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## Calculation of optimal parameters for a vibratory finishing machine for decorative elements with an active working tool

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The calculation of optimal parameters for a new design vibration installation with an active working body is proposed using numerical methods. By modeling the system as a mass-spring-damper system and applying harmonic Fourier analysis of the vibrations, a mathematical model of the dynamic interaction between the working body and the part was obtained, taking into account additional friction forces, the angle of inclination, and elastic and damping forces. The key determined parameters—phase shift between the vibrations of the working body, the part, and the abrasive. An example of calculating optimal parameters using the gradient descent method is provided. Based on the calculation results, comparative time graphs of the phase shift of vibrations of the part and the working body, as well as amplitude, were constructed.

**Keywords:** vibratory finishing, tumbling, active working tool, mathematical model of dynamic interaction between the working tool and workpiece in a vibratory machine, optimal parameters, phase shift of vibrations of the working tool and workpiece

## Розрахунок оптимальних параметрів установки для віброабразивної обробки декоративних елементів з активним робочим органом

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Запропоновано розрахунок оптимальних параметрів числовим методом віброустановки нової конструкції з активним робочим органом. Наведена нова принципова схема обладнання є модифікацією відомої УВВ-04, що містить керований механічний збуджувач кутових коливань, до якої введено додатково робочу камеру обробки з активним робочим органом, оберти якого регулюються окремим приводом. Методом моделювання системи як масово-пружинної демпферної системи та гармонійного Фур'є-аналізу коливань отримано математична модель динамічної взаємодії робочого органу і деталі, з врахуванням додаткових сил тертя, кутом нахилу, пружних і демпфуючих сил, що часто ігноруються в попередніх дослідженнях, і більш точно описує динамічні взаємодії у системі та дозволяє встановити шляхи оптимізації. Модель може бути розширена та вдосконалена з урахуванням конкретних вимог і особливостей оброблюваних деталей. Визначено ключові параметри - фазовий зсув між коливаннями робочого органу, деталі та абразиву. Наведено приклад розрахунку оптимальних параметрів методом чисельного розв'язання системи диференціальних рівнянь за допомогою методу Рунге-Кутта. Запропоновано алгоритм визначення оптимальних параметрів з використанням методу градієнтного спуску. Цільова функція оптимізації включає мінімізацію фазового зсуву та амплітуди коливань робочого органу та деталі. По результатам обчислень побудовано часові графіки залежностей величини фазового зсуву коливань деталі та робочого органу, амплітуди. Проведено їх порівняльний аналіз до і після оптимізації, який засвідчив досягнення мінімальних величин фазового зсуву коливань, максимальної амплітуди зі збереженням стійкості системи, синхронізацію коливань робочого органу і деталі. Це є передумовою забезпечення рівномірного контакту між робочим органом та деталлю, покращує передачу енергії, приводить до зменшення енерговитрат та собівартості вібротехнологічного процесу з підвищенням ресурсу міцності та надійності обладнання. Запропонований метод може бути використаний в прикладних задачах на виробництві.

**Keywords:** віброабразивна обробка, галтовка, активний робочий орган, математична модель динамічної взаємодії робочого органу і деталі віброустановки, оптимальні параметри, фазовий зсув коливань робочого органу і деталі

## Introduction

Vibro-abrasive processing is a method that uses mechanical vibrations to treat parts to improve their quality. It is widely used in various industries, including the production of decorative elements, such as deburring after casting and mechanical processing of jewelry made from silver and gold, metal, ceramic (vases, figurines), and plastic interior items. Vibration installations with active working bodies ensure effective mechanical treatment of parts, improving the quality and productivity of processes. One of the key characteristics of such installations is their ability to generate high-frequency vibrations, which are transmitted to the treated materials through working bodies.

