UDC 621.833.65

Strilets O.R., *PhD, Associate Professor*
ORCID 0000-0003-3834-7176   oleh.strilets@gmail.com
Strilets V.M., *PhD, Professor*
ORCID 0000-0003-2098-2315   v.m.strilets@gmail.com
Stepaniuk A.A., *PhD, assistant*
a.a.stepaniuk@nuwm.edu.ua
National University of Water and Environmental Engineering, Rivne

TENSION AND DEFORMATIONS OF STAR-SHAPED SPRING VERTICES WITH STRANGULATED ENDS OF THE FLEXIBLE COUPLING

The article examines the structure of overload flexible coupling that contains internal and external hubs, connected by a star-shaped spring with the circular vertices inserted in grooves on the outer surface of internal hub and inner surface of the external hub. Position of the star-shaped spring is fixed on the inner hub by the wedge, which allows the spring to be made either solid or of separate circular vertices with strangulated ends. Geometric synthesis of star-shaped spring with vertices of circular shape, depending on its size was conducted. It is assumed that star spring vertices are the double-hinge arches of circular form. Calculations of analytical solutions using the methods of structural mechanics were done. Analytical expressions proved workability of the star-shaped spring of flexible overload coupling during the torque transmission.

Key words: *flexible overload coupling, star-shaped spring, circular vertices, statics calculation, deformation.*

Стрілець О.Р., *к.т.н., доцент*
Стрілець В.М., *к.т.н., професор*
Степанюк А.А., *к.т.н., асистент*
Національний університет водного господарства та природокористування, м. Рівне

НАПРУЖЕННЯ І ДЕФОРМАЦІЇ ВИСТУПУ ЗІРКОПОДІБНОЇ ПРУЖИНИ З ЗАЩЕМЛЕНИМИ КІНЦЯМИ ПРУЖНОЇ МУФТИ

Описана будова і принцип роботи запобіжної муфти, що містить внутрішню і зовнішню півмуфти, з'єднані між собою зіркоподібною пружиною з виступами круглої форми, встановленою в заглибини на зовнішній поверхні внутрішньої та внутрішній поверхні зовнішньої півмуфти. Зіркоподібна пружина може бути виконана суцільною або складена з окремих виступів з защемленими кінцями. Проведено геометричний синтез зірковоподібної пружини з виступами кругової форми, в залежності від необхідних розмірів. Прийнято, що виступ зіркоподібної пружини являє собою арку з защемленими кінцями і для неї, методами будівельної механіки, проведені розрахунки. Отримані аналітичні вирази дозволяють робити висновки про роботоздатність зірковоподібної пружини пружної муфти при передачі нею обертального моменту.

Ключові слова: *муфта запобіжна пружна, зіркоподібна пружина, виступ кругової форми, статичний розрахунок, деформація.*

Introduction. Drives of hoisting, transport, building, road, land reclamation machines and others often include different couplings, which are quite responsible mechanical devices that can determine the reliability and durability of all equipment. The main purposes of couplings are the shafts connection and torque transmission. Besides, elastic safety couplings perform such responsible functions as compensation of the harmful effects of shaft ends geometric axes offset, resulting from inaccurate production, installation or design features and operating conditions of drives; amortization of vibrations, jolts and shocks arising during operation; protection of machine elements from overload; facilitation of the machine start-up. A variety of couplings maintenance functions contributed to the development of a large number of constructions. But in all cases the work of couplings has still many shortcomings that need to be eliminated due to their improvement.

Review of recent sources of research and publications. The modern technical literature, for example [1-4] describes the safety cam couplings with shear pin or others that through the direct contact of their half-sleeves transmit torque toughly, and it negatively affects the work of drives elements and machine in general. The analysis of the shortcomings of known coupling designs enabled to develop the new design of safety elastic couplings with star-shaped springs at the level of patents [5-8].

Specifing problems unsolved earlier. Theoretical and experimental research for new safety elastic couplings has not yet been held. Part of such research has been made for couplings with a parabolic [9] and a circular vertices [10] and with hinged fastening of ends.

