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Sergii Pichugin*
National University «Yuri Kondratyuk Poltava Polytechnic»
https://orcid.org/0000-0001-8505-2130

Development of the statistical approach to the calculation of building structure

Abstract. The current method of calculating building structures by limit states (previously by allowable stresses) is essentially semi-probabilistic. Meanwhile, in parallel with this dual method, for many years, fully probabilistic (statistical) approaches to the calculations have been developed. The aim of the article is to consistently review the approaches developed over the course of ninety years to the implementation of general probabilistic methods for calculating building structures, starting from the 1930s to the present. It is emphasized that domestic specialists are actively developing probabilistic methods, the results of which are implemented in regulatory documents. The prospects of the results of this scientific direction in creating a new generation of building design standards are noted.

Keywords: probabilistic methods, statistical methods, design codes, safety factor, limit states

*Corresponding author E-mail: pichugin.sf@gmail.com

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Introduction

The current method of calculating building structures by limit states (previously by allowable stresses) is essentially semi-probabilistic. The main parameters of the method – load values, strength characteristics of materials, dimensions of elements, etc. – are of a variable statistical nature. They are studied and described by probabilistic methods, on the basis of which deterministic values of design parameters are justified. Meanwhile, in parallel with this dual method, for many years, fully probabilistic (statistical) approaches to the calculations of building structures have been developed. It so happened that for a long time they could not be implemented into the methodology of building design, despite the promising results of this scientific direction.

Review of research sources and publications

The beginning of the development of probabilistic (statistical) calculation of building structures can be considered the end of the 1930s, when a real scientific offensive on the safety factor, the basis of the method of allowable stresses, was carried out by domestic and foreign researchers [1-3]. The probabilistic nature of the safety factor, presented in the canonical form of the product of factors, was revealed, and numerical values of the guarantee of the non-destructibility of the structure were obtained [4,5]. The 1950s-1960s were marked by unsuccessful attempts by supporters of probabilistic methods to introduce the concept of strength reserve and safety characteristics into design practice [6,7]. Since the 70s of the last century, domestic specialists have been actively developing probabilistic methods [8-11], their results have been partially implemented in regulatory documents of Ukraine [12]. In recent years, publications have appeared with proposals for further implementation of probabilistic approaches in the calculations of building structures [13,14].

Definition of unsolved aspects of the problem

It should be emphasized that due to the fact that the current building design standards use fixed parameters, the selection of which is based on a probabilistic basis, over the many years of operation of the allowable stress method, and later the limit state method, the main attention of the developers of the standards was paid to the accumulation and statistical processing of data on individual components of the calculation methodology. As a result, the achievements of the developers of general probabilistic variants of the calculation methodology for building structures, which contain valuable scientific results that are still relevant today, were ignored.

Problem statement

The goal and objectives of the study are a consistent review of approaches developed over the past ninety years to the implementation of general probabilistic methods for calculating building structures.

Basic material and results

It is known that the basic principle of engineering calculation is the condition of non-destructiveness, according to which the greatest force acting on the structure during its service life S_{act}, must be less than or, at the very least, equal to the smallest possible ultimate resistance of the structure material R_{mat} during this time. The condition of non-destructiveness can be written in expanded form in conjunction with the method of allowable stresses:

$$\max S_{act} = nS \le cR = \min R_{mat} \qquad (1)$$

where n is the safety factor relative to the calculated forces (stresses); c is the transition factor from the actual material stress to the standard one.

If we transfer the factor n to the right side of the equation, we obtain the following expression:

$$S \le (c/n)R = kR \qquad (2)$$

W here k is the calculated safety factor for the standard resistance of the material.

In his small but extremely meaningful work "Fundamentals of Statistical Consideration of the Safety Factor of Structures", the main content of which is given in the monograph [5], the classic of metal structures N.S. Strilecki noted that the fulfilment of inequality (1) can be predicted only with a certain probability. The conclusion was substantiated that by following a statistical path, studying and comparing the facts of the operation of a homogeneous group of structures and materials in structures, it is possible to establish the law of the appearance of these factors and extrapolate this law into the future, if there are sufficient grounds for this.

