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ULTIMATE ELONGATION OF CONCRETE

It is shown the results of experimental studies of the deformation and concrete fracture process under the axial tension and under the conditions of non-homogeneous state of stress when changing the deformation gradient over the cross section. The test samples were loaded by the short duration load to the point of structural and with load alleviation under the stresses close to destruction. The basic influencing factors on the ultimate elongation of studied types of concrete are found out and the analytical dependences for their description are proposed. The experiments have shown that under the non-uniform stress state the ultimate elongation is not constant; it varies within a wide range and depends on the critical strain gradient. For solving applied problems, the suitable, for practical use, universal form of connection load – deformation was offered, which truly reflects the unity of the relation between stress and deformation of concrete, which is in the condition of homogeneous and non-homogeneous stress states.

***Keywords**: stress-strain diagram of cracked concrete, ultimate elongation of concrete, axial tension, the deformation gradient.*

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ГРАНИЧНІ ДЕФОРМАЦІЇ РОЗТЯГНУТОГО БЕТОНУ

Викладено результати експериментальних досліджень закономірностей деформування і процесу руйнування бетону при осьовому розтягу і при наявності градієнта деформацій по перерізу. Дослідні зразки завантажувалися короткочасним навантаженням до руйнування і з розвантаженням при напруженнях, близьких до руйнування. Визначені фактори,що впливають на граничні деформації розтягнутого бетону різних видів та запропоновано аналітичні рівняння для їх опису. Дослідами встановлено, що при неоднорідному напруженому стані граничні деформації не постійні, а змінюються в широкому діапазоні та залежать від градієнта деформацій. Для вирішення прикладних задач запропоновано універсальну форма зв'язку «навантаження – деформації», яка достовірно відображає єдність зв'язку між напруженнями та деформаціями бетону що знаходиться в умовах однорідного та неоднорідного напружених станів.

***Ключові слова:** діаграма напруження – деформації розтягнутого бетону, гранична деформація розтягнутого бетону, осьовий розтяг, градієнт деформацій.*

Introduction. Planning of concrete and reinforced-concrete constructions is carried out on the basis of data of materials' durability and udeformations, which were got by the test of samples, as a rule, under the single-axis loading in laboratory conditions that are only the models of the real elements of constructions. Thus, the law of geometrical similarity was reckoned in, supposing the complete identity of samples' properties and the process of concrete deformation and destruction. However, the mechanism of concrete deformation in the conditions of the inhomogeneous high-pressure state is not identical to the axial tension and does not allow credibly to estimate deformations and durability. Therefore the experimental researches of the influence of different significant factors on durability and ultimate deformations of the cracked concrete have practical value.

Analysis of the last researches and publications. Operating rules [1], during calculation of durability of concrete constructions' standard cross-sections and crack strength of reinforced-concrete constructions, set the limit state, coming from the presentations, and dopted from the strength theory of elastic and inelastic materials. For the estimation of the stress and stress state of cross-sections, including the uniaxiality fabric hypothesis (the diagram $\sigma_{ct} - \varepsilon_{ct}$ under the axial tension is identical to the connection for the conditions of the inhomogeneous high-pressure state) and the supposition that the limit state is reached upon condition the extreme fibers achieve the ultimate strain of tension $\varepsilon_{ct2} = -2f_{ct} / E_{cto}$ Thus, it is considered that ultimate tensility is invariable.

Experimental data, given in literature [2 – 4], show that under the non-homogeneous tension the ratios $\varepsilon_{ct2} / \varepsilon_{ct1}$ and $6M_u / bh^2 f_{ct}$ (for the beams of rectangular cross-section) are non-constant, change in a wide range and depend, mainly, on the gradient of ultimate strain.

There isn't the very opinion at explaining of physical nature of variety of these sizes, the researchers offer different hypotheses.

It is supposed that the effective mechanism of stress grading acts in tension sides of the concrete beams, allowing the plastic flow to develop deep into the cross-sections. Such stress redistribution, maybe, takes place due to the nonlinear creep of concrete. However, direct experiments do not confirm this suspicion: nonlinearity of concrete creep under tension is rather weak, and elastic strains of the non-homogeneous cracked concrete are higher, than under axial tension. There is also the hypothesis about the possibility of concrete behavior in the «cracked» state and the statistical theory of concrete brittle strength. They describe the stress-strain state of beam's normal cross-sections in different ways, and, accordingly, offer the different computational schemes of ultimate states [2, 4, 6]. Analogical principle is characteristic and for the conditions of non-homogeneous pressure [5, 8 – 10].