Problem statement. To conduct a preliminary analysis of power parameters of safety elastic couplings including star-shaped springs with strangulated ends of vertices.

Main material and results. This article deals with static calculations of star-shaped springs of safety elastic couplings [5-9] in operation, i.e. during the transfer of sustainable torque. One of these couplings is shown on Fig. 1.
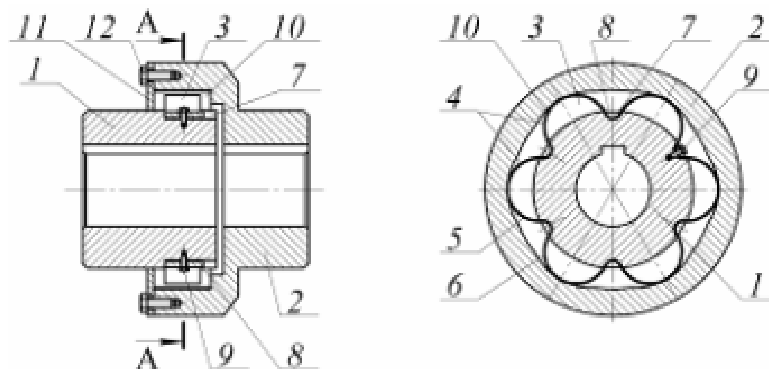

Figure 1 – Scheme of safety elastic couplings with star-shaped spring with strangulated ends

Safety coupling with the star-shaped spring has internal 1 and external 2 half-sleeves, connected by a star-shaped spring 3 with vertices 4. Side surfaces 5 and 6 of the vertices 4 made convex apart from the axis of symmetry, and their internal ends set in recesses 7 on internal half-sleeve 1 and fixed by wedges 8 with some tension and screws 9. The tops of the vertices 4 are located in the recesses 10 of external half-sleeve 2, moreover, those recesses 10 are made with radius greater than the tops rounding radius.

Safety coupling with the star-shaped spring assembles in the following order. First, internal ends of vertices 4 set in recesses 7 on internal half-sleeve 1 and fixed with wedges 8 and screws 9. Next, assembled internal half-sleeve 1 with star-shaped spring 3 set in the external half-sleeve 2 so that the vertices 4 are in contact with the recesses 10. Safety coupling with the star-shaped spring with strangulated ends of vertices is ready.

Safety coupling with composed star-shaped spring works as follows. When rotating internal sleeve 1 transmits torque through star-shaped spring 3 to the external sleeve 2, the overload mode star-shaped spring 3 deforms, decreasing in its diameter along its outer contour. The surfaces 5 and 6 of vertices 4 bent in direction of its convexity, providing deformation of star-shaped spring 3 within the limits of its elastic deformation until they are out of the recesses 10 of external sleeve 2. Rounding radius of recesses 10 are larger than radius of peaks 4 so vertices 4 slide over the cylindrical surface of the outer external 2 until torque is reduced to the nominal value. Star-shaped spring 3 can be solid or composed of separate vertices.

Fig. 2 shows a model of elastic couplings with star-shaped spring, created in the "KOMPAS - 3D", assembled (Fig. 2, a) and disassembled (Fig. 2, b).
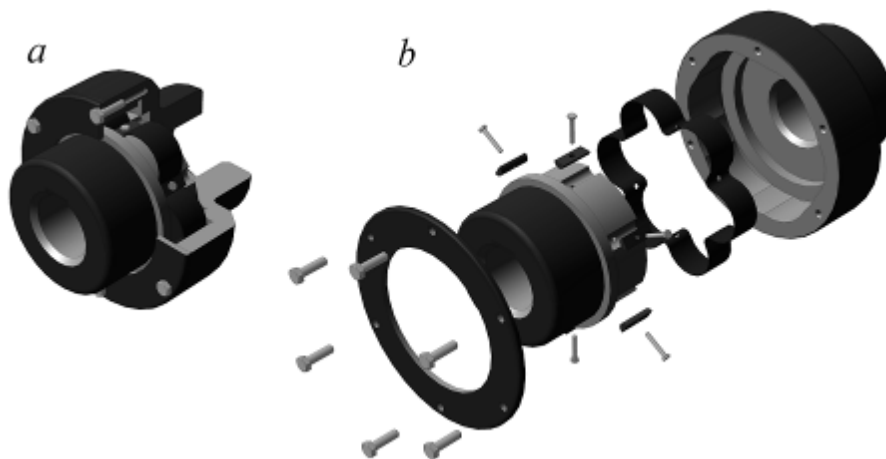

