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Determination of the vibrating table kinetic energy

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The article determines the total kinetic energy of the vibrating table, in which the vibrator is fixed on a lever vertically in the center under the vibrating plate. To find the total kinetic energy, a kinematic diagram of the vibrating table under study was drawn up, and the functional dependence of the vibrating table total kinetic energy on the factors acting on it obtained. A graph of the change in the vibrating table kinetic energy under study was drawn depending on the lever length, on which the vibrator was fixed. The analysis of the resulting dependence indicates an increase in the kinematic energy of the vibrating table with an increase in the lever length. The obtained kinetic energy is necessary for the development of an above-mentioned equipment mathematical model, which will be compiled later using Lagrange equations of the second kind

Keywords: vibrating table, lever, vibrator, imbalance, degree of freedom, kinetic energy

Визначення кінетичної енергії вібраційного столу

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В статті проводиться визначення загальної кінетичної енергії технологічного комплекту обладнання для виробництва бетонних виробів (вібростолу), у якого вібрезбуджувач закріплюється на важелі вертикально по центру під віброплитою. Дане обладнання використовується для виготовлення малогабаритних бетонних виробів. При виконанні досліджень були використанні методи математичної фізики та фізико-математичне моделювання методами прикладної механіки. Для визначення положення і опису вільних рухів матеріальних тіл, з яких складається розглядувана механічна система, була застосована ортогональна вібраційна система відліку з трьох систем координат. Для знаходження загальної кінетичної енергії, що є сумою кінетичних енергій декількох матеріальних тіл - плити, корпусу вібрезбуджувача, дебаланса і ємності з бетонною сумішшю, була складена кінематична схема досліджуваного вібростолу і отримана функціональна залежність загальної кінетичної енергії вібростолу від діючих на нього факторів. Був побудований графік зміни кінетичної енергії досліджуваного вібраційного столу залежно від довжини важеля, на якому закріплювався вібрезбуджувач, із врахуванням номінальних чисельних значень величин параметрів вібростолу. Аналіз одержаної залежності вказує на збільшення кінематичної енергії вібростолу при збільшенні довжини важеля, що підтверджує ефективність важільного закріплення вібрезбуджувача. Це в свою чергу призводить до зменшення енерговитрат під час віброущільнення бетонних виробів за рахунок зменшення потужності використовуваного для ущільнення вібрезбуджувача. Отримана функціональна залежність кінетичної енергії також необхідна для складання в подальшому математичної моделі вищезгаданого обладнання за допомогою рівнянь Лагранжа другого роду

Ключові слова: вібростіл, важіль, вібратор, дебаланс, ступінь вільності, кінетична енергія

Introduction

Vibration is the most common method of compacting concrete composites [1,2]. More than 90% of all construction products made of concrete and reinforced concrete are made using this method of concrete mix compaction [3]. This is explained by the fact that in the process of vibrational action on concrete mixtures, favorable conditions for thixotropic rarefaction and the most compact placement of aggregate particles are created [4,5].

The modern development of construction requires the introduction of the latest technologies and the installation of engineering equipment for various purposes according to the criteria of energy minimization and high efficiency of the technological process. There are a huge variety of vibration machines used in construction, which differ in design and purpose. And, above all, great attention should be paid to the development and implementation of energy-saving technologies and equipment.

A review of various vibration equipment designs shows, that in the manufacture of a wide concrete products range equipment with the required operating parameters used, with the help of which high-quality vibration compaction of the concrete mix is achieved. In order to find out the influence of individual parameters of the developed by us technological set of equipment for the manufacture of concrete products [6] on the movement of its working body and energy consumption, it is necessary to determine the kinetic energy and make a mathematical model of this mechanical system.

Review of the research sources and publications

Extensive use of vibration technology, numerous theoretical and experimental studies of the dynamics of vibrating machines made it possible to identify the features of their work, to explain and put into practice the peculiar effects that occur during the action of vibration on mechanical systems [7,8,9]. Therefore, a lot of research and development is devoted to this issue [10,11], which reveals such advantages of vibration equipment as high sealing efficiency, simplicity of design, high reliability and relatively low metal consumption and energy intensity. One of the priority directions of construction vibration equipment development is to reduce energy costs in production. Energy saving technologies along with increasing productivity and improving product quality occupy a priority place in modern construction.

In our case, we need to create a vibrating table, which by placing a vibration exciter on the vertical lever under the vibrating plate would save energy in the manufacture of concrete products by reducing the power of the vibrator while maintaining the required parameters of vibration compaction.

Definition of unsolved aspects of the problem

It is necessary to make a mathematical model of the developed and constructed vibration table according to the kinematic scheme, that has not been used before, having previously determined its kinetic energy.

Problem statement

The purpose of this work is to obtain the functional dependence of mechanical system total kinetic energy, that simulates a vibrating table with a vibrating exciter placed on a vertical lever under the vibrating plate. This functional dependence of kinetic energy is necessary for the analysis of technological factors, in particular the length of the lever, having an impact on the energy intensity of this mechanical system, as well as for further compilation of a technological equipment set mathematical model for the manufacture of concrete products (vibrating table) using the Lagrange equations of the second order.