All this information about the specific of deformation of heterogeneously cracked concrete and heterogeneously compressed concrete, point to iniquity of some hypotheses adopted from the theory of elastic and inelastic materials for the estimation of its stress and strain state.

Selection of problematical parts of general issue and setting the task of researches. The requirement of reliability at designing of concrete and reinforced concrete constructions actualizes caring out the special experimental researches according to this issue with the purpose of studying such questions:

– to investigate experimentally numerical influence of significant factors on common denominator of concrete resistance under non-homogeneous tension;

– to estimate ultimate tensility ε_{ct2} under bending in comparison with maximum axial tensility ε_{ct1} and to develop recommendations on the determination ε_{ct2} taking into account influencing factors;

– to investigate the features of destructions' development (strength and contractible) in the cracked concrete;

– to estimate the actual deformed state of normal cross-sections of concrete beam and construct a calculation model.

Basic material and results. For the quantitative estimation of influence of different factors (macrostructure and viscoelastic properties of concrete, type of the tense state, gradient of ultimate deformations, loading conditions) on ultimate tensility of concrete, the special experimental researches are carried out.

For the design of macrostructure and viscoelastic properties the model fine-grained concrete (MC), normal heavy concrete (HC) and light ceramsite concrete (LC) of wide range of compression breaking strengths are used from 10 to 80 MPa. Every class of investigational types of concretes contained from 3 to 24 samples.

The preproduction test was carried out under axial tension and in the conditions of the non-homogeneous state of stress. The wide range of gradients of ultimate deformations was designed by the deflection test of similar beams of cross-sections h = (1 – 60) cm, siding b = (1/3 and 3)h and bearing distance l = 6 h. Nonrigid behaviors of concrete under axial tension are determined at the test of samples-cylinders with the diameter 12 cm, length 40 cm. The samples (cylinders and beams) were tested in the short run by the step load till the destruction with the aggregate exposure on each state. For development of deformation of simple bending the beams were loaded by two concentrated forces in three spans, the cylinders – with the use of the device allowing to except centering error in the process of load application. For the selection of completely recoverable deformations of concrete expansion, the part of samples was off-loaded after every stage of loading. Deformations were being measured on the long bases and locally.

The tests verified the idea about the mechanism of concrete deformation in the conditions of axial uniform extension as about the process of starting and development of dispersed cracks that unite in the cross-section of destruction in a critical crack with the load growth. In the cracked concrete the tiny cracks appear long before the destruction and do not cause its bearing-capacity failure. There are the micro-discontinuities in concrete under axial tension with the loads $N_{crc} = (0.3 - 0.8)N_u$ in the tests. Body shrinkage-related stresses are able to break down soundness of coating surface or in isolated point reach the value similar to the limiting values before load application. In this case the behavior of limit equilibrium of ties is arrived even under the light external load, and there are micro-fractures of local character. Thus, cracked, under axis stress, concrete works with cracks and deforms on the length of sample very homogeneously ensuing incipient micro-fractures. The character of concrete's local deformations along the length of the centrally cracked sample with the growth of the load is shown on fig. 1.

The deformations, measured by devices with a different base, are different and incomparable. Devices, measuring local deformations, where cracks are formed in the base, show deformations that outnumber the values by several timeds according to the devices on a large base. In the next cross-sections with a progressive crack the local deformations can have even an opposite sign (reduction deformations), that is the result of initial surface-tension (for example, devices 11 and 15, fig. 1).

Inhomogeneity of the structure and micro-crack forrmation (shrinkable and power destructions) in the cracked concrete is responsible for the large nonuniformity of deformations both on length and on cross-section of samples, and, as a result, stochastic changeability of measurable deformations of concrete. In this case such concept, as «maximum tensility» usually used in the theory of concrete strength, is conditional. In the article the category «maximum tensility» of concrete under the axial tension ε_{ct1} is interpreted as peak deformations (the end of rising branch), corresponding to the concrete tension f_{ct} and the deformations, measured by devices on a large base (smoothed deformations).

For all tested types of concrete the diagrams $\sigma_{ct} - \varepsilon_{ct}$ are plotted under axial tension and the association between the parameters of concrete ε_{ct} and f_{ct} is set (fig. 2).