Figure 2 – Models of elastic safety coupling: *a* – assembled; *b* – disassembled

Fig. 3 shows the scheme of the dimensions for the geometric synthesis of star-shaped springs, where: D - inner diameter of external sleeve; D_1 - outer diameter of the internal sleeve; R - radius circular performance; l - pitch (span) of circular vertices; h - the height of segment; r - radius of recesses in the internal sleeve; R_1 - radius of recesses in the external sleeve; δ - the height of recesses in the external sleeve; z - number of vertices.
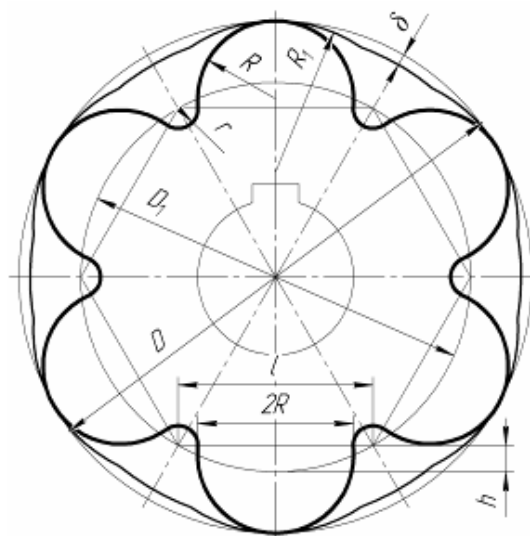

**Figure 3 – Scheme for geometric synthesis
of star-shaped springs with circular vertices**

The relationship between these dimensions can be described by

$$l = D_1 \, sin\frac{180°}{z} \, ; \tag{1}$$

$$h = R - \sqrt{R^2 - \frac{l^2}{4}} \, ; \tag{2}$$

$$r = \frac{l - 2R}{2} \, ; \quad R_1 = (1,8 - 2,2)R \, ; \tag{3}$$

$$D = D_1 + R - h - \delta \, . \tag{4}$$

The length of the of the workpiece for manufacturing of the solid star-shaped springs is

$$L_{заг} = \pi(R + r)z \, . \tag{5}$$

When designing couplings it is recommended to take following parameters construction:

$$D_1 \leq 1,75 \, d,$$

where d – shaft diameter;
$R = (0,4\ldots0,45) \, l$;
$z = 6$.

Set problem is solved with the following assumptions: vertices of star-shaped spring deform the same along its axis of symmetry; acting load in vertices lies on this line of symmetry in the plane of star-shaped spring that is perpendicular to rotation axis of coupling, and equals

$$F = \frac{2T_p}{Dzf} \, , \tag{6}$$

where T_p – estimated torque, transmitted by coupling;
D – outer diameter of star-shaped spring;
z – number of star-shaped spring vertices;
f – coefficient of friction.

The vertices deform similarly. All calculations are reduced to calculation of circular arch with strangulated ends using the method of Mohr [11].

Scheme for calculations of star-shaped spring vertex is shown in Fig. 4.