To achieve a certain goal, it is necessary to solve the following tasks:

1. Create a kinematic diagram of the vibrating table with a vibration exciter placed on a vertical lever under the vibrating plate.
2. To determine the kinetic energy of individual material bodies composing the vibro-table.
3. Formulate the functional dependence of the total kinetic energy of the vibro-table on the factors affecting it.
4. Build a graphical dependence of the change in the kinetic energy of the vibrating table under study depending on the length of the lever on which the vibration exciter is fixed.

Basic material and results

To clarify the general trend of vibration table for the manufacture of small-sized concrete products individual parameters influence on the movement of his working body, as well as the mutual influence of the vibrating table individual nodes movement let us consider it as a mechanical system, get this mechanical system a mathematical model, and solve it.

The general view of the technological set of equipment for the concrete products manufacture (vibrating table) is shown in Fig. 1.

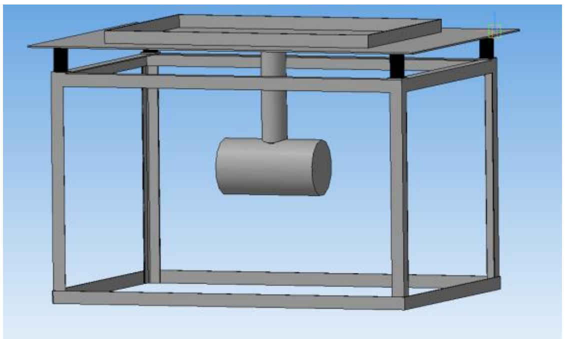

Figure 1 – General view of the technological set of equipment for the manufacture of concrete products

Technological set of equipment (subsequently vibrating table) consists (see Fig. 2) from the plate *1* with dimensions $2a_1 \times 2b_1$ in projections and thickness $2\delta_1$, which rests on a fixed supporting surface with the help of four elastic elements with rigidity C each. To the plate 1 lever 2 is rigidly attached with length l_{lever}, to the lower end of which is also rigidly attached mechanical centrifugal imbalanced oscillator exciter (vibrator), debalancing shaft axis of rotation 4 which, at rest, the vibrating table is parallel to the longer axis of symmetry of the plate 1. On shaft 4 imbalance 5 rigidly fixed, the rotation of which generates, provides and determines the working technological movement of the vibrating table under consideration. On plate 1 symmetrically placed and rigidly fixed technological tank for concrete products molding.

Undoubtedly, the primary source of operation for the vibrating table used in the production of small-sized concrete products is the rotation of the unbalanced shaft 4 with imbalance 5 of exciter, parameters and characteristics of which completely determine the structure, magnitude and effectiveness of the dynamic action on the processed medium [12].

To obtain a mathematical model, let us use the Lagrange equations of the second order [13]

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}=Q_i \qquad (i=1,2,...,s), \qquad (1)$$

where T – kinetic energy of a mechanical system.

To find kinetic energy T let us consider in Figure 3 the kinematic scheme of the vibration table, as a mechanical system consisting of a plate 1, which is the working body of the vibrating table and which we will consider as an absolutely solid body in the form of a homogeneous rectangular parallelepiped with mass m_1. The external restraints that limit the movement of the slab are elastic elements (springs), with distances between vertical longitudinal axes equal to $2a$ and $2b$ (see. Fig. 2). Lever weight 2 ignored. The exciter consists of its body 3, which we take for an absolutely solid homogeneous hollow circular cylinder with a mass m_3, inside which the debalanced shaft 4 has the ability to rotate (the mass of which is also neglected) with an imbalance *5* located on it, with mass m and eccentricity e. Central longitudinal axis of the body 3 of exciter determines the rotation axis position of its debalancing shaft 4.

In a first approximation, the container fixed on the slab, together with the concrete mix to be molded, will be considered as a homogeneous solid with mass m_6 in the form of a rectangular parallelepiped dimensions $2a_6 \times 2b_6 \times 2h_6$, which is rigidly attached to the plate 1 (see Fig. 2).

Thus, the considered vibration device is modeled by a mechanical system consisting of four material bodies.

During the direct vibration action, which determines the process of manufacturing (molding) concrete products, material bodies 1,3 and 6 carry out complex spatial movements that can be considered as free. The movement of a free solid can be decomposed in many ways [14] for two movements: a) translational motion together with an arbitrarily selected fixed point of the body, which is called the pole; b) spherical motion around this pole.

To determine the position and description of the free motions of material bodies of the mechanical system under consideration, we apply an orthogonal vibration reference frame [15], which consists of three coordinate systems: fixed $Oxyz$ and movable $Cx'y'z'$ and $Cx_1y_1z_1$. Start of count O of system $Oxyz$ let us associate with the center of plate 1 inertia C in the position of mechanical system static equilibrium, which is shown in the Figure 2, by aligning the corresponding coordinate axes with the main central axes of plate inertia.