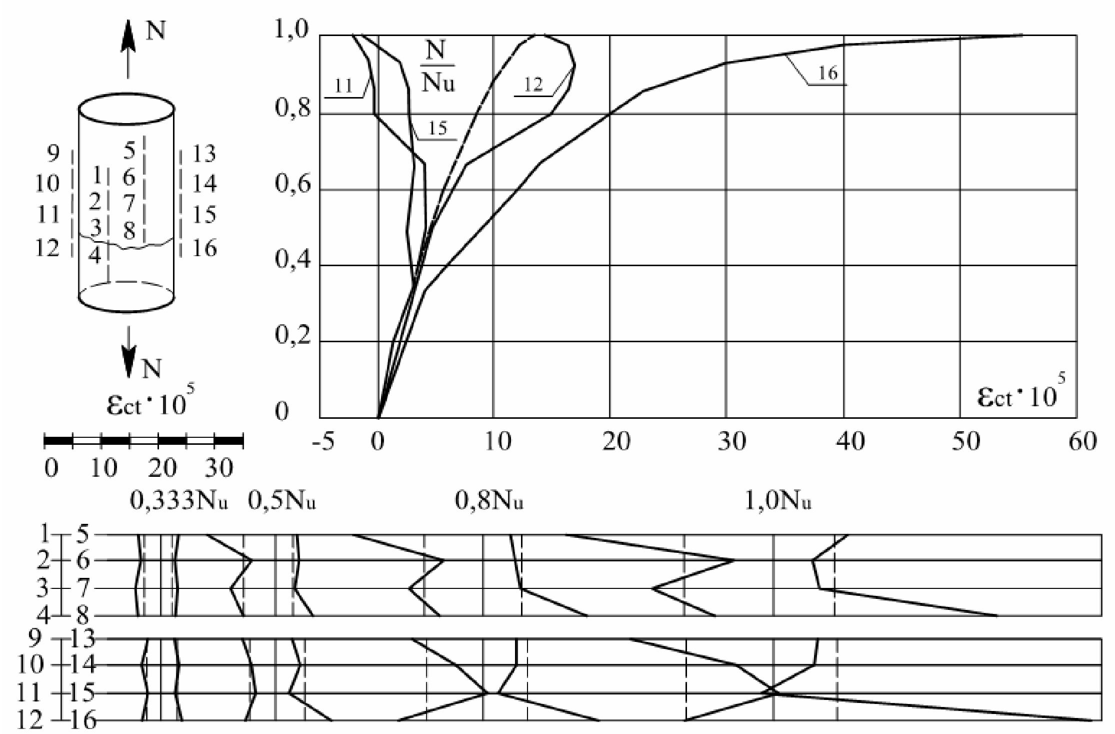

Figure 1 – Strain distribution along the length of the centrally cracked sample

—— local deformations; ------ normal deformations.

Deformation of concretes under axial tension with the load growth has nonlinear character. Nonlinearity is characteristic not only for total deformations but also for immediate elastic ones. It decreases with the growth of concrete class and less expressed for MC and LC, than for HC. The size of pseudo-plastic deformations, measured by devices on a large base, is insignificant, and the destruction of samples, regardless of type and class of concrete, takes place brittle.

For the solving the applied tasks, the analytical dependences, comfortable for practical use, are offered, describing the diagrams $\sigma_{ct} - \varepsilon_{ct}$ under axial tension definitely:

$$\sigma_{ct} = f_{ct}\left[1-\left(1-{}^{\varepsilon_{ct}}\!/\!_{\varepsilon_{ct1}}\right)^{1/n}\right]; \tag{1}$$

$$\varepsilon_{ct} = \varepsilon_{ct1}\left[1-\left(1-{}^{\sigma_{ct}}\!/\!_{f_{ct}}\right)^{n}\right]; \tag{2}$$

$$\frac{N}{N_u} = 1-(1-\frac{\varepsilon_{ct}}{\varepsilon_{ct1}})^{1/n}. \tag{3}$$

The physical meaning of n expands: $n = f_{ct}/\varepsilon_{ct1}E_{cto} = v_{ctu}$, where v_{ctu} – is limiting factor of concrete elasticity under axial tension.