For the first time, the structure of the safety factor (2) was presented in the form of a product of factors, which was called *the canonical structure of the safety factor*. An essential feature of these factors is that they can be considered independent of each other. Each of the coefficients that characterizes any feature of the structure's operation depends on a large number of reasons and circumstances that may occur during the service of the structure, and therefore it can best be described using a statistical method.

N.S. Strilecki rightly showed that the condition of non-destructibility (1) requires the combination of the extreme values of the distribution curves nS and cR. However, since these curves are asymptotic, the exact fulfilment of the specified condition is impossible, because the extreme values of the curves are unknown. Thus, the fulfilment of the condition of non-destructibility is possible only with a certain accuracy. For this purpose, it is necessary to conditionally cut off the specified curves at a certain point and connect the cut-off curves. The measure of the accuracy of such a connection is, obviously, the rejected areas of the curves at the actual point of intersection or the product of these areas. By rejecting them, we can take them for practical zero and by connecting the curves, we can state that the condition of non-destructibility is fulfilled and our structures are practically non-destructible. Thus, the product of the rejected areas $\omega_1 \cdot \omega_2$ can be considered as a measure of the inaccuracy of the statement that the structure is non-destructible, and the value Γ can be considered as a measure of the accuracy of the statement that the structure is indestructible.

$$\Gamma = 1 - \omega_1 \cdot \omega_2 \qquad (3)$$

Therefore, this value Γ (3) was called by N.S. Strilecki the value of *the guarantee of the indestructibility of the structure.* He emphasized that the value of the guarantee of the indestructibility is a completely conditional value associated with the fulfillment of requirement (1). An estimate of the approximation of this approach is given in the publication [4].

Back in 1938, N.S. Strilecki first determined the numerical values of the guarantee of the indestructibility, that is, he obtained the first results of the implementation of the statistical method of calculating structures. Steel trusses under a cold reinforced concrete roof were considered, statistical data on snow and wind loads for 35 years (1885...1930) were taken into account. An analysis of trusses made of St3 steel with allowable stress $[\sigma]$ = 1400 kgf/cm^2 and statistical characteristics of the yield strength was carried out: mathematical expectation $\overline{\sigma}$ = 2700 kgf/cm^2 and standard $\hat{\sigma}$ = 148 kgf/cm^2. The areas of the tail parts of the curves were $\omega_1 = 2{,}5 \cdot 10^{-4}$; $\omega_2 = 3 \cdot 10^{-4}$, which gave the value of the guarantee of indestructibility Γ=1 – $8.5 \cdot 10^{-8}$. Therefore, steel trusses calculated according to the 1934 design codes had very high values of the guarantee of non-destructiveness. Hence, if we consider the initial data to be correct, we could conclude that there are objective prerequisites for increasing the allowable stresses for metal structures compared to the 1934 design codes.

At the height of World War II in 1942, these considerations were substantiated by N.S. Strilecky and taken into account in the "Instructions for the Design and Application of Steel Structures in Wartime Conditions (U-28-42)". In them, the allowable stresses of steel structures were increased by 200 kgf/cm^2 and were taken for structures made of St3 steel to be 1600 kgf/cm^2 (12.5% more) while maintaining the mechanical characteristics of steels without changing them (the standardized lowest yield strength for St3 steel was 2200 kgf/cm^2). This was a real triumph of scientific and technical thought when "at the tip of a pen" such a significant increase in the design strength of steel was achieved, especially necessary during the war. These changes reduced the safety factor from 1.58 to 1.36 (under the action of the main loads). Despite the rather small value of the safety factor, which was a record, such an increase in stresses turned out to be possible, as the corresponding analysis and subsequent accident-free operation showed. The value of the guarantee of indestructibility at a safety factor of 1.36 was, according to various estimates, Γ=1-6·10^-7^; 1-3·10^-6^; 1-5·10^-6^. Thus, all values of Γ remained quite close to unity, they turned out to be only millionths less than unity for light-type metal structures. After the end of the war, the increased allowable stress was left unchanged.