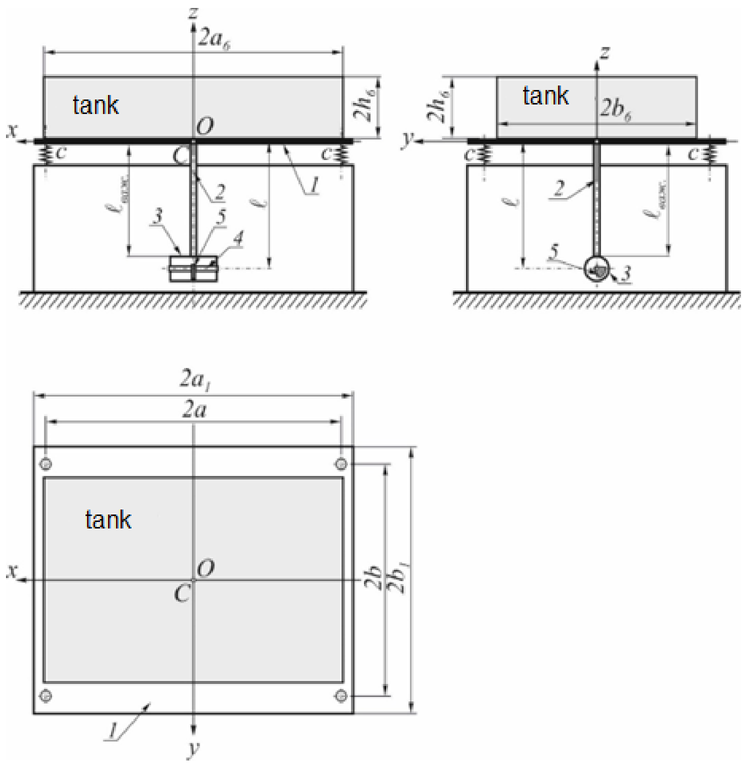

Figure 2 – Vibrating table for the manufacture of concrete products

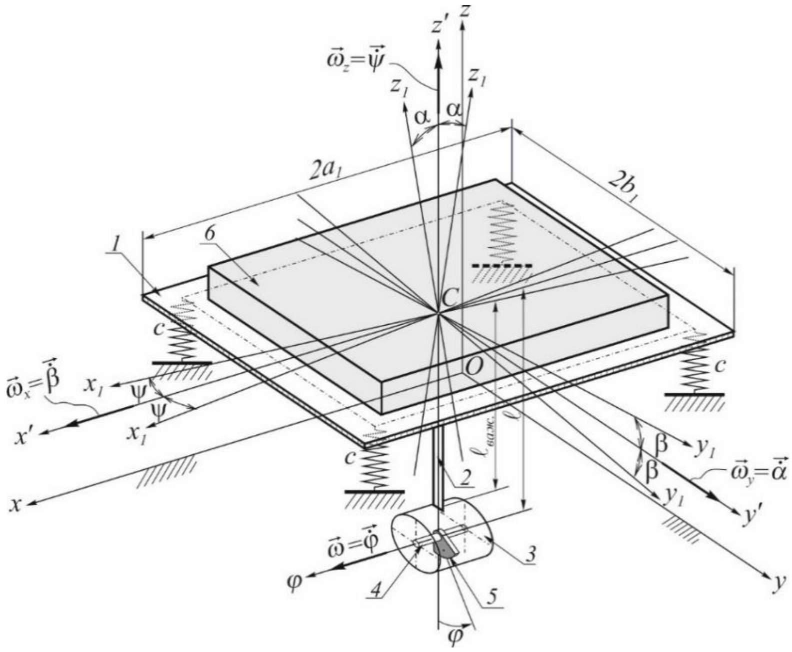

Figure 3 – Kinematic scheme of the vibrating table

The origin of both moving coordinate systems moves with the point C; In this case, the system $Cx'y'z'$ moves progressively, so that its axes remain parallel to the axes of the fixed coordinate system, and the system $Cx_1y_1z_1$ rigidly bound to the plate 1. In the static equilibrium position of a mechanical system, all three coordinate systems coincide, and when a mechanical system moves, a point C uniquely characterizes the motion of the center of inertia of a plate relative to a fixed coordinate system $Cxyz$.

Euler's angles of rotation of the moving system relative to the fixed one will be replaced by vibrational angles of rotation α, β and ψ (Fig. 4), where:

- α specifies the plate rotation angle 1 (or movable system $Cx_1y_1z_1$) in the frontal coordinate plane $Cx'z'$ around the axis Cy' or (which is the same) around a fixed axis Oy with angular velocity $\dot{\alpha}$;
- β – plate 1 rotation angle in the profile plane $Cy'z'$ around the axis Ox with angular velocity $\dot{\beta}$;
- ψ – plate 1 rotation angle in the horizontal plane $Cx'y'$ around the axis Oz with angular velocity $\dot{\psi}$.