Figure 4 – Scheme for calculations of star-shaped spring vertex

According to [11] this arch with strangulated ends (Fig. 3) is three times statically undefined system. The most suitable equivalent and main systems are shown in Fig. 5.
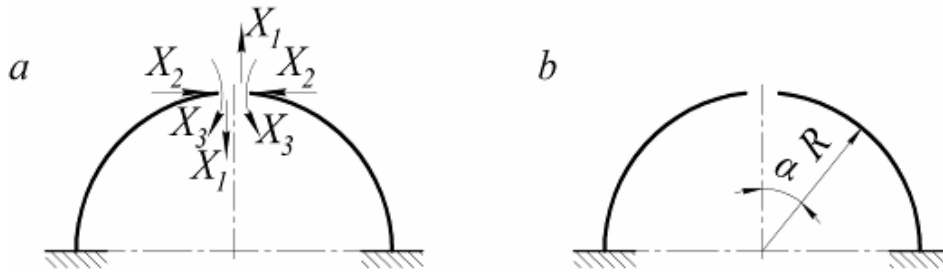
Figure 5 - Equivalent (*a*) and main (*b*) systems of star-shaped spring vertex

Superfluous bonds are taken to unidentify X_1. X_2, X_3. Arch is deformed identically to star-shaped spring vertex. Unidentified forces in the equivalent system are determined from zero equality condition for displacements in directions x_1. x_2, x_3.

For this th following the canonical equation of the forces method is done:

$$\delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 + \delta_{1F} = 0;$$

$$\delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 + \delta_{2F} = 0;$$ (7)

$$\delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 + \delta_{3F} = 0,$$

where δ_{11}, δ_{22}, δ_{33} – displacement in the directions x_1. x_2, x_3, caused by forces $X_1 = 1$. $X_2 = 1$, $X_3 = 1$, respectively;

δ_{12}, δ_{13}– displacement in the direction of the X_1, caused by $X_2 = 1$ and $X_3 = 1$, respectively;

δ_{21}, δ_{23} – displacement in the direction of the X_2, caused by $X_1 = 1$ and $X_3 = 1$, respectively;

δ_{31}, δ_{32} – displacement in the direction of the X_3, caused by $X_1 = 1$ and $X_2 = 1$, respectively;

δ_{1F}, δ_{2F}, δ_{3F} – displacement in the direction of X_1. X_2, X_3, respectively, caused by the external load F.

Main system can be derived from the equivalent system after its liberation from external load F and unidentified forces X_1. X_2, X_3, that replace redundant links. Main system is shown in Fig. 4, *b*, where R is the circle radius.

All this displacement are defined using the Mohr integrals

$$\delta_{11} = \sum\int_0^S \frac{M_1^2 ds}{EJ}; \qquad \delta_{22} = \sum\int_0^S \frac{M_2^2 ds}{EJ}; \qquad \delta_{33} = \sum\int_0^S \frac{M_3^2 ds}{EJ};$$

$$\delta_{12} = \sum\int_0^S \frac{M_1 M_2 ds}{EJ}; \qquad \delta_{13} = \sum\int_0^S \frac{M_1 M_3 ds}{EJ}; \qquad \delta_{21} = \sum\int_0^S \frac{M_2 M_1 ds}{EJ};$$

$$\delta_{23} = \sum\int_0^S \frac{M_2 M_3 ds}{EJ}; \qquad \delta_{31} = \sum\int_0^S \frac{M_3 M_1 ds}{EJ}; \qquad \delta_{32} = \sum\int_0^S \frac{M_3 M_2 ds}{EJ};$$ (8)

$$\delta_{1F} = \sum\int_0^S \frac{M_1 M_F ds}{EJ}; \qquad \delta_{2F} = \sum\int_0^S \frac{M_2 M_F ds}{EJ}; \qquad \delta_{3F} = \sum\int_0^S \frac{M_3 M_F ds}{EJ},$$

where E – elasticity modulus of the first kind for spring material;

J – axial inertia moment of the section (see. Fig. 4), equals $J = b\Delta^3/12$;

