horizontal r_{br} = 0,65 m and vertical h_{br} = 0,70 m [16]. Its formation is modeled by applying forced horizontal and vertical displacements the 8 nodes (from 865 to 983) located on a path lower part of the borehole. The fragment of scheme deformation basis for constructing the enlarged base is presented in Figure 3.

The higher level of soil compaction is near and under the enlarged base. On the distance 0,15 m from the lateral surface of the enlarged base the value ρ_d increased from 1.54 g/cm^3 to 2,07 g/cm^3. Radius of a sufficient compaction zone, where ρ_d = 1,60 g/cm^3, according to modeling is $r_s \approx$ 1,00 m, and from [16] r_s = 0,96 m. Modulus of soil deformation in borders of the sufficient compaction zone increased by 2,3 times its size in the middle of the zone – in 3.5 times. The above parameters around the pile soil are used for modeling of pile work under the loading. The cavity, obtained by drilling and compacted, «fill» by construction materials (crushed stone and concrete) also additional FE and nodes are inserted (there are respectively
13 and 43), they are modelling pile body and the enlarged base. The load is applied by steps (firstly – 300 кN, and after – steps by 100 кN) in the form of concentrated force to axial node upper side of pile.

As output data of this task for soil base SSS modeling, which could be presented as transversely-isotropic medium, correlations of soil deformation modulus values according to previous investigations were taken [6, 7, 11] on the areas which are composed by loess soils with natural humidity, $n_{E,\alpha=90}$ = 0,8: for loam, placed over the enlarged base, $E_\Theta = E_r$ =4,65 MPa; E_z =5,8 MPa ; for clay, placed under the enlarged base, $E_\Theta = E_r$ =11,2 MPa; E_z =14,0 MPa. Comparison of the «load – settlement» behavior $S = f(P)$ of cast-in-situ pile for isotropic soil base (position 2) and for the transversely-isotropic medium (position 3) is presented in Figure 4.

Figure four presents that settlement according to modeling is on 15 – 20% exceeded results of static tests, but with increasing of the load this difference significantly reduce. The value of pile settlement in the 2 case, if $E_\theta = E_r < E_z$, is about for 10% bigger when $E_\Theta = E_r = E_z$.

So the result doesn’t deny famous patterns of soil mechanics and conclusions of other researchers [2, 3, 8]. Also it should be noted that according to modeling as for isotropic medium and for anisotropic soil in any FE don’t reach its ultimate limit state.