In the 1940s, proposals for the introduction of fully statistical design methods appeared. The apologist for this approach was A.R. Rzhanitsyn, who had been campaigning for the statistical method since 1947 [6]. He recalled that the inaccuracy of the normative calculation for allowable stresses is associated with the following factors:

• the spread of values of the characteristics of building materials, due to the existing technology of their manufacture;

• deviations from the calculated values of the acting loads, determined by natural influences that do not depend on human will (for example, wind load);

• inaccuracies in the geometric dimensions of structures, due to the methods used for manufacturing and assembling structures.

To obtain data on these statistical inaccuracies, a mass experiment is required with the composition of experimental distribution curves. The nature of these curves can also be determined theoretically. Assuming the approximation of the calculation, it is also possible to use statistical characteristics: the average value, the standard, sometimes the asymmetry coefficient.

With this approach, the condition for failure-free operation (non-destructibility) of the structure has the following form:

$$\tilde{Y} = \tilde{R} - \tilde{S} \ge 0 \qquad (4)$$

where $\tilde{R}$ is generalized random bearing capacity of the structure; $\tilde{S}$ is generalized random load on the structure; $\tilde{Y}$ is characteristic, which was introduced by A.R. Rzhanitsyn and called *the strength reserve*.

The mathematical expectation and the standard of the strength reserve are defined as for a linear function:

$$\bar{Y} = \bar{R} - \bar{S}\,; \quad \hat{Y} = \sqrt{\hat{R}^2 + \hat{S}^2} \qquad (5)$$

The parameter equal to $\beta = \bar{Y}/\hat{Y}$ was called *the safety characteristic* (A.R. Rzhanitsyn [6]) or *the safety index* (C.A. Cornell [7]), they establish the probability of failure (Q) and failure-free operation (P) especially simply in the case of a normal distribution of the strength reserve Y:

$$Q(Y < 0) = 0{,}5 - \Phi(\beta), \quad P(Y \ge 0) = 0{,}5 + \Phi(\beta), \qquad (6)$$

where $\Phi(\beta)$ is Laplace function.

A.R. Rzhanitsyn formulated the basic concept of this method as follows: “Having obtained with a certain degree of accuracy the desired statistical set, for example, the distribution of the bearing capacity of a structure, we can stop at such a minimum value of this bearing capacity, which has a certain reasonable, very small probability of its occurrence. This value can be taken as the permissible bearing capacity, according to which the load on this structure should be assigned” [1]. For that time (1940–1950), this approach was perceived as bold and truly revolutionary.

In this case, the safety factor k was related to the safety characteristic by the following formula

$$\beta = (k-1)\Big/\sqrt{V_r^2 k^2 + V_q^2}\;; \qquad (7)$$

where k is the safety factor equal to the ratio of the average expected bearing capacity to the average expected working stresses; V_r, V_q are the coefficients of variation of the bearing capacity and load; β is the safety characteristic, which depends on the probability of failure.

In the design codes, the coefficients of uniformity and load are defined as

$$k_0 = 1 - \gamma V_r\,; \quad k_n = 1 + \gamma V_q\,.;$$

After substituting into formula (7) the expressions of V_r, V_q through the coefficients of uniformity and load, the following equation was obtained for the safety factor:

$$k = \frac{1 + \sqrt{1 - k_0 k_n (2 - k_0)(2 - k_n)}}{k_0 (2 - k_0)} \qquad (8)$$

If we accept $k = k_n / k_0$, then this gives inflated safety factors for the same safety characteristic. A.R. Rzhanitsyn confirmed this with a numerical example of calculating a steel roof span [1].

The preparation and start of using calculations based on limit states (1950s) activated supporters of fully statistical design methods. The same A.R. Rzhanitsyn rightly emphasized the shortcomings of the new method [5]:

• all limit states are considered the same for all types of buildings and all elements;

• incorrect consideration of the joint statistical spread of several quantities included in the calculation formulas, with the adjustment of new results to the results according to the old codes.

As an alternative, a statistical method was proposed with the following main provisions:

• the conditional probability of passing through the limit state is normalized, the parameter of probability is the safety characteristic β, equal to the number of standard deviations from the mean value;

• approximate differentiated values $\beta = 2 - 4{,}5$ are proposed for buildings and structures of different capital and responsibility (apparently for the first time);

• taking the appropriate β, the load coefficient k_n, uniformity coefficient k_0 and the resulting safety factor k are determined by formula (8).