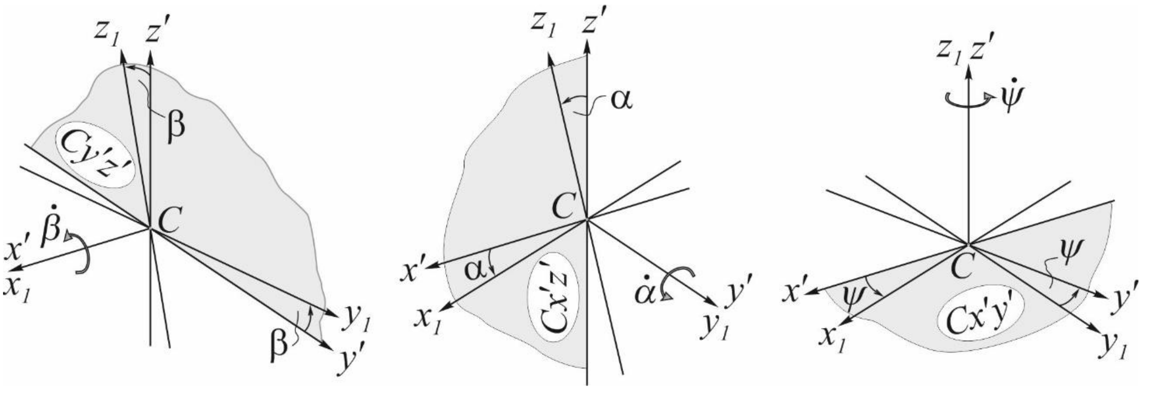

Figure 4 – Vibration count system

When moving a mechanical system, vibration angles α, β and ψ acquire only small values, which distinguishes them favorably from Eulerian angles, in which only the nutation angle is small, and the angles of precession and self-rotation may not be small. Vibration angles are of the same order and are given by periodic trigonometric functions; corresponding angular velocities $\dot{\alpha}$, $\dot{\beta}$ and $\dot{\psi}$ are same order with angular velocity $\omega = \dot{\phi}$.

Such assumptions make it possible to significantly simplify the process of determining kinetic energy and obtain a simpler mathematical model, which describes with a high degree of accuracy the position and movement of any point and individual material body of the table under study.

Since the mechanical system that simulates the vibrating table consists of four material bodies, its kinetic energy

$$T = T_1 + T_3 + T_5 + T_6\ , \qquad (2)$$

where T_1, T_3, T_5, T_6 – accordingly, the kinetic energies of each of the bodies.

To find T_I let's choose the center of inertia beyond the pole C of plate 1; then by Koenig's theorem [16]

$$T_1 = \frac{m_1 \cdot v_c^2}{2} + \frac{I_1 \Omega_1 \cdot \omega_1^2 \Omega_1}{2} + \ , \qquad (2)$$

Where v_c – point speed module C;

$\omega_1 \Omega_1$ – plate 1 instantaneous angular velocity module around the axis Ω_1 of instant rotation, which in the considered position of the mechanical system passes through a point C;

$I_1 \Omega_1$ – axial moment of plate inertia 1 relative to the instantaneous axis Ω_1.

Due to the fact that the origin of the moving reference frame is chosen in the plate 1 inertia center, then [17]

$$T_1 = \frac{m_1}{2} v_c^2 + \frac{1}{2}\left(I_{1x_1}\omega_{1x}^2 + I_{1y_1}\omega_{1y}^2 + I_{1z_1}\omega_{1z}^2\right)\ ,$$

where I_{1x_1}, I_{1y_1}, I_{1z_1}– accordingly, the plate 1 inertia moments relative to coordinate axes Cx_1, Cy_1 and Cz_1; ω_{1x}, ω_{1y} i ω_{1z} – projections of instantaneous angular velocity $\vec{\omega}_{1\Omega_1}$ on the corresponding axes of the fixed system $Oxyz$.

Next, we note that $\omega_{1x} = \dot{\beta}$, $\omega_{1y} = \dot{\alpha}$ and $\omega_{1z} = \dot{\psi}$, and with the coordinate method of determining the point motion [18] module $v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2 + v_{Cz}^2}$, where $v_{Cx} = \dot{x}_C$, $v_{Cy} = \dot{y}_C$ and $v_{Cz} = \dot{z}_C$ – vector projections $\vec{v}_C$ to the axis $Oxyz$. Then

$$v_C^2 = \dot{x}_C^2 + \dot{y}_C^2 + \dot{z}_C^2.$$

Substituting all the above values, we finally get that

$$T_1 = \frac{m_1}{2}(\dot{x}_C^2 + \dot{y}_C^2 + \dot{z}_C^2) + \\ + \frac{I_1 x_1}{2}\dot{\beta}^2 + \frac{I_1 y_1}{2}\dot{\alpha}^2 + \frac{I_1 z_1}{2}\dot{\psi}^2\ , \qquad (3)$$

To find T_3 we choose the center of inertia for the pole C_3 of the exciter body 3. Similar to the definition T_I we get that

$$T_3 = \frac{m_3}{2} v_{C_3}^2 + \frac{1}{2}\left(I_{3x_3}\omega_{3x}^2 + I_{3y_3}\omega_{3y}^2 + I_{3z_3}\omega_{3z}^2\right).$$

Since the plate 1, lever 2 and body connected rigidly, then according to the concepts of theoretical mechanics these three different physical bodies are one material body, because of which

$$\omega_{3x} = \omega_{1x} = \dot{\beta}\ ,$$

$$\omega_{3y} = \omega_{1y} = \dot{\alpha}\ ,$$

$$\omega_{3z} = \omega_{1z} = \dot{\psi}\ .$$

Absolute speed [19] vector $\vec{v}_3$ of body mass center 3

$$\vec{v}_{C_3} = \vec{v}_{C_3 e} + \vec{v}_{C_3 r},$$

where $\vec{v}_{C_3 e}$ and $\vec{v}_{C_3 r}$ – respectively, the vectors of portable and relative velocities of the point C_3.

Since portable movement is free, then

$$\vec{v}_{C_3 e} = \vec{v}_{C_3 e1} + \vec{v}_{C_3 e2},$$

where $\vec{v}_{C_3 e1}$ – pole velocity; $\vec{v}_{C_3 e2}$– point velocity C_3 in its spherical motion around the pole.