One of the main issues in using vibration installations is achieving system stability. Resonance phenomena that may occur during operation lead to significant vibration amplitudes, which can cause damage to working bodies and parts, as well as reduce processing efficiency. To avoid such phenomena, it is necessary to optimize system parameters such as vibration frequency, oscillation amplitude, and rotation speed of the working body. An example of calculating the optimal parameters of a vibration installation with an active working body is provided. The focus is on the mathematical modeling of the system, phase shift analysis, and studying the dynamic behavior of the working body and the treated parts. Special emphasis is placed on the treatment of small parts up to 15 mm, as they require a particular approach to selecting the geometric shapes and parameters of the working bodies. It is shown that the correct choice of vibration frequency and oscillation amplitude of the working body can significantly increase the productivity and stability of the vibration installation.

## Review of the research sources and publications

Sofronas et al. were among the first researchers to develop a comprehensive model for the vibration processing process in 1979. They used a statistical tool known as the surface response methodology to study the effect of hardness, imprint width, processing time, abrasive media size, and vibration frequency on reducing the height of burrs, rounding edges, and reducing surface roughness.

Hashimoto, in defining the basic principles of vibration processing, proposed mathematical models using differential equations to calculate surface roughness and material removal. Hashimoto suggested that in the steady state, the vibration processing process has a constant material removal rate. He used two processing chambers with variable capacity, a vibration frequency of 21 Hz, and an amplitude of 5 mm for his experiments. The experimental results confirmed his mathematical models.

Naeini et al. developed a discrete element model (DEM) to simulate the motion of multiple spherical steel particles in a two-dimensional vibration system. DEM is used to simulate collisions and measure the velocities of particles, the working chamber, and the part. The DEM model is based on a linear contact model with the following parameters: normal and

tangential stiffness ( $k$ ), damping in the normal and tangential directions ( $\beta$ ), and the coefficient of sliding friction ( $\mu$ ). It was noted that further increasing the accuracy of the model can be achieved by considering the stiffness and damping parameters in the normal and tangential directions as different rather than equal. Experimentally, it was found that the normal contact forces between the medium and the sensor were approximately 10 times higher than the tangential forces. It was established that vibration processing is based on such phenomena as plastic deformation from the impacts of the medium and material removal by abrasive particles due to the relative motion of the medium and the part.

Kumar et al. [4] developed a simple one-dimensional simulator for the vibration processing process. Titanium parts with different placements in the processing chamber were used to test material removal rates, surface roughness, and measure contact forces. During the study, it was found that surfaces located deeper in the abrasive medium and perpendicular to the vibrational motion demonstrated higher material removal rates. The flow of the medium can be imagined as layers—the medium deeper in the working chamber moves with the weight of the medium above it and thus strikes the part with greater force than the medium layer closer to the surface, leading to higher material removal rates.

In the work of Wang et al., a special sensor was developed to measure the normal contact forces between abrasive particles and the part in the working bowl-type processing chamber. Signal analysis from the force sensor using the Fourier series spectral decomposition method found that most of the energy transferred from the medium to the part occurred at the machine's operating frequency. Comparative results were obtained when the sensor was placed facing forward and backward on the surface of the part that rotated around the bowl. It was concluded that the amount of material removed from the workpiece by abrasive impacts was relatively constant for all surfaces of the part. Tests were also conducted with a fixed part, and it was found that the contact forces were higher compared to an unfixed part. Since the vibration processing time is significantly reduced in this case, fixation becomes increasingly relevant. It was also found that the average impact forces and average impulse showed an increasing trend with an increase in the size of the working chamber in a dry environment, while in a wet environment, this increase was insignificant. Wang et al. also noted that with a change in lubrication from dry to wet using a detergent, the quality of work decreased, and the medium's speed relative to the part also decreased, leading to the conclusion that the lubrication condition has a greater relative impact on the medium's speed than on the part's speed. The rigidity and roughness varied mainly depending on the degree of lubrication, the size of the medium, and its roughness. This is because plastic deformation with each impact depends on the interaction between the medium and the part.

One of the key studies [1] was conducted by Jia L. and Wang C. They investigated multi-frequency controlled synchronization of four induction motors in a vibration system using the fixed frequency ratio method. In their research, the authors found that the phase shift has a significant impact on the synchronization efficiency of the motors. Specifically, it was shown that proper phase shift adjustment allows for greater stability and efficiency of the system's operation, which is critical for industrial applications requiring high precision in part processing.