At this time, the authoritative specialist B.I. Belyaev [5] presented a program article devoted to the author's statistical method. In it, he rightly noted that all the quantities included in the calculation of structures (loads, yield strength of steel, geometric characteristics of the cross-section of elements, coefficient for centrally compressed elements) are random. However, he categorically stated that all their deviations from the average values obey the normal distribution law. This statement, which is valid for the yield strength, crane loads, geometric characteristics, random imperfections, was unreasonably extended to snow and wind loads. The developed general method was illustrated by the example of a centrally compressed steel rod with a cross-sectional area *F* and the sum of the acting loads $\sum P$. An auxiliary quantity was introduced that characterizes the degree of use of the bearing capacity of the rod material, similar to the strength reserve according to A.R. Rzhanitsyn:

$$R = \sigma_T - \sigma = \sigma_T - \sum P/F \qquad (9)$$

where σ_T is the yield strength of the steel.

The quantity *R* as a function of random variables is also a random variable with a mean value equal to zero (which cannot be agreed with). Using the linearization operation, which later became widespread, the deviation of the function *R* was calculated with respect to the deviations of its arguments:

$$\Delta R = \sqrt{\left(\frac{\partial R}{\partial \sigma_T}\right)^2 \Delta\sigma_T^2 + \sum\left(\frac{\partial R}{\partial P}\right)^2 \Delta P^2 + \left(\frac{\partial R}{\partial F}\right)^2 \Delta F^2} .$$

The notation $\sigma = k\sigma_T$ is introduced. Then it turns out that

$$\Delta R = \sigma_T - \sigma = \sigma_T(1-k),$$

and the equation for determining the coefficient *k* takes the final form:

$$(1-k)^2 = k^2\left[\frac{\sum \Delta P^2}{(\sum P)^2} + \frac{\Delta F^2}{F^2}\right] + \frac{\Delta\sigma_T^2}{\sigma_T^2}; \qquad (10)$$

Here *k* is the reciprocal of the safety factor, which guarantees that in the elements of a steel structure during its unlimited long-term operation (an unfounded statement) the stresses from the loads will not reach the yield point with the same probability as deviations $\Delta\sigma, \Delta P, \Delta F$ exceeding their maximum possible values accepted in the calculation may appear.

Expressions were obtained for cases of transverse bending, central and eccentric compression (tension), which have a similar form. As a general result, the values of the normative stress for various cases of the action of forces on the structure were proposed. For their justification, the well-known criterion "three sigma" was used, which corresponds to the probability of two-sided excess - 0.00272, one-sided - 0.00136. The obtained differentiated values of the coefficient $k \le 1$ are given (without sufficient justification). Thus, in their proposals B.I. Belyaev conservatively remained within the framework of the allowable stress method, trying to show that his method is more economical than the limit state method.

Later, B.I. Belyaev applied the general approach outlined above to reinforced concrete bending and compression elements, bulky formulas were obtained, and numerical examples performed according to them showed savings of 11 - 22% [5].

The proposals of A.R. Rzhanitsyn and B.I. Belyaev to build a system of reserve factors entirely on the basis of the statistical method did not receive support either in the 1950s or later, although it was recognized as expedient to use mathematical statistics as an important auxiliary tool.

B.I. Belyaev continued the struggle for his statistical method of calculating building structures (1965) [5], considering in detail the following remarks of opponents of the statistical method:

a) a very long work is required to collect initial statistical data for a reliable choice of the laws of distribution of random arguments, especially the “tail parts”;

b) random parameters that determine the operation of the structure are divided into “constant” ones that do not change with time (resistance of materials, geometric parameters, constant loads), and temporary loads that act repeatedly; it is believed that they are so fundamentally different that for their joint consideration it is impossible to accept the laws of probability theory;

c) in the statistical method a single safety characteristic $\beta = 3$ is accepted, although for different distribution laws it corresponds to a different probability of exceedance;

d) the destruction of a structure cannot be a mass event, and the statistical interpretation of its probability loses its meaning; in addition, homogeneous conditions for the operation of a structure are rarely feasible (remarks of V.V. Bolotin).