Since a point is chosen as the pole C, then

$$\vec{v}_{C_3 e1} = \vec{v}_C = \vec{\imath} \cdot \dot{x}_C + \vec{\jmath} \cdot \dot{y}_C + \vec{k} \cdot \dot{z}_C.$$

To find $\vec{v}_{C_3 e2}$ let's apply vibration angles and depict body 3 inertia center C_3 (see Fig. 5) in three orthogonal projections, where $\ell = CC_3 = \ell_{\text{важ.}} + (\delta_1 + R_3)$; R_3 – outer body radius 3.

Let's write $\vec{v}_{C_3 e2}$ as the sum of three addends

$$\vec{v}_{C_3 e2} = \vec{v}_{3xy} + \vec{v}_{3xz} + \vec{v}_{3yz},$$

where $\vec{v}_{3xy}$, $\vec{v}_{3xz}$ and $\vec{v}_{3yz}$ – vector projections $\vec{v}_{C_3 e2}$ to planes xy, yz and zx

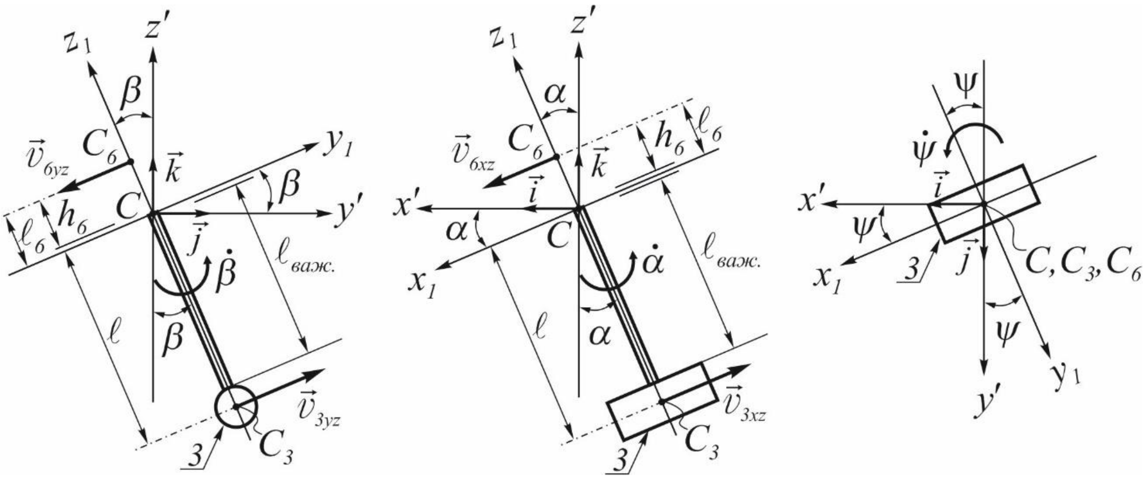

Figure 5 – To determining a portable speed $\vec{v}_{C_3 e2}$ and $\vec{v}_{C_6 e2}$

From Figure 5 it is obvious that the velocity modules $v_{3yz} = \dot{\beta} \cdot \ell$, $v_{3xz} = \dot{\alpha} \cdot \ell$ and $v_{3xy} = \dot{\psi} \cdot 0 = 0$, and the corresponding projections on the specified planes:

$$\vec{v}_{3yz} = \vec{j} \cdot v_{3yz} \cdot cos\beta + \vec{k} \cdot v_{3yz} \cdot sin\beta = \\ = \vec{j} \cdot \dot{\beta} \cdot l \cdot cos\beta + \vec{k} \cdot \dot{\beta} \cdot l \cdot sin\beta ,$$

$$\vec{v}_{3xz} = -\vec{i} \cdot v_{3xz} \cdot cos\alpha + \vec{k} \cdot v_{3xz} \cdot sin\alpha = \\ = -\vec{i} \cdot \dot{\alpha} \cdot l \cdot cos\alpha + \vec{k} \cdot \dot{\alpha} \cdot l \cdot sin\alpha .$$

Adding, we get that

$$\vec{v}_{C_3e_2} = -\vec{i} \cdot \dot{\alpha} \cdot l \cdot cos\alpha + \vec{j} \cdot \dot{\beta} \cdot l \cdot cos\beta + \\ + \vec{k} \cdot (\dot{\alpha} \cdot l \cdot sin\alpha + \dot{\beta} \cdot l \cdot sin\beta) ,$$

$$\vec{v}_{C_3e} = \vec{i}\,(\dot{x}_c - \dot{\alpha} \cdot l \cdot cos\alpha) + \vec{j}(\dot{y}_c + \dot{\beta} \cdot l \cdot cos\beta) + \\ + \vec{k}\,(\dot{z}_c + \dot{\alpha} \cdot l \cdot sin\alpha + \dot{\beta} \cdot l \cdot sin\beta) .$$

Since the plate 1, lever 2 and body 3 connected rigidly, then any movement of the point C_3 relative to the point C absent, which is why $\vec{v}_{C_3r} = 0$, and

$$\vec{v}_{C_3} = \vec{i}\,(\dot{x}_c - \dot{\alpha} \cdot l \cdot cos\alpha) + \vec{j}(\dot{y}_c + \dot{\beta} \cdot l \cdot cos\beta) + \\ + \vec{k}\,(\dot{z}_c + \dot{\alpha} \cdot l \cdot sin\alpha + \dot{\beta} \cdot l \cdot sin\beta) .$$