Blechman I.I. in his book [2] considers nonlinear dynamic effects in vibration mechanics and their applications. He developed a general approach to analyzing phase shifts in various types of vibration systems, including those used in industry. His work emphasizes the importance of accounting for nonlinear effects, which can significantly impact the system's behavior and stability.

The study by Andrievsky B. et al. [5] focuses on the creation of an educational-research complex for the study of vibration devices and processes. They found that the use of active working bodies in vibration installations allows achieving high accuracy and efficiency in part processing due to the control of phase shift and oscillation frequency.

#### **Definition of unsolved aspects of the problem**

However, most previous studies focused on individual parameters and did not consider a comprehensive approach to optimizing the entire system. This limited the ability to achieve maximum results, as the interaction between different parameters could affect the process efficiency. In particular, this pertains to the impact of phase shift between the oscillations of the working body and the part on processing efficiency and system stability.

#### **Problem statement**

Development of a methodology for determining the optimal parameters of a vibration unit with an active working body to ensure maximum processing efficiency of parts and maintain system stability. The main focus is on the analysis of phase shift between the oscillations of the working body and the part and its impact on system stability.

#### **Basic material and results**

Development of a methodology for determining the optimal parameters of a vibration unit with an active working body to ensure maximum processing efficiency of parts and maintain system stability. The main focus is on analyzing the phase shift between the oscillations of the working body and the part, and its impact on system stability.

Optimization in the context of vibratory abrasive processing refers to the process of adjusting and correcting system parameters, such as vibration frequency and amplitude, rotation speed of the working body, and the mass of the processed parts and abrasive, to achieve the best operational characteristics. The main goals of optimization may include:

Reducing the phase shift of oscillations to synchronize the movements of the working body and the processed parts, ensuring more effective interaction

between them and improving surface processing quality.

Improving processing quality, reducing defects, and enhancing the surface finish of parts.

Increasing productivity, shortening processing time, and increasing the number of processed parts per unit of time.

Avoiding operation in resonance modes to prevent significant oscillation amplitudes that can lead to mechanical damage to the unit.

Enhancing reliability, allowing the unit to operate for extended periods without unstable modes or breakdowns.

In the context of vibration systems, processing accuracy is influenced by the following factors:

Dimensional tolerances. Accurate processing ensures that part dimensions meet specifications with minimal deviations.

Shapes, angles, and the mutual arrangement of surfaces must be precisely manufactured according to drawings. Suboptimal vibrations can lead to deformations or deviations from the specified geometric parameters.

Surface roughness and the repeatability of the technological process.

#### *Algorithm for Selecting Optimal Parameters*

##### *1. Set Initial Values:*

Establish initial values for the amplitude  $A_{AA}$ , vibration frequency  $\omega$ , and angular rotation frequency.

##### *2. Solve the System of Differential Equations:*

Solve the system of differential equations for the mathematical model using the chosen initial parameters over a specified time interval with numerical methods (e.g., Runge-Kutta method).

##### *3. Conduct Analysis:*

Analyze the results by plotting the oscillations of the working body, the part, and the phase shift.

##### *4. Apply Optimization Algorithm:*

Use an optimization algorithm (e.g., gradient descent). The objective function for optimization includes minimizing the phase shift and the amplitudes of oscillations of the working body and the part. It can be represented as the sum of the quadratic deviations of phase shifts and amplitudes.

$$J(\theta) = \sum_{i=1}^n (\Delta\phi_i^2 + A_i^2) \quad (1)$$

where  $\Delta\phi_i$  - phase shift,  $A_i$  - amplitude of oscillations,  $n$  - number of data points

5. Repeat the Numerical Integration of the system of equations with the new parameter values.

6. Plot Graphs for the Optimized Parameters, compare them with the initial data. Perform an analysis and evaluation.

Consider the installation, the schematic diagram of which is shown in Fig. 1 [14]. The setup is based on a device with a controlled vibration exciter for angular oscillations [13], located in the laboratory of construction machinery at the Department of Industrial Engineering of "Poltava Polytechnic." The

working body performs a combined rotational and vibrational movement.