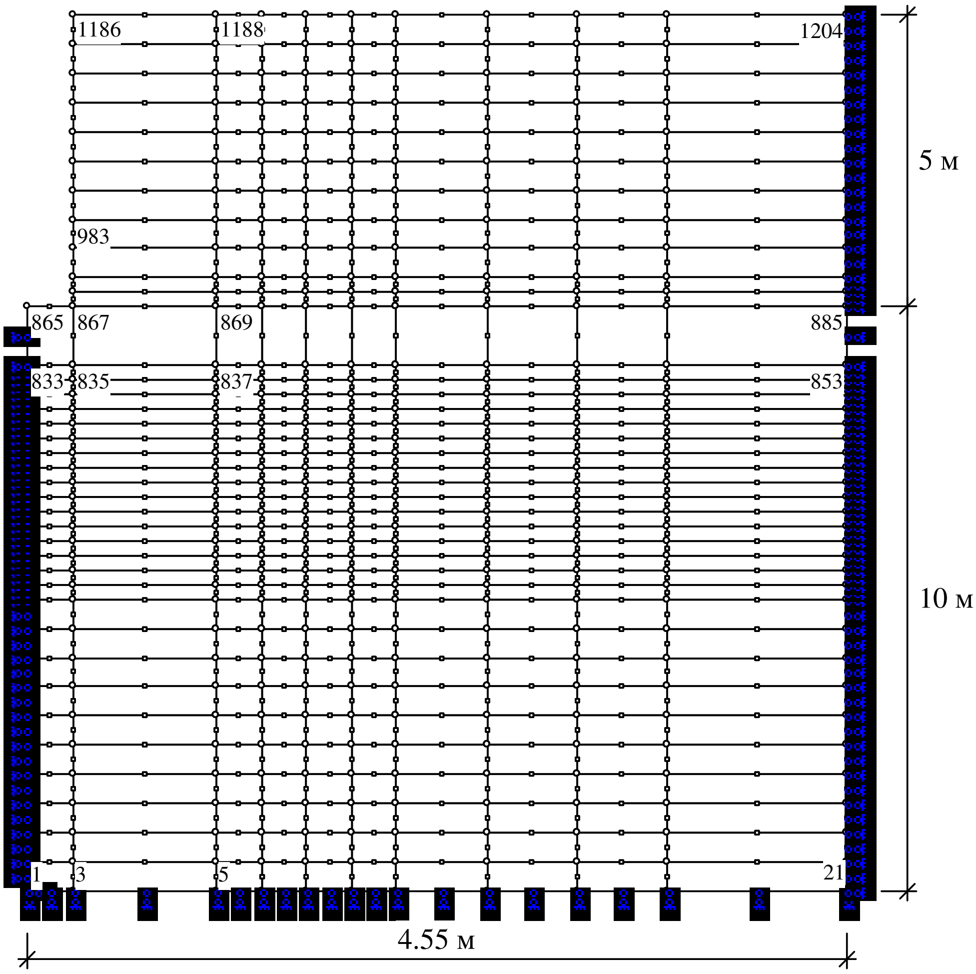

Figure 2 – Examples of initial design scheme area division of the FE after the formation of leading borehole and before the expansion placement

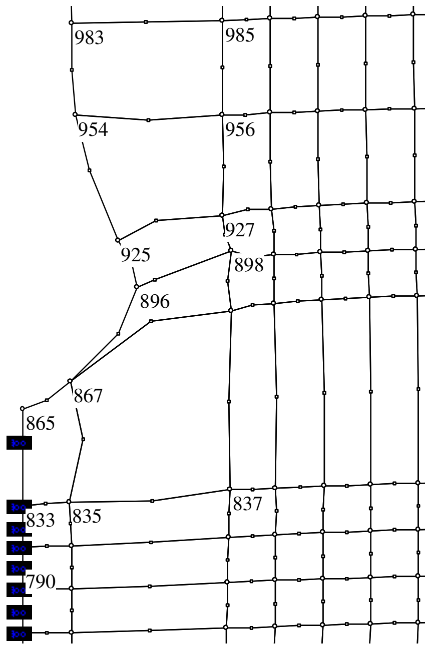

Figure 3 – The fragment of scheme of first stage modeling of pile soil base deformation with leading borehole after arrangement of an enlarged base

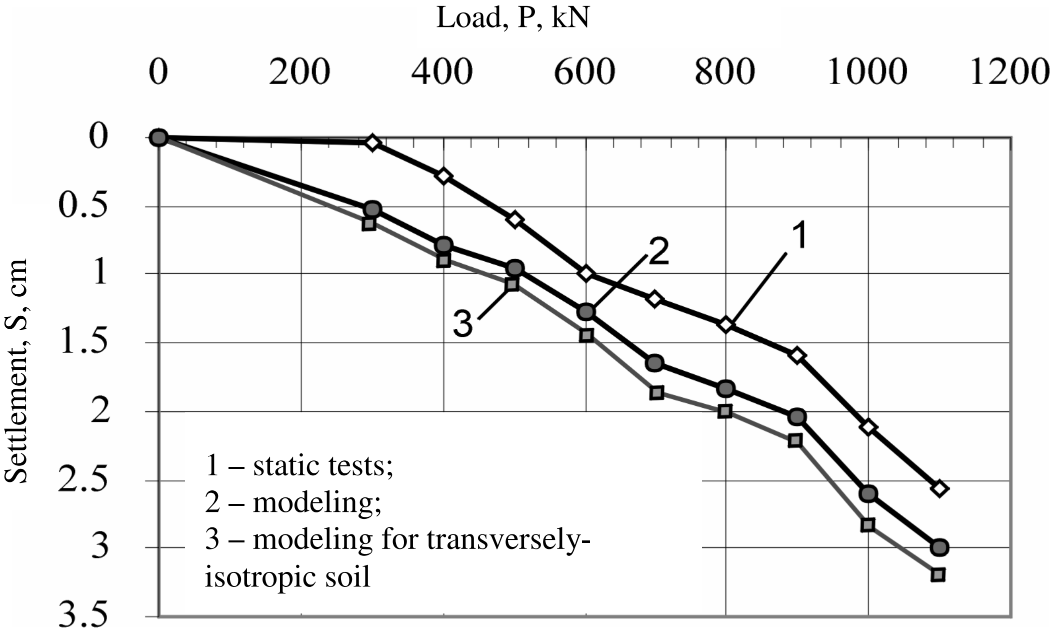

Figure 4 – Graphs of depending the settlement on loading onto cast-in-situ pile with leading borehole and enlarged base:

1 – static tests; 2 – modeling; 3 – modeling for transversely-isotropic soil $n_{E,\,\alpha=90^{\circ}} = 0{,}8$

Conclusions. For values of the soil deformation in isotropic plane E_r and E_θ in transverse plane E_z for axis-symmetrical problem of FEM in the physically and geometrically non-linear presentation it is possible to estimate SSS of orthotropic soil base of foundation, which are arranged or work with soil compaction.

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