Squaring the resulting expression and substituting the value $v_{C_3}^2$ into the formula of kinetic energy T_3 for body 3, we get

$$T_3 = 0.5m_3\left(\dot{x}_c^2 + \dot{y}_c^2 + \dot{z}_c^2\right) - m_3 \cdot l \cdot \dot{x}_c \cdot \dot{\alpha} \cdot \cos\alpha + \\ + m_3 \cdot l \cdot \dot{y}_c \cdot \dot{\beta} \cdot \cos\beta + m_3 \cdot l \cdot \dot{z}_c \cdot \dot{\alpha} \cdot \sin\alpha + \\ + m_3 \cdot l \cdot \dot{z}_c \cdot \dot{\beta} \cdot \sin\beta + 0.5m_3 \cdot l^2 \cdot \dot{\alpha}^2 + \\ + 0.5I_{3x_3} \cdot \dot{\beta}^2 + 0.5I_{3y_3} \cdot \dot{\alpha}^2 + 0.5I_{3z_3} \cdot \dot{\psi}^2. \quad (4)$$

Choosing center of Inertia C_6 as pole of material body 6, similar to the definition T_3 we consistently establish that

$$T_6 = 0.5m_6\left(\dot{x}_c^2 + \dot{y}_c^2 + \dot{z}_c^2\right) - m_6 \cdot l_6 \cdot \dot{x}_c \cdot \dot{\alpha} \cdot \cos\alpha - \\ - m_6 \cdot l_6 \cdot \dot{y}_c \cdot \dot{\beta} \cdot \cos\beta + m_6 \cdot l_6 \cdot \dot{z}_c \cdot \dot{\alpha} \cdot \sin\alpha - \\ - m_6 \cdot l_6 \cdot \dot{z}_c \cdot \dot{\beta} \cdot \sin\beta + 0.5m_6 \cdot l_6^2 \cdot \dot{\alpha}^2 - \\ - m_6 \cdot l_6 \cdot \dot{z}_c \cdot \dot{\alpha} \cdot \dot{\beta} \cdot \sin\alpha \cdot \sin\beta + 0.5m_6 \cdot l_6^2 \cdot \dot{\beta}^2 + \\ + 0.5I_{6x_6} \cdot \dot{\beta}^2 + 0.5I_{6y_6} \cdot \dot{\alpha}^2 + 0.5I_{6z_6} \cdot \dot{\psi}^2. \quad (5)$$

To determine T_5 let's decompose the complex movement of imbalance 5 on portable with the body 3 and relative movement with respect to the specified body. Portable movement is free. The relative motion of the imbalance is rotation with angular velocity $\omega = \dot{\phi}$. By Koenig's theorem

$$T_5 = 0.5m \cdot v_{c_5}^2 + 0.5I_5' \cdot \dot{\varphi}^2.$$

where I'_5– moment of imbalance inertia 5 relative to the axis passing through its center of mass parallel to the axis of rotation of the debalancing shaft 4.

Vector $\vec{v}_{C_5}$ of absolute velocity of the imbalance center of mass 5

$$\vec{v}_{C_5} = \vec{v}_{C_5e} + \vec{v}_{C_5r}, \quad (6)$$

Since portable movement is free, then

$$\vec{v}_{C_5e} = \vec{v}_{C_5e1} + \vec{v}_{C_5e2}, \quad (7)$$

where $\vec{v}_{C_5e1}$ – pole velocity;
$\vec{v}_{C_5e2}$– center of mass imbalance velocity 5 in its spherical motion around the pole.

To simplify rather complex and cumbersome calculations, we apply a coordinate method for determining motion and its kinematic characteristics.

Since a point is chosen as the pole C_3, then

$$\vec{v}_{C_5e_1} = \vec{v}_{C_3} = \vec{i}\,(\dot{x}_c - \dot{\alpha} \cdot l \cdot cos\alpha) + \\ + \vec{j}(\dot{y}_c + \dot{\beta} \cdot l \cdot cos\beta) + \\ + \vec{k}\,(\dot{z}_c + \dot{\alpha} \cdot l \cdot sin\alpha + \dot{\beta} \cdot l \cdot sin\beta) ,$$

To find $\vec{v}_{C_5e2}$ we will use vibration angles and for clarity we will depict schematically in the figure 6 the imbalance *5* center of inertia C_5 in three orthogonal projections, where $e = C_3C_5$ – its eccentricity.