Let  $x(t)$  – be the position of the working body at time (t),  $x_d(t)$  – be the position of the part at time (t),  $x_a(t)$  – be the position of the abrasive at time (t).

The main parameters of the vibration setup are:

- $m$  — mass of the working body
- $m_d$  — mass of the processed parts
- $m_a$  — mass of the abrasive
- $k$  — stiffness coefficient of the spring connecting the working chamber to the frame
- $k_d$  — elasticity coefficient of the workpiece
- $k_a$  — elasticity coefficient of the abrasive
- $c$  — damping coefficient of the working body
- $c_d$  — damping coefficient of the parts

- $c_a$  — damping coefficient of the abrasive
- $A$  — amplitude of vibration oscillations
- $\omega$  — frequency of vibrations
- $\Omega$  — rotational frequency of the working element
- $\mu$  — coefficient of friction
- $g$  — acceleration due to gravity
- $\theta$  — angle of inclination of the installation
- $\phi$  — phase shift of vibrations between the part and the working tool

The mathematical model of the vibratory machine with an active working element is a key tool for analyzing and optimizing its parameters. Using Newton's second law, we can write the system of equations that describe the behavior of the system as follows:

$$\begin{cases} m \frac{d^2 x}{dt^2} = mA\omega^2 \sin(\omega t) - k(x - x_d) - c \left( \frac{dx}{dt} - \frac{dx_d}{dt} \right) \\ m_d \frac{d^2 x_d}{dt^2} = mA\omega^2 \sin(\omega t + \phi) + \mu m_d g \cos(\theta) + k(x - x_d) + c \left( \frac{dx}{dt} - \frac{dx_d}{dt} \right) - k_d x_d - c_d \frac{dx_d}{dt} \\ m_a \frac{d^2 x_a}{dt^2} = k_a x_a + c_a \frac{dx_a}{dt} - k_d (x_d - x_a) - c_d \left( \frac{dx_d}{dt} - \frac{dx_a}{dt} \right) \end{cases} \quad (2)$$

where:

-  $m \frac{d^2 x}{dt^2}$  – inertial force of the working body

-  $mA\omega^2 \sin(\omega t + \phi)$  – the magnitude of the vibration force

-  $m_d \frac{d^2 x_d}{dt^2}$  – inertial force of the part

-  $\mu m_d g \cos(\theta)$  – the friction force of the part due to the angle of inclination

-  $c_d \frac{dx_d}{dt}$  – damping force acting on the part

-  $k_d x_d$  – elastic force acting on the part.

-  $c \left( \frac{dx}{dt} - \frac{dx_d}{dt} \right)$  – damping force of interaction with the working body

-  $k(x - x_d)$  – strength of elasticity of interaction with the working body.

-  $m_a \frac{d^2 x_a}{dt^2}$  – inertial force of the abrasive.

-  $k_a x_a(t)$  – elastic strength

-  $c_a \frac{dx_a}{dt}$  – damping force

-  $k_d (x_d - x_a)$  – elastic force acting between the abrasive and the part

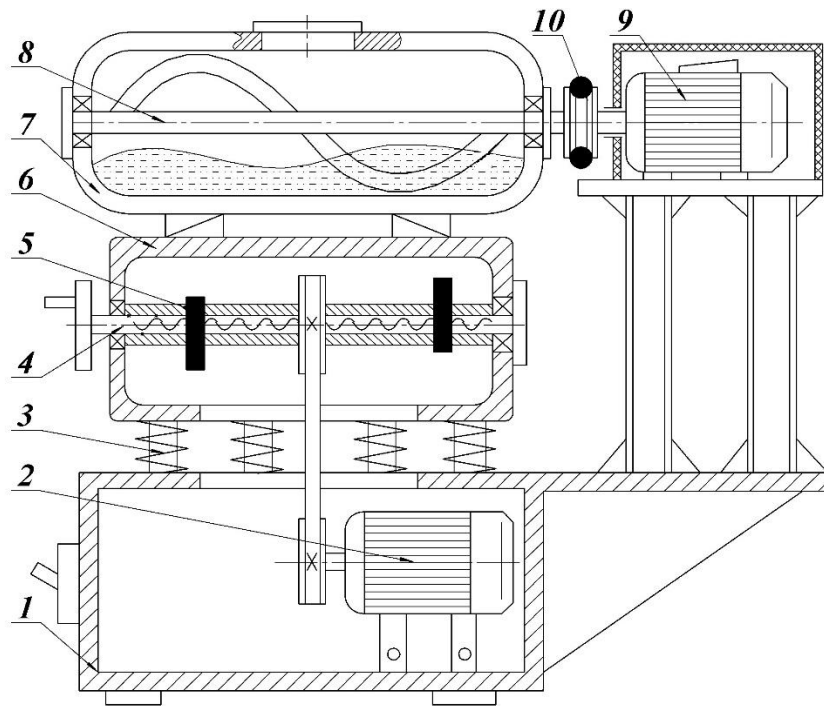
The resonant frequency is determined by the formula:

$$\omega_{\text{res}} = \sqrt{\frac{k}{m}} \quad (3)$$

where  $k$  – spring stiffness,  $m$  – the mass of the working body. This is the frequency at which the amplitude of the system's vibrations reaches its maximum. The optimal vibration frequency should be lower than the resonance frequency to avoid excessive vibration amplitudes and ensure the stable operation of the system

$$\omega_{\text{opt}} < \omega_{\text{res}} \quad (4)$$

The excitation force  $F_0$  is selected to ensure the required vibration amplitude without overloading the system. Typically, the vibration amplitude for vibrosystems in the processing of small parts ranges from a few millimeters to several centimeters. This ensures effective contact between the working tool and the part without excessive wear or overloading the system.



**Figure 1 – Principal diagram of the vibro-abrasive processing setup:**  
**1 – framework; 2 – vibration motor; 3 – spring; 4 – lead screw; 5 – debalances; 6 – frame;**  
**7 – working chamber; 8 – active working element; 9 – working element motor; 10 – coupling**

For this, the formula of the amplitude of oscillations in the harmonic mode is used:

$$A = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \quad (5)$$

where:

- A – amplitude of oscillations,
- $\omega$  – frequency of oscillations.

#### *Selection of the Working Tool Shape*

For processing small parts (up to 15 mm), it is essential to choose a working tool shape that ensures uniform treatment of the parts and minimizes system vibrations. Here are some possible shapes of working tools and their characteristics:

##### *1. Cylindrical Shape:*

Advantages: Simple to manufacture, good interaction with the parts.

Disadvantages: High vibration levels during operation, potential for uneven treatment.

Recommendations: Suitable for rough surface treatment where high precision is not required.2.

##### *Conical Shape:*

Advantages: Better load distribution, reduced vibration amplitude.

Disadvantages: Complex manufacturing, potential for parts to get stuck in sharp corners.

Recommendations: Optimal for medium complexity surface treatment, providing good stability.3.

##### *Spherical Shape:*

Advantages: Minimizes vibrations, ensures uniform load distribution, prevents parts from getting stuck.

Disadvantages: High manufacturing complexity.

Recommendations: Best choice for processing small parts requiring high precision.

#### *Selection of the Working Tool Dimensions*

##### *1. Diameter of the Working Tool:*

Recommendations: The diameter should be sufficient to ensure adequate contact area with the parts. An optimal diameter for parts up to 15 mm is 30-50 mm, which provides sufficient stability and uniform treatment.

##### *2. Length of the Working Tool:*

Recommendations: The length should correspond to the length of the working chamber to maximize the use of chamber space and ensure effective mixing of the parts.

##### *3. Spiral Pitch (for Screw-Type Working Tools):*

Recommendations: The spiral pitch should be chosen to ensure sufficient movement of parts along the axis of the working tool without imposing excessive loads on the system. The optimal spiral pitch for parts up to 15 mm is 20-30 mm.

Selecting a screw-type working tool will ensure a continuous flow of parts and abrasive along the working chamber, promoting uniform treatment, even load distribution on the parts, preventing localized force peaks, and facilitating intensive mixing.