Equations (1) and (2) were got at the solving of differential equation (4):

$$E_{ct} = \frac{d\sigma_{ct}}{d\varepsilon_{ct}} = E_{cto}(1-\frac{\sigma_{ct}}{f_{ct}})^{1-n}. \tag{4}$$

Equation (4) of the module of deformations under short standard tension satisfies to the basic phenomenological requirements to the arc

$$\sigma_{ct}-\varepsilon_{ct}\ (\sigma_{ct}=0\ -\ \frac{d\sigma_{ct}}{d\varepsilon_{ct}}=E_{cto}\ \text{and}\ \sigma_{ct}=f_{ct}\ -\ \frac{d\sigma_{ct}}{d\varepsilon_{ct}}=0\ ).$$

Formula (4) corresponds to concrete downloading with $V_{\sigma_{ct}} = const$ so it does not describe the descending arm of diagram $\sigma_{ct} - \varepsilon_{ct}$ ($V_{\varepsilon_{ct}} = const$).

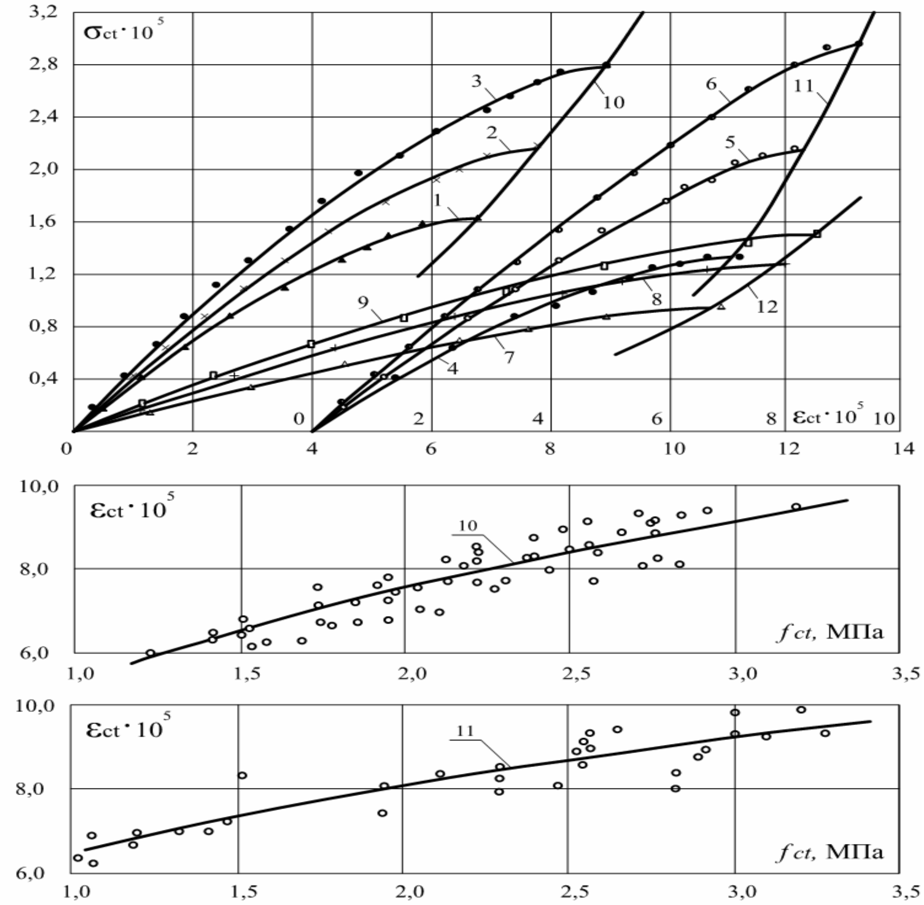

Figure 2 – Diagrams $\sigma_{ct} - \varepsilon_{ct}$ under axial tension (*a*) and relation of maximum tensility ε_{ct1} from concrete strength under axial tension f_{ct} (*b*)
1, 2, 3 are MC, accordingly, strength $f_{cm.cube10*10}$ = 23,7; 41,8 and 59,5 MPa;
4, 5, 6 – HC, $f_{cm.cube10*10}$ = 28,0; 47,5, 76,5 MPa;
7, 8, 9 – LC, $f_{cm.cube10*10}$ = 11,1; 19,4, 29,9 MPa;
10, 11, 12 – relation $\varepsilon_{ct1}(f_{ct})$ for MC, HC, LC.