Objections of B.I. Belyaev to the above remarks.

a) *Laws of distribution of random variables*. A significant part of the calculated variables is normally distributed (resistance of materials, volumetric weights of materials, geometric parameters of the section, random distortions). Temporary loads (atmospheric, temperature, crane) can be considered as statistical sets of maxima described by the theory of distributions of extreme members of the sample. For this purpose, it is proposed to use the indicative law, which is successfully used by meteorologists for wind speeds.

b) *Multiple loading of the structure with temporary loads*. It is proposed to replace them with equivalent single loads with changed average and standard.

c) *The value of the safety characteristic*. It is justified by examples that β can be applied according to the normal law for both indicative and lognormal distributions.

d) *The method of calculating structures is not based on the statistics of structural failures*. The operation of structures made of materials with random properties under the action of random loads is certainly a mass phenomenon. “This phenomenon is repeatedly realized in practically homogeneous conditions (material, load)”.

B.I. Belyaev concluded his publication with an optimistic conclusion: “We can hope that objections to the statistical method of calculating building structures will be removed and this progressive method will find application in design practice” [5]. As we now know, these hopes were not destined to come true.

In 1967, A.R. Rzhanitsyn published a brief review (bibliography of 29 sources) of the development of probabilistic methods for calculating structures. He noted that the current codes for calculating structures do not reflect the random nature of loads fully enough, and he built a bridge to the theory of reliability of construction objects: "The entire set of probabilistic calculations of structures for strength is sometimes called the theory of reliability of buildings and structures, adding here the issue of changes in suitability over time due to corrosion, aging of the material, etc. The theory of reliability in construction has not yet gained much development, but is successfully used to solve problems in mechanical engineering and the operation of various devices."

Later (1973) A.R. Rzhanitsyn continued the development of the probabilistic method and solved the problem of determining the economically justified reliability of a structure [5]. The costs associated with the construction of a structure and its possible damage during a given service life are determined as

$$R = C + VY\,; \qquad (11)$$

where C is the initial cost of construction of the structure; V is the probability of its damage; Y are the losses caused by this damage, which include the cost of restoration and the losses caused by the disruption of the operation process.

Next, the minimum mathematical expectation of these costs is found, the condition of which has the form:

$$\partial R/\partial C = 1 + V\partial Y/\partial C + Y\partial V/\partial C = 0. \qquad (12)$$

Usually, losses Y can be considered independent of the initial cost of the structure, therefore $\partial Y/\partial C = 0$. Then, taking into account $V = 1 - P$ (P is security), we get $\partial C/\partial P = Y$. This condition allows us to choose optimal security for each limit state. The developed methodology was illustrated by a numerical example for a pavement span.

Moving on to the 1990s, we would like to emphasize that with the collapse of the USSR, the new states had the opportunity to move away from the crude Soviet construction codes and develop their own, more adequate regulatory documents. Ukrainian specialists, unlike Russian developers of codes, prepared the fundamental State Construction Codes DBN B.1.2-14-2009 “General Principles for Ensuring Reliability and Constructive Safety of Buildings and Structures” [15], which took into account the acquired international experience and significantly and deeply developed the basic principles of the probabilistic method of structural calculation. For the first time, the possibility of using probabilistic methods to assess the level of structural reliability was allowed in the presence of sufficient statistical information for unique and especially important structures.

The above-described probabilistic model based on random variables (4) is implemented in domestic codes [15] in a slightly different form and with different notations. The design conditions for the realization of failure in a generalized form are written in the form of a *workability function* g, which takes into account the parameters $\tilde{x}_i$ that characterize the random values of the effects $\tilde{F}$, strength characteristics $\tilde{f}$, geometric characteristics $\tilde{a}$, time T and other factors:

$$\mathrm{g}(\tilde{x}_1,...,\tilde{x}_n) < 0. \qquad (13)$$

The main reliability indicator is the *probability of failure* $P_f(T_{ef})$, i.e. the probability that a failure of a given type will occur within a set time

$$\mathrm{P_f}(\mathrm{T_{ef}}) = \mathrm{Prob}\{\mathrm{g}(\tilde{\mathrm{x}}_1,...,\tilde{\mathrm{x}}_\mathrm{n}) < 0/\mathrm{T_{ef}}\}, \qquad (14)$$

where the symbol *Prob{A/T}* defines the probability of event A occurring during time T.