#### *Example of Numerical Calculation of Optimal Parameters*

For the calculation example, let's consider the processing of ABS plastic parts, specifically decorative elements, with the goal of deburring, rounding sharp edges, and preparing the surface for painting. The abrasive medium used is ceramic granules. Below are the initial data and processing mode parameters:

- $m$  — 2 kg
- $m_d$  — 0,3 kg
- $m_a$  — 1 kg
- $k$  — 1000 N/m
- $k_d$  — 500 N/m
- $k_a$  — 300 N/m
- $c$  — 10 Nc/m
- $c_d$  — 5 Nc/m
- $c_a$  — 3 Nc/m
- $A$  — 0.002 m
- $\omega$  — 50 rad/s
- $\Omega$  — 30 rad/s (287 RPM)
- $\mu$  — 0,1
- $g$  — 9,81 m/s<sup>2</sup>
- $\theta$  — 0 rad
- $\phi$  — 5 rad

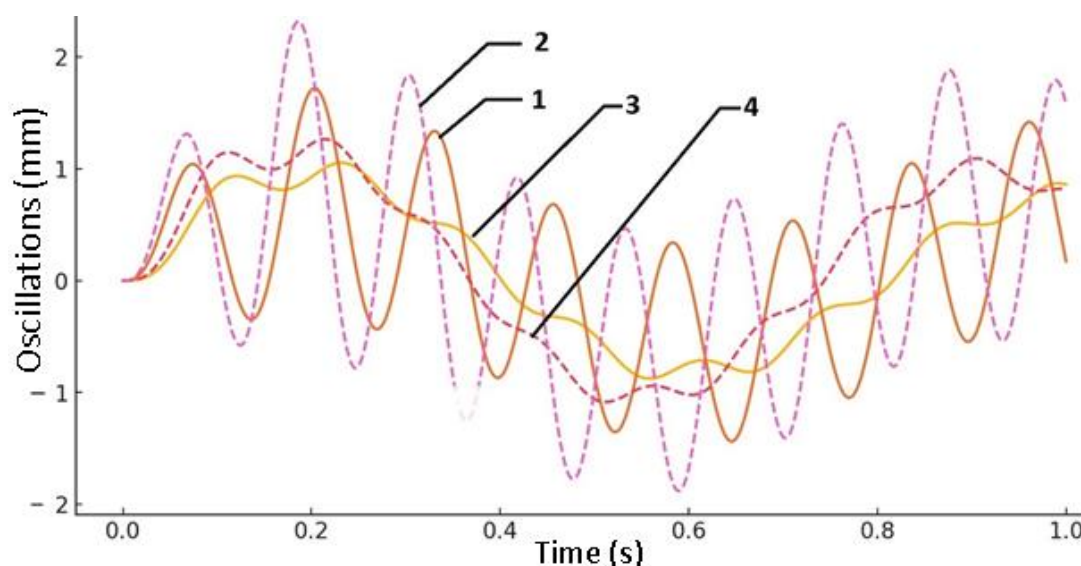
We will perform the calculation of the main parameters of the setup, such as the vibration amplitude of the working tool and parts, and the phase shift, using the MathCAD software package. Then, the proposed gradient descent optimization algorithm will be applied to determine the optimal parameters.

In the graph shown in Fig. 2, it is observed that the amplitude of oscillations for both the working tool and the parts increased after optimization. This indicates an increase in the energy transmitted to the part, which enhances productivity, as the processing becomes more intense, removing more material per unit time. However, to maintain high processing accuracy in practice, it is essential to carefully control the amplitude and frequency of vibrations to avoid excessive impact on the quality of the manufactured parts.