M_1, M_2, M_3 and M_F – bending moments from the forces X_1. X_2, X_3 and F, respectively: $M_1 = \pm X_1 R \sin\alpha$; $M_2 = \pm X_2 R(1 - \cos\alpha)$; $M_2 = X_3$; $M_F = F_1 R \sin\alpha$, where $F_1 = F/2$.
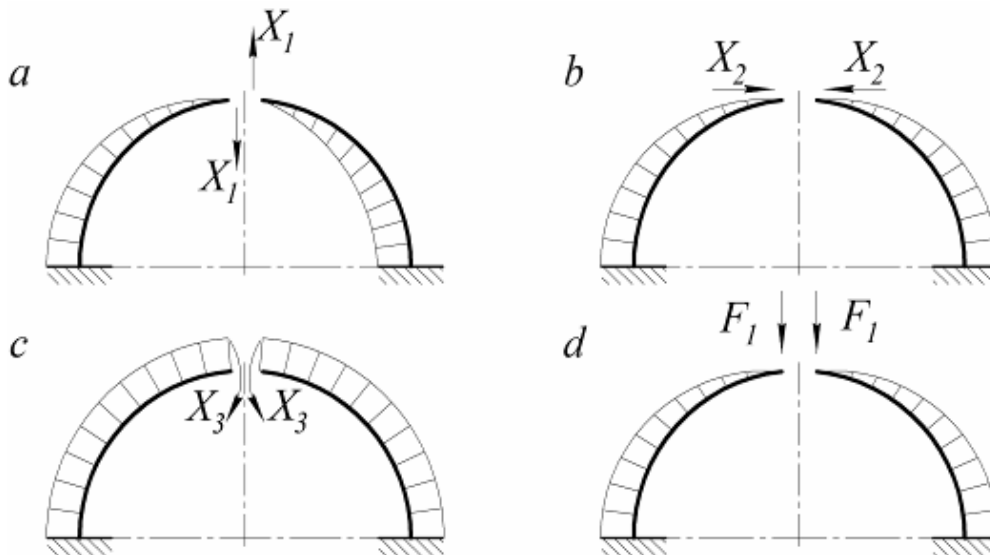
Fig. 6 shows the constructed diagrams of bending moments

Figure 6 – Diagrams of bending moments from forces X_1. X_2, X_3 and F

From the analysis of integrand values of moments from expressions (7) and diagrams (Fig. 6) we have that $\delta_{11} = 0$, $\delta_{12} = \delta_{21} = 0$, $\delta_{13} = \delta_{31} = 0$ and $\delta_{1F} = 0$. Thus, the system of canonical equations (7) is reduced to

$$\begin{aligned} \delta_{22}X_2 + \delta_{23}X_3 + \delta_{2F} = 0 \\ \delta_{32}X_2 + \delta_{33}X_3 + \delta_{3F} = 0 \end{aligned} . \tag{9}$$

Using expressions (3) and bending moment diagrams (see Fig. 6), assuming that limit integration for curved sections is $s = D\,d\,\alpha$, and for angle α that varies from 0 to $\pi/2$, it is got:

$$\delta_{22} = \frac{R^3}{EJ}\int_0^{\pi/2}(1-\cos\alpha)^2 d\alpha = \frac{(3\pi - 8)R^3}{4EJ} ;$$

$$\delta_{23} = \delta_{32} = \frac{R^2}{EJ}\int_0^{\pi/2}(1-\cos\alpha)d\alpha = \frac{(\pi - 2)R^2}{2EJ} ;$$

$$\delta_{33} = \frac{R}{EJ}\int_0^{\pi/2} d\alpha = \frac{\pi R}{2EJ} ; \tag{10}$$

$$\delta_{2F} = \frac{FR^3}{EJ}\int_0^{\pi/2}(1-\cos\alpha)\sin\alpha\, d\alpha = \frac{FR^3}{2EJ} ;$$

$$\delta_{3F} = \frac{FR^2}{EJ}\int_0^{\pi/2}\sin\alpha\, d\alpha = \frac{FR^2}{EJ} .$$

To get a system of equations (4) relatively to unidentified X_2 and X_3 it is used:

$$X_2 = \frac{\delta_{23}(\delta_{22}\delta_{3F} - \delta_{32}\delta_{2F})}{\delta_{22}(\delta_{22}\delta_{33} - \delta_{23}^2)} + \frac{\delta_{2F}}{\delta_{22}} ; \qquad X_3 = \frac{\delta_{32}\delta_{2F} - \delta_{22}\delta_{3F}}{\delta_{22}\delta_{33} - \delta_{23}^2} . \tag{11}$$