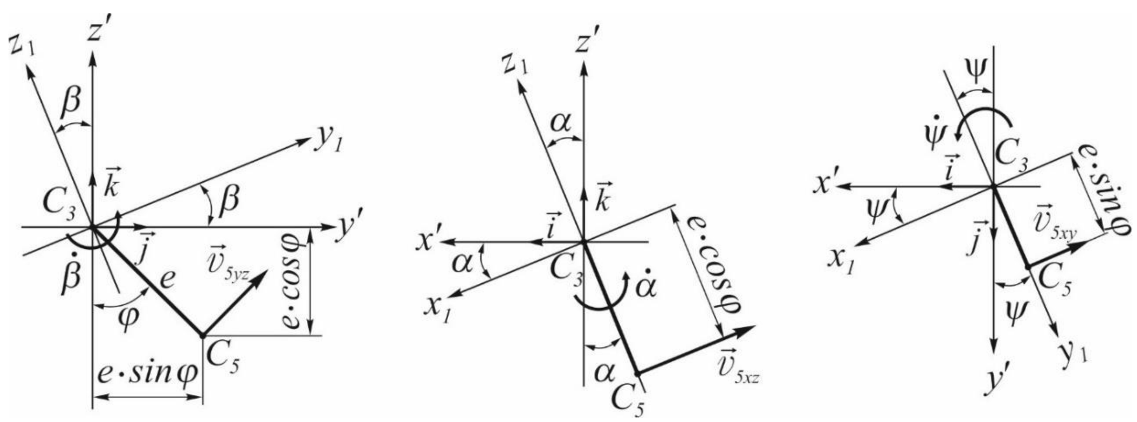

Figure 6 – Decomposition $\vec{v}_{C_5e2}$ into three components

Let's write now $\vec{v}_{C_5e2}$ as the sum of three terms:

$$\vec{v}_{C_5e2} = \vec{v}_{5xy} + \vec{v}_{5xz} + \vec{v}_{5yz}, \qquad (8)$$

where $\vec{v}_{5xy}$, $\vec{v}_{5xz}$ and $\vec{v}_{5yz}$ – velocity vector $\vec{v}_{C_5e2}$ projections to coordinate planes *xy*, *yz* and *zx*.

From Figure 6 it is obvious that the velocity modules

$$v_{5yz} = \dot{\beta} \cdot e, \quad v_{5xz} = \dot{\alpha} \cdot e \cdot \cos\phi \quad \text{and} \quad v_{5xy} = \dot{\psi} \cdot e \cdot \sin\phi,$$

and the corresponding projections on the specified planes

$$\vec{v}_{5yz} = \vec{j} \cdot v_{5yz} \cdot \cos\phi + \vec{k} \cdot v_{5yz} \cdot \sin\phi = \vec{j} \cdot \dot{\beta} \cdot e \cdot \cos\phi + \vec{k} \cdot \dot{\beta} \cdot e \cdot \sin\phi,$$

$$\vec{v}_{5xz} = -\vec{\iota} \cdot v_{5xz}\cos\alpha + \vec{k} \cdot v_{5xz}\sin\alpha = -\vec{\iota} \cdot \dot{\alpha} \cdot e\cos\phi\cos\alpha + \vec{k} \cdot \dot{\alpha} \cdot e\cos\phi\sin\alpha,$$

$$\vec{v}_{5xy} = -\vec{\iota} \cdot v_{5xy}\cos\psi - \vec{j} \cdot v_{5xy}\sin\psi = -\vec{\iota} \cdot \dot{\psi} \cdot e\sin\phi\cos\psi - \vec{j} \cdot \dot{\psi} \cdot e\sin\phi\sin\psi.$$

Substituting these values in formula (8), and the values of $\vec{v}_{C_5e1}$ and $\vec{v}_{C_5e2}$ in formula (7), we have

$$\vec{v}_{C_5e} = \vec{\iota} \cdot \left(\dot{x}_C - \dot{\alpha} \cdot \ell \cdot \cos\alpha - \dot{\alpha} \cdot e \cdot \cos\phi \cdot \cos\alpha - \dot{\psi} \cdot e \cdot \sin\phi \cdot \cos\psi\right) +$$
$$+\vec{j} \cdot \left[\dot{y}_C + \dot{\beta} \cdot \ell \cdot \cos\beta + \dot{\beta} \cdot e \cdot \cos\phi - \dot{\psi} \cdot e \cdot \sin\phi \cdot \sin\psi\right] +$$
$$+\vec{k} \cdot \left[\dot{z}_C + \dot{\alpha} \cdot \ell \cdot \sin\alpha + \dot{\alpha} \cdot e \cdot \cos\phi \cdot \sin\alpha + \dot{\beta} \cdot \ell \cdot \sin\beta + \dot{\beta} \cdot e \cdot \sin\phi\right].$$

Since the relative motion of the imbalance is rotational, then (see Fig. 7) the module $v_{C_5r} = C_3C_5 \cdot \omega = e \cdot \dot{\phi}$, and a vector

$$\vec{v}_{C_5r} = \vec{j} \cdot v_{C_5r} \cdot \cos\phi + \vec{k} \cdot v_{C_5r} \cdot \sin\phi = \vec{j} \cdot \dot{\phi} \cdot e \cdot \cos\phi + \vec{k} \cdot \dot{\phi} \cdot e \cdot \sin\phi.$$