The codes [15] also allow for the characterization of reliability by the *failure range* β, approximately corresponded to the probability P_f by the relation

$$\beta = \Phi^{-1}(1 - P_f)\,, \qquad (15)$$

where $\Phi(z)$ is the function of the normalized probability distribution of workability g.

When using the normal probability distribution in calculations, the function $\Phi(z)$ can be defined as an integral of probabilities

$$\Phi(z) = 0{,}5\pi^{-1}\int_{-\infty}^{z}\exp[-u^2/2]du\,. \qquad (16)$$

Regulatory requirements for reliability are formulated using the calculation condition for the realization of failure (13) and the probability of its implementation (14) in the form

$$P_{f,i}(T_{ef}) = \mathrm{Prob}\{g_i(\tilde{x}_1,...,\tilde{x}_n) < 0/T_{ef}\} \le P_i^{ex}\,, \qquad (17)$$

where g_i is the workability function with respect to the failure of the i-th type; P_i^{ex} is *the appropriate value of the probability of failure of the* i-th type, which is accepted according to the DBN codes [15].

If the failure range β is used, then instead of (17) the condition is accepted

$$\beta \ge \beta_i^{ex}\,, \qquad (18)$$

where *the appropriate value* β_i^{ex} for the i-th type of failure is taken in accordance with the recommendations of DBN [15] or in accordance with the accepted appropriate probability of failure.

A recommendation was given for structures whose failure leads only to economic losses to assign values P_i^{ex} and β_i^{ex} based on the condition of minimizing the total costs of their manufacture, installation, operation and elimination of losses from a possible failure.

For mastering practical probabilistic calculations using the above algorithm, the monograph [6] is recommended.

DBN V.1.2-14-2009 also provides a variant of probabilistic calculation taking into account the time factor. This modern approach was based on the results of research conducted by the scientific school "Reliability of Building Structures", which has been working for many years at the National University " Yuri Kondratyuk Poltava Polytechnic" [12]. A probabilistic model of random processes was successfully applied, in which in expression (3) $\widetilde{S}(t)$ is effort (or stress) in the structure in the form of a random process; $\widetilde{R}(t)$ is a random process or a random value $\widetilde{R}$ of the bearing capacity; $\widetilde{Y}(t)$ is a random process of the structure strength reserve. Under such conditions, the failure of the structure is interpreted as the emission of a random effort $\widetilde{S}(t)$ for a random level of the bearing capacity $\widetilde{R}(t)$ or as the emission of a random process $\widetilde{Y}(t)$ into the negative region.

If the load and bearing capacity are described by stationary or quasi-stationary stochastic processes, the estimate of the probability of structural failure can be determined from the number of emissions $N_+(t)$ as

$$Q(t) \cong N_+(t) = \frac{\omega_q f_Y(\beta) t}{\beta_\omega \sqrt{2\pi}} . \qquad (19)$$

This formula was obtained in [6], it adopts the following notations: ω_q is an effective frequency of the random process of the strength reserve; $f_Y(\beta)$ is the ordinate of the density distribution function of the strength reserve $\widetilde{Y}(t)$, which corresponds to the value of the safety characteristic β ; *t* is a design life; β_ω is the broadband coefficient of the random process $\widetilde{Y}(t)$.