According to test results (fig. 2b) ultimate deformation under short axial tension for investigational concretes changes in a range $\varepsilon_{ct} = (6{,}8 \div 12{,}5)10^{-5}$. Variability of elements ε_{ct1} is high; however, there is certain conformity in its arithmetic mean values, depending on a type and size of aggregate, cement content and class of concrete. Lightweight concretes have more deformability, than high weight concrete. MC and HC have, practically, identical values ε_{ct1}. However, there is tendency to increase ε_{ct1} with decreasing of aggregate size. They are more in MC than in HC. The researches show that total ultimate deformations of tension for all tested concretes increase with increasing the strength. Between these characteristics there is correlation relationship that is approximated by equation:

$$\varepsilon_{ct1} = A f_{ct}^{\alpha} , \qquad (5)$$

where f_{ct} in MPa; *A* and α – parameters of this concrete type (for HC – A = $5{,}35 \cdot 10^{-5}$, α= 1/2; MC – A = $6{,}41 \cdot 10^{-5}$, α= 1/3; LC – A = $10{,}91 \cdot 10^{-5}$, α= 1/3).

It should be noted that restrictions $\varepsilon_{ct1}(f_{ct1})$ have private character, as the properties and component type of concrete mixture, storage instructions and age of samples, technological factors, conditions of deformation of standards etc. influence on a valueε_{ct1}.

Inhomogeneous tension brings substantial change in behavior of concrete deformation.

The results of beams' tests show that their deformation, as well as under axial tension, is accompanied by progressive development of destructions in the cracked concrete long before its destruction. From the first stages of load application new micro-cracks are formed in a sample with force action with the simultaneous opening of solidification shrinkage cracks. With light load the speed of destruction development is low, it dies out, and does not result momentary drop of bearing strength of concrete beams. With load increasing the micro-cracks gradually develop and, closing up, they form macro-cracks, their accelerated development results to crushing of sample. It is set experimentally, that the width of micro-crack opening in concrete beams before crushing is a considerable size, i.e. the beams in the tension side work with cracks. Local deformations along the length of beam in the area of simple bending, both on the most tensile face and on the depth of section, are very different. The devices, the micro-cracks are formed in their base, show sharp increasing of local deformations of tension («pseudoplastic» deformations by O. Berg or «separated» deformations by M. Holmyanskiy). In nearby cross-sections increasing of deformations discontinued or decreased. The deformations of shortening in the tension side to the moment of destruction sometimes were $\varepsilon_{ct} = (4\div6)10^{-5}$.

According to the size and speed of local deformation it is possible to judge what will happen with destruction of beam cross section. The cross-sections, where were a few critical cracks forming along the length of zone of simple bending, but, as a rule, only in one of them the crack was, resulting in destruction in future, progressing under the load $(0{,}7–0{,}9)M / M_u$. Critical cracks divided the zone of simple bending into separate blocks.

The character of local deformations changing is different in the compressive side of the beam. The non-uniformity of local deformations changing along the length of the beam in the compressive side is inconspicuous. It is conditioned, mainly, by of concrete structure heterogeneity. However, under the high load levels there is some growth acceleration of local deformations of shortening, located in those cross-sections, where the critical cracks (moving of zero axis) develop in the tension side.

The development of micro-cracks in concrete with the deformation gradient takes place otherwise, than under the uniaxial state of stress. The cracked concrete can not be examined as a solid uniform body, following only the laws of elastic-plastic deforming [2, 4, 5, 7]. This material is heterogeneous with broken soundness of coating surface and works in the «cracked» state. The theory of equilibrium cracks (Hriffits, Ervin), elaborated for the materials such as concrete, confirms this idea.

The analysis of concrete centerline deformations shows that normal sections are plane with the growth of load, i.e. the tests confirm acceptability Bernoulli hypothesis for normal sections only the case if the deformations in the tension side are measured (averaged) on a sufficiently large base. This behavior is observed under all levels of the fraction loading.

Micro-crack formation in the cracked concrete causes some displacement of neutral layer with the load growth toward the compressive face. Its position is determined according to the normal total centerline deformations of the compressed and tension fibers of beams and specified according to the completely recoverable deformations of fibers, came up under unloading:

$$\xi_u = \frac{x}{h} = \frac{\varepsilon_c}{\varepsilon_{ct2} + \varepsilon_c} \quad . \tag{6}$$

For comparison the relative height of the compressive side ξ_u is also calculated through beam deflections and compressive deformations ε_c, their variableness is small:

$$\xi_u = \frac{\varepsilon_c l^2}{8hf^{\max}} . \tag{7}$$

The differences in averaging values ξ_u, determined by different methods (6) and (7), under other equal terms, are scant; and the value ξ_u is 0,446 for a fine-grained concrete; 0,407 – ceramsite concrete; 0,425 – heavy-weight concrete.