The final expression for determination of resultant bending moment in the vertex of circular shape with the strangulated ends it is

$$M_\Sigma = M_2 X_2 + M_3 X_3 + M_{2F} + M_{3F} . \tag{12}$$

Fig. 7 shows the diagram of resultant bending moment in circular vertex with strangulated ends
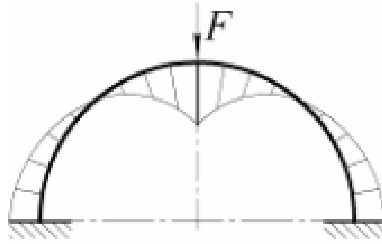
**Figure 7 – The resultant diagram of bending moment
in circular vertex with strangulated ends**

When $\alpha = 0$, it is the maximum bending moment on the axis of symmetry of circular vertex.

To determine the displacement δ it is used Mohr method and the basic system (see. Fig. 5, b). In the direction of displacement δ it is exerted power unit ($X = 1$) and from it, bending moment diagram $M_x = R / 2$ is done,
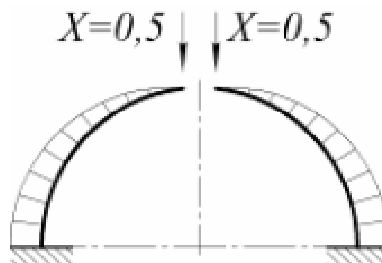

Figure 8 – Diagram of bending moment M_4 from the force X

Then use the moments' value M_Σ and M_4, and obtain:

$$\delta = \frac{R}{2EJ}(M_2 X_2 + M_3 X_3 + M_{2F} + M_{3F}). \tag{13}$$

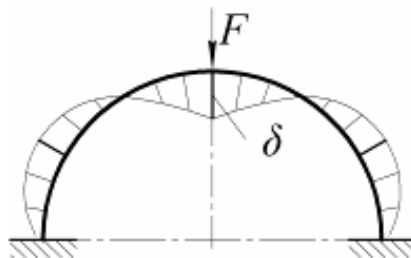The nature of the deformation of of vertex with strangulated ends of star-shaped spring is shown in Fig. 9.

**Figure 9 – Diagram of deformation of vertex
with strangulated ends of star-shaped spring**

Conclusions. Analytical dependence (13) between deformation and the load in star-shaped spring with the circular vertices with strangulated ends may be used in designing of new safety elastic couplings.

Expression (12) allows to determine the maximum value of bending moment for dangerous intersection and to find a tension for it by formulas.

The proposed method of theoretical research of dependence between the load and deformation in star-shaped springs with circular vertices with strangulated ends can be used for springs with different number of vertices and is the basis for further safety elastic couplings studies.

References

1. *Малащенко В.О. Муфти приводів. Конструкції та приклади розрахунків / В.О. Малащенко. – Львів : Видавництво НУ «Львівська політехніка», 2006. – 196 с.*
   Malashchenko V.O. Mufty pryvodiv. Konstruktsiyi ta pryklady rozrakhunkiv / V.O. Malashchenko. – L'viv : Vydavnytstvo NU «L'vivs'ka politekhnika», 2006. – 196 s.
   ISBN 966-553-497-1

2. *Bhandary V.B. Design of Machine Elements. New Delhi : Tata McGrew-Hill Education Private Ltd., 2010 – 934 p.*
   ISBN 007-068-179-1

3. *Ganesh Babu K., Srithar K. Design of Machine Elements. 2nd edition. New Delhi : Tata McGrew-Hill Education, 2010 – 544 p.*
   ISBN 007-067-284-9

4. *Smith E. H. Mechanical Engineer's Reference Book. 12*th *edition. Oxford: Reed Educational and Professional Publishing Ltd, 1994 – 1190 p.*
   ISBN 148-310-257-2

5. *Пат. 90494 Україна, МПК F16 D3/00. Муфта запобіжна пружна / В. А. Федорук, В. М. Стрілець, О. Р. Стрілець, А. В. Цимбалюк. – опубл. 26.05.2014, Бюл. №10.*
   Pat. 90494 Ukrayina, MPK(2014.01) F16 D3/00. Mufta zapobizhna pruzhna / V. A. Fedoruk, V. M. Strilets', O. R. Strilets', A. V. Tsymbalyuk. – opubl. 26.05.2014, Byul. #10.