If $\widetilde{R}(t)$ and $\widetilde{S}(t)$ are normally distributed, then $\widetilde{Y}(t)$ also has a normal distribution, and the formula for $Q(t)$ takes the following form

$$Q(t) = \omega_q \exp\left(-0{,}5\beta^2 t\right) / \left(2\pi\beta_\omega\right) . \qquad (20)$$

On the base of the formula (20) and the normative value of the failure probability $[Q]$, it is easy to determine the corresponding safety characteristic

$$\beta = \left\{2\ln\left[\omega_q t / \left(2\pi [Q] \beta_\omega\right)\right]\right\}^{1/2} . \qquad (21)$$

The presented method is included in the codes [15] in the following form: the probability of failure of the structure during the established service life T_{ef} is defined as

$$P_f(T_{ef}) = K_0 f_\gamma(\beta) T_{ef} . \qquad (22)$$

Here it is denoted: $f_\gamma(\beta)$ is the density of the normalized distribution of random values of the value of the strength reserve $\widetilde{Y} = \widetilde{R} - \widetilde{S}$ at a value corresponding to the failure range (safety characteristic) β ; K_0 is the frequency characteristic, which is calculated by the formula

$$K_0 = \frac{(1+\theta^2 k^2)}{3} \left[\frac{\sum_{i=1}^{n} (\hat{s}_i a_i K_i^{tr} \omega_i)^2}{2\pi (1+\theta^2 k^2)(1+k^2)(\hat{r}^2 + \hat{s}^2)} \right]^{1/2}$$

In this formula it is indicated: ω_i is effective frequency of the *i*-th impact; K_i^{tr} is trend coefficient, which takes into account seasonal changes of the *i*-th impact (for example, snow and wind loads); θ is ratio of the effective frequency of the highest frequency of the loads taken into account (for example, crane) to the second in decreasing effective frequency (for example, wind load frequency); $k = \hat{s}_0 \Big/ \sqrt{\hat{s}^2 + \hat{r}^2 - \hat{s}_0^2}$ is coefficient, which characterizes the contribution of the standard of the highest frequency load taken into account to the standard of the strength reserve.

The current view on the prospects of transition to a new generation of design standards built on a probabilistic basis is set out in a monograph published by A.V. Perelmuter together with the author of the article [14]. It is emphasized that it is generally accepted that safety margins are intended to compensate for five main types of failure causes:

1) loads have higher values than expected;

2) the material has worse properties than expected;

3) the theory of the analysed failure mechanism is imperfect;

4) the possible manifestation of unknown and therefore unaccounted for causes of failure;

5) possible human errors (for example, in the project).

The first two options can be generally classified as variability of design parameters; therefore, they are available for probabilistic assessment. The last three types of failure causes operate not with probabilities, but with possibilities, they are difficult or even impossible to present in probabilistic terms, and therefore they belong to the category of non-statistical uncertainty.

The first and perhaps most important argument in favour of probabilistic methods is that they can be the basis for economic optimization. There is, for example, no way to numerically estimate the difference in safety between the case of using a safety factor of 2.0 and a safety factor of 3.0. And without a measurable effect (such as a reduction in the expected number of accidents), it is impossible to establish the value of the change in risk, and therefore economic optimization of the project is impossible. On the contrary, statistical analysis determines as a result the probability of an accident, which allows calculating the expected profit for a safer project. This is what is needed to optimize the ratio between the risks of losses and revenues. These considerations are taken into account in DBN B.1.2-14-2018 "General principles for ensuring the reliability and structural safety of buildings and structures" [16] in the procedure for predicting possible emergency situations and assessing the risk of damage in the following expanded format:

$$P = P(H) \times P(A/H) \times P(T/H) \times P(D/H) \times C, \quad (23)$$

where *P(H)* is the probability of occurrence of a hazardous event or phenomenon; *P(A/H)* and *P(T/H)* are the probabilities of encountering a hazard with an object in space and time, respectively; *P(D/H)* is the probability that threat *H* will cause damage *D*; *C* are the relative losses.

The second argument in favour of probabilistic approaches is their supposed ability to provide a more integrated assessment of the safety of the system as a whole. For example, logical-probabilistic assessment takes into account the comparison of the behaviour of different elements of the system, which allows us to indicate the significance, contributions and specific contribution of elements to the overall reliability indicator. Probabilistic approaches therefore seem suitable for identifying critical elements and building a maintenance scheme [17].

It should be emphasized that there are two different interpretations of failure probabilities calculated using statistical analysis. One of them considers the calculated probabilities as relative indices of failure probabilities, which can be compared both with some normalized value and with the corresponding values for alternative project options.