The test results of geometrically similar beams of experimental concrete types of the wide strength range show that total ultimate deformations of fibre tension under the bend ε_{ct2} are higher, than under the axial tension ε_{ct1} of analogical equally efficient concrete, and substantially depend on the gradient of ultimate deformations *grad(*ε_{ct1}*/h)* (depth of beams). With increasing of beam depth the deformation ε_{ct2} decreases and under high values (h > 40 cm) tends to ε_{ct1}. For the beams of low depth (h < 2 cm) the ultimate deformation ε_{ct2}, more than 4 times ε_{ct1} (fig. 3a, б).

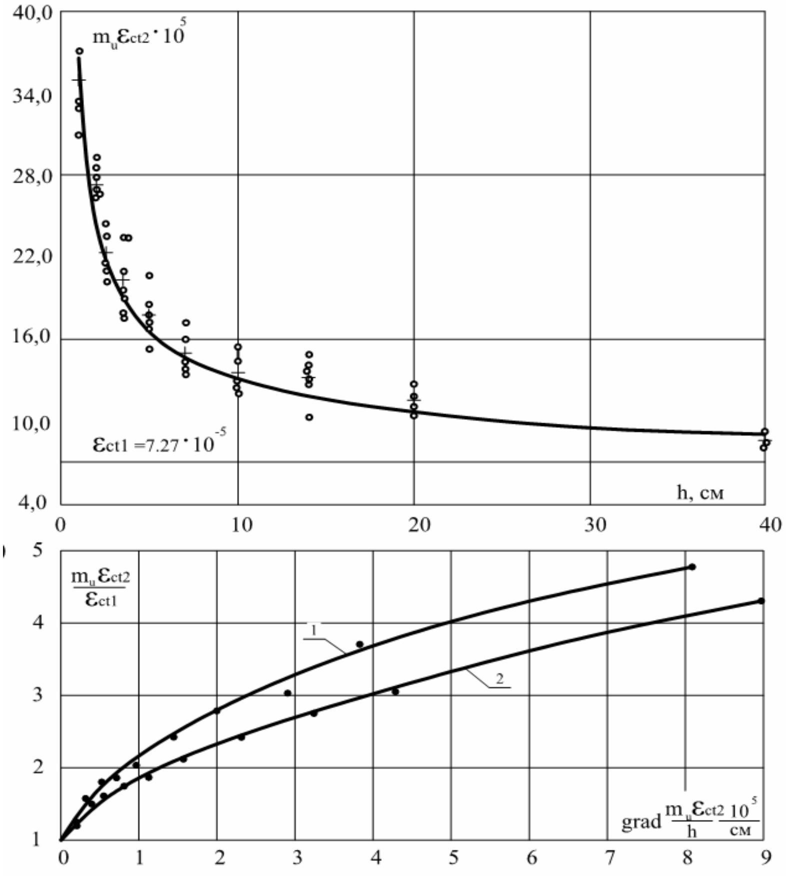

Figure 3 –The relation of ultimate deformations of beams fibre tension under bending ε_{ct2} from the depth of beams (a) and the gradient of ultimate deformations along the cross-section (b).
1, 2 – MC, accordingly, the strength f_{cm} = 23,7; 41,8 MPa.

The increment of ultimate deformations of border fibers of the tension side under bending (considering deformations ε_{yc}) above ultimate deformation under axial tension is related to the depth of beam by hyperbolic dependence:

$$(m_u\varepsilon_{ct2} - \varepsilon_{ct1} + \Delta\varepsilon_{yc})h^{\alpha} = S = const . \tag{8}$$

The value of parameter S is determined by the test of standard beam, where $\varepsilon_{yc} = 0$. The depth of standard beam corresponds to the diameter of cylinder (12 cm), according to it ε_{ct1} is determined. In the relation (8) the coefficient $m_u = \xi_u / (1-\xi_u)$ takes into account the limit displacement of neutral axis of beams, and the value $\Delta\varepsilon_a$ – the difference of shrinking deformations of upperbound tension of cylinder and beam of this depth, caused by the inequality of concrete shrinkage. This type of function (8) also allows to estimate the influence of shrinkage-related prestress in the samples of different sizes on ultimate tensility of concretes. For massive elements (h > 40 cm) and concretes with the enlarge cement content the shrinking deformations ε_{yc} tend toε_{ct1}.