6. *Пат. 98075 Україна, МПК F16 D3/52. Муфта запобіжна пружна / В. М. Стрілець, О. Р. Стрілець, М. М. Козяр, В. А. Федорук. – опубл. 10.04.2015, Бюл. №7.*
   Pat. 98075 Ukrayina, MPK F16 D3/52. Mufta zapobizhna pruzhna / V. M. Strilets, O. R. Strilets, M. M. Kozyar, V. A. Fedoruk. – opubl. 10.04.2015, Byul. #7.

7. *Пат. 98076 Україна, МПК F16 D3/52. Муфта запобіжна з складеною зіркоподібною пружиною / О. Р. Стрілець, М. М. Козяр, В. А. Федорук, В. М. Стрілець. – опубл. 10.04.2015, Бюл. №7.*
   Pat. 98076 Ukrayina, MPK F16 D3/52. Mufta zapobizhna z skladenoyu zirkopodibnoyu pruzhynoyu / O. R. Strilets', M. M. Kozyar, V. A. Fedoruk, V. M. Strilets'. – opubl. 10.04.2015, Byul. #7.

8. *Пат. 107981 Україна, МПК(2006.01) F16 D3/52. Муфта запобіжна пружна / О. Р. Стрілець, В. А. Федорук, В. М. Стрілець, О. В. Лисанець. – опубл. 24.06.2016, Бюл. №12.*
   Pat. 107981 Ukrayina, MPK(2006.01) F16 D3/52. Mufta zapobizhna pruzhna / O. R. Strilets', V. A. Fedoruk, V. M. Strilets', O. V. Lysanets'. – opubl. 24.06.2016, Byul. #12.

9. *Стрілець В. М. Обґрунтування кінематико-силових параметрів зіркоподібної пружини запобіжної пружної муфти / В. М. Стрілець, О. В. Лисанець, О. Р. Стрілець // Збірник наукових праць. Серія: Галузеве машинобудування, будівництво. – П. : Полт.НТУ, 2015. – Вип. 3(45). – С. 40 – 46.*
   Strilets V. M. Obgruntuvannya kinematyko-sylovykh parametriv zirkopodibnoyi pruzhyny zapobizhnoyi pruzhnoyi mufty / V. M. Strilets, O.V. Lysanets, O. R. Strilets // Zbirnyk naukovykh prats'. Seriya: Haluzeve mashynobuduvannya, budivnytstvo. – P. : Polt.NTU, 2015. – Vyp. 3(45). – S. 40 – 46.
   http://znp.pntu.edu.ua/uk/3_45_2015

10. *Стрілець О. Р. Силові параметри пружної запобіжної муфти з зіркоподібною пружиною з круговими виступами / О. Р. Стрілець, В. М. Стрілець, А. А. Степанюк // Вісник національного університету водного господарства та природокористування. Технічні науки. – Рівне: НУВГП, 2016. – Вип. 2. – С. 120–131.*
    Strilets O. R. Sylovi parametry pruzhnoyi zapobizhnoyi mufty z zirkopodibnoyu pruzhynoyu z kruhovymy vystupamy / O. R. Strilets, V. M. Strilets, A. A. Stepanyuk // Visnyk natsio nal'noho universytetu vodnoho hospodarstva ta pryrodokorystuvannya. Tekhnichni nauky. – Rivne: NUVHP, 2016. – Vyp. 2. – S. 120–131.

11. *Поліщук М. П. Будівельна механіка: навчальний посібник / М. П. Поліщук. – Рівне: НУВГП, 2004. – 237с.*
    Polishchuk M. P. Budivel'na mekhanika: navchal'nyy posibnyk / M. P. Polishchuk. – Rivne: NUVHP, 2004. – 237s.