Within the second interpretation, the results of statistical analysis are perceived as objective values of the probability of failure. According to this view, these probabilities should be used not as simple relative indicators, but as qualitative estimates of the objective frequency of possible events. However, failure probabilities often contain their definitions based on expert assessments, and such assessments are inevitably subjective. In addition, some phenomena are excluded from the analysis. In these cases, we cannot compare the probability of failure of one facility with another. For example, comparing the safety of a nuclear power plant with the safety of a flood protection system based only on the statistical analysis of such two systems is an unreliable and incorrect decision, since the system of their uncertainties is different and it is difficult or even impossible to compare them.

One of the important features of probabilistic methods is that they can take into account potential negative factors only to the extent that their probabilities can be reliably quantified. In practice, these difficulties can lead to a one-sided bias in attention to only those hazards for which there are reasonable probability estimates. Therefore, probabilistic analysis tends to neglect potential events for which it is impossible to obtain a probability value for their realization.

Finally, we will quote from the [14]: "Returning to the fact that the safety margin, as mentioned above, was designed to compensate for the five main sources of failure, it can be considered that in the first two cases it is better to use probabilistic information. The main advantage in assigning safety margin factors on a non-statistical basis concerns the other three sources of failure. Therefore, the probabilistic approach should be only one of several tools for risk assessment. It is clear that both approaches have their own advantages, and it is unconstructive to view them as competitors, since neither can reveal the whole truth about risk and safety".

Conclusions

The long-term experience of developing probabilistic (statistical) approaches to the calculations of building structures is analyzed. The range of tasks for which statistical solution models are inherent is outlined. The prospects of the results of this scientific direction in creating a new generation of design standards are emphasized.

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Пічугін С.Ф. *
Національний університет «Полтавська політехніка імені Юрія Кондратюка»
https://orcid.org/0000-0001-8505-2130

Розвиток статистичного підходу до розрахунку будівельних конструкції

Аннотація. Сучасний метод розрахунку будівельних конструкцій за граничними станами (раніше за допустимими напруженнями) є по суті напівймовірнісним. Основні параметри методу – значення навантажень, міцнісні характеристики матеріалів, розміри елементів тощо – мають змінний статистичний характер. Вони описуються ймовірнісними методами, на основі яких обґрунтовуються детерміновані параметри проектування, які в подальшому використовуються в розрахунках. Тому основна увага розробників норм приділялася накопиченню і статистичній обробці даних щодо окремих компонентів методики розрахунків. Між тим, паралельно з цим двоїстим методом протягом багатьох років розвивалися повністю ймовірнісні (статистичні) підходи до розрахунків, які містять цінні наукові результати, актуальні і в теперішній час. Метою статті є послідовний розгляд розроблених на протязі дев'яноста років підходів до впровадження загальних ймовірнісних методів розрахунку будівельних конструкцій. Початком цього процесу можна вважати кінець 1930-х років, коли відбувся справжній науковий наступ на коефіцієнт запасу, основу методу допустимих напружень, який здійснили вітчизняні і закордонні дослідники, було виявлено імовірнісний характер коефіцієнта міцності, представленого в канонічній формі добутку факторів, і отримано числові значення гарантії неруйнівності конструкції. 1950-1960-ті роки ознаменувалися безуспішними спробами прихильників імовірнісних методів впровадити в практику проектування концепцію резерву міцності та характеристики безпеки. Починаючи з 1970-х рр. вітчизняні фахівці активно розробляють імовірнісні методи, результати яких впроваджуються в нормативні документи. Відмічено, що в останні роки з'явилися публікації з пропозиціями щодо подальшого впровадження імовірнісних підходів у розрахунки. Окреслено коло задач будівельного проєктування, для яких притаманні саме статистичні моделі розв'язання. Підкреслено перспективність результатів цього наукового напрямку у створенні нового покоління норм проєктування будівельних конструкцій.

Ключові слова: ймовірнісні методи, статистичні методи, норми проєктування, коефіцієнт запасу, граничні стани

*Адреса для листування E-mail: pichugin.sf@gmail.com

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