It is important to notice that this relation of ultimate deformations under heterogeneous tension from the gradient of deformations also belongs to elastic deformations, got during beam unloading that directly connect with the tensions. This circumstance testifies that the actual tensely-deformed state of the tension side of concrete beam does not rise to calculation, got according to the complete diagram of axial tension taking into account a descending branch. Connection between tensions and deformations under heterogeneous tension is different comparing to that under axial tension. At the same time, the diagrams «load – deformations» of the bending elements are well described by equation (9), analogical to the connection, «load – deformations» (3) of axial tension:

$$\frac{M}{M_u} = 1 - \left(1 - \frac{\varepsilon_{ct}}{\varepsilon_{ct2}}\right)^{1/n}, \qquad (9)$$

where $n = f_{ct} / \varepsilon_{ct1} E_{cto} = \nu_{ctu}$.

The elastic ratio under bending is approximately equal to the elastic ratio under axial tension. Equation (9) estimates the deformations of beams for all levels of loading. Taking into account the succession of hypothesis of plane cross-sections, it allows to define the connection between an external moment and deformation of any fibre of the tension side (fig. 4).

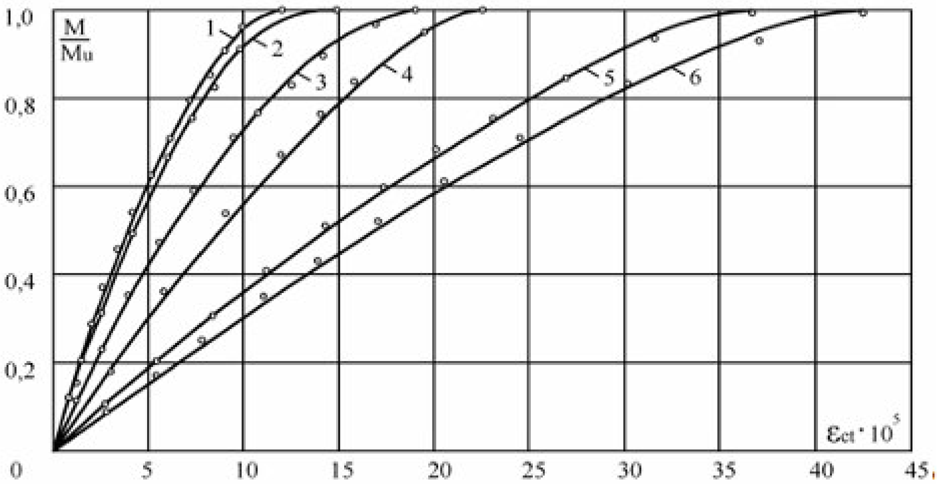

Figure 4 – Diagrams «load – deformations» of the tension fibers of the beams of different depth

1, 2, 3, 4, 5, 6 – accordingly, the beams from MC with the strength f_{cm} = 23,7 MPa
Depth of sections 40; 20; 7; 5; 1,9; 1,0 sm

Conclusions. Experimental data show that deformation of concrete beams is accompanied by development of micro- and macro-cracks in the cracked concrete (beams work in the «cracked state»). Shrinkable and power destructions, creating in the concrete, cause the very difficult field of deformations in the tension side of beam and allow to suppose that the mechanism of beam's fiber deformation is not identical to the axial ladening (the hypothesis of uniaxiality fiber works is not acceptable). Total ultimate deformations of the concrete (including elastic ones) under non-homogeneous tension are not constant. They depend on the gradient of ultimate deformations and are changed in a wide range $\varepsilon_{ct2} / \varepsilon_{ct1} = 1 \div 5$. Such conformities give reason to take equivalent cross-section with the depth of crack development $t = h\,(1–2\xi_u)$ in the tension side, the three-cornered form of deformation epure and border ε_{ct2}, depending on the depth of cross-section of beams as calculated deformed cross-sectional beams in the ultimate state. Bernoulli hypothesis is acceptable to this cross-section and the position of neutral axis is known.

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