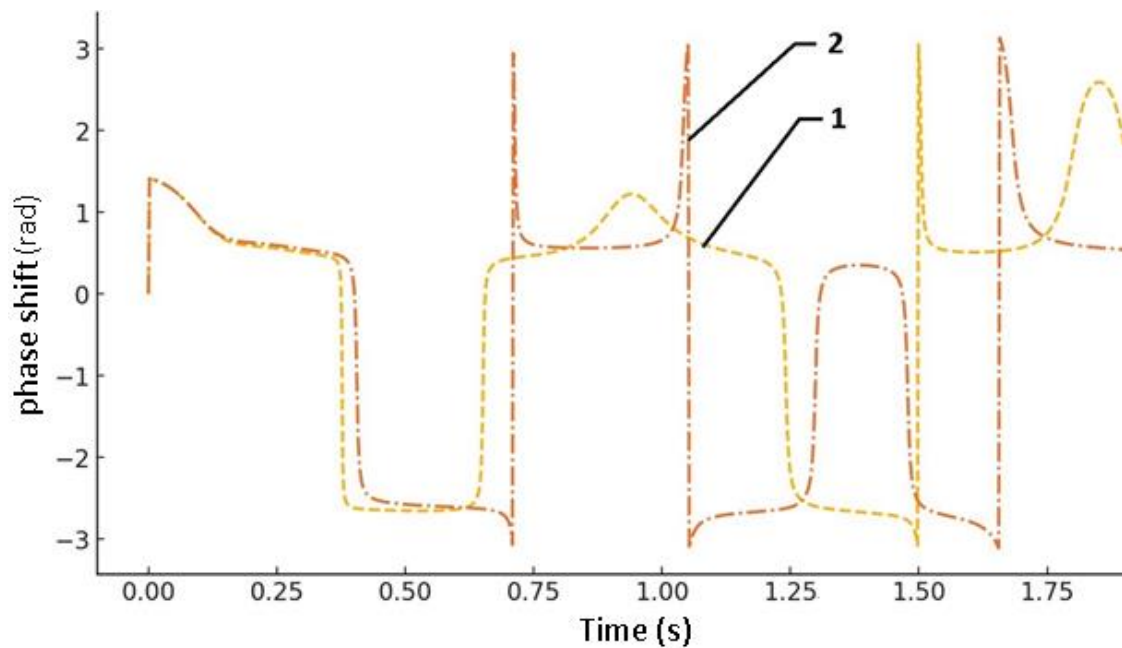
The graph in Fig. 3 shows a reduction in the phase shift between the working tool and the part after optimization. This decreases relative movements, thereby improving processing accuracy. The phase shift can be further optimized by adjusting the stiffness and damping of the system.

1. The vibration amplitude  $A$  was increased from 0.002 m to 0.003 m.
2. The vibration frequency  $\omega$  was increased from 50 rad/s to 55 rad/s.
3. The rotational frequency  $\Omega$  was increased from 30 rad/s to 35 rad/s.
4. The spring stiffness  $k$  was increased from 1000 N/m to 1200 N/m to enhance force transmission efficiency and system stability.

Further experimental studies are necessary to validate the theoretical results obtained and to refine the mathematical model.



**Figure 2 – The graph of oscillations based on the calculation results:  
1 - workpiece before optimization; 2 - workpiece after optimization;  
3 - working element before optimization; 4 - working element after optimization.**



**Figure 2 – The graph of the phase shift magnitude between the oscillations of the workpiece and the working element based on the calculation results: 1 - before optimization; 2 - after optimization.**

### Conclusions

Thus, the following has been established:

1. A mathematical model for analyzing the dynamic interaction between the working body and the workpiece in a vibro-abrasive machine has been proposed, taking into account frictional and damping forces. This allows consideration of all dynamic characteristics of the system and determination of the phase shift between the working body and the workpieces.

2. Precise adjustment of the optimal frequency and vibration amplitude parameters can be achieved through the use of numerical methods for integrating the system of differential equations in the model, combined with the application of optimization algorithms.

3. The constructed graphs based on the calculation results and their analysis demonstrated a reduction in the phase shift between the working body and the workpiece after optimization. This increases the accuracy of vibro-processing and improves the quality of the workpiece surfaces, while also reducing equipment wear.

4. Minimizing the phase shift between the working body and the workpiece reduces relative movements, thereby increasing processing accuracy. The phase shift can be optimized by adjusting the system's stiffness and damping.

5. Ensuring stable system operation without significant vibrational oscillations allows for maintaining high processing accuracy. Consistency in the frequency characteristics and amplitude contributes to precise processing

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