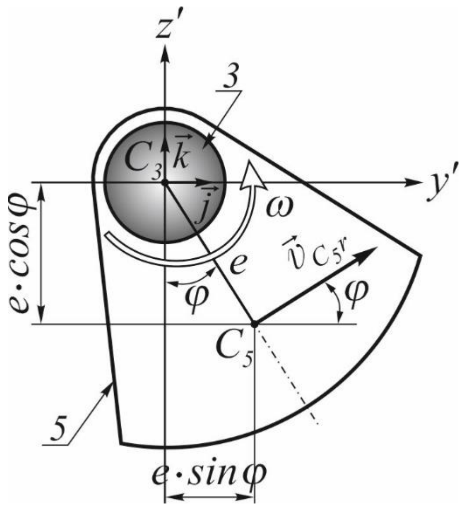

Fig. 7. To determine the relative speed $\vec{v}_{C_5r}$ of an imbalance

After substitution $\vec{v}_{C_5e}$ and $\vec{v}_{C_5r}$ into formula (6), squaring and substituting the value $v_{C_5}^2$ into the formula of kinetic energy T_5, we get

$$T_5 = \frac{m}{2} \cdot (\dot{x}_C^2 + \dot{y}_C^2 + \dot{z}_C^2) -$$
$$-m \cdot \ell \cdot \dot{x}_C \cdot \dot{\alpha} \cdot \cos\alpha - m \cdot e \cdot \dot{x}_C \cdot \dot{\alpha} \cdot \cos\alpha \cdot \cos\phi - m \cdot e \cdot \dot{x}_C \cdot \dot{\psi} \cdot \cos\psi \cdot \sin\phi +$$
$$+m \cdot \ell \cdot \dot{y}_C \cdot \dot{\beta} \cdot \cos\beta + m \cdot e \cdot \dot{y}_C \cdot \dot{\beta} \cdot \cos\phi -$$
$$-m \cdot e \cdot \dot{y}_C \cdot \dot{\psi} \cdot \sin\psi \cdot \sin\phi + m \cdot e \cdot \dot{y}_C \cdot \dot{\phi} \cdot \cos\phi +$$
$$+m \cdot \ell \cdot \dot{z}_C \cdot \dot{\alpha} \cdot \sin\alpha + m \cdot \ell \cdot \dot{z}_C \cdot \dot{\beta} \cdot \sin\beta +$$
$$+m \cdot e \cdot \dot{z}_C \cdot \dot{\alpha} \cdot \sin\alpha \cdot \cos\phi + m \cdot e \cdot \dot{z}_C \cdot \dot{\beta} \cdot \sin\phi + m \cdot e \cdot \dot{z}_C \cdot \dot{\phi} \cdot \sin\phi -$$
$$-\frac{m}{2} \cdot \ell^2 \cdot \dot{\alpha}^2 \cdot \cos2\alpha + m \cdot \ell \cdot e \cdot \dot{\alpha}^2 \cdot \cos\phi - \frac{m}{2} \cdot e^2 \cdot \dot{\alpha}^2 \cdot \cos^2\phi \cdot \cos2\alpha +$$
$$+\frac{m}{2} \cdot \ell^2 \cdot \dot{\beta}^2 + m \cdot \ell \cdot e \cdot \dot{\beta}^2 \cdot \cos(\beta - \phi) + \frac{m}{2} \cdot e^2 \cdot \dot{\beta}^2 - \frac{m}{2} \cdot e^2 \cdot \dot{\psi}^2 \cdot \sin^2\phi +$$
$$+m \cdot \ell^2 \cdot \dot{\alpha} \cdot \dot{\beta} \cdot \sin\alpha \cdot \sin\beta + m \cdot \ell \cdot e \cdot \dot{\alpha} \cdot \dot{\beta} \cdot \sin\alpha \cdot \sin\beta \cdot \cos\phi +$$
$$+m \cdot \ell \cdot e \cdot \dot{\alpha} \cdot \dot{\beta} \cdot \sin\alpha \cdot \sin\phi + m \cdot \ell \cdot e \cdot \dot{\alpha} \cdot \dot{\psi} \cdot \cos\alpha \cdot \cos\psi \cdot \sin\phi +$$
$$+m \cdot \ell \cdot e \cdot \dot{\alpha} \cdot \dot{\phi} \cdot \sin\alpha \cdot \sin\phi + \frac{m}{2} \cdot e^2 \cdot \dot{\alpha} \cdot \dot{\beta} \cdot \sin\alpha \cdot \sin2\phi +$$
$$+\frac{m}{2} \cdot e^2 \cdot \dot{\alpha} \cdot \dot{\psi} \cdot \cos\alpha \cdot \cos\psi \cdot \sin2\phi + \frac{m}{2} \cdot e^2 \cdot \dot{\alpha} \cdot \dot{\phi} \cdot \sin\alpha \cdot \sin2\phi -$$
$$-m \cdot \ell \cdot e \cdot \dot{\beta} \cdot \dot{\psi} \cdot \cos\beta \cdot \sin\psi \cdot \sin\phi + m \cdot \ell \cdot e \cdot \dot{\beta} \cdot \dot{\phi} \cdot \cos(\beta - \phi) -$$

$$-\frac{m}{2}\cdot e^2\cdot\dot\beta\cdot\dot\psi\cdot\sin\psi\cdot\sin2\phi+m\cdot e^2\cdot\dot\beta\cdot\dot\phi-$$

$$-\frac{m}{2}\cdot e^2\cdot\dot\psi\cdot\dot\phi\cdot\sin\psi\cdot\sin2\phi+\frac{I_5\cdot\dot\phi^2}{2}. \qquad (9)$$

Substituting values (3), (4), (5) and (9) to the formula (2), and having performed legitimate transformations, we get that the kinetic energy of the vibrating table

$$T=\frac{M}{2}\cdot\dot x_C^2+\frac{M}{2}\cdot\dot y_C^2+\frac{M}{2}\cdot\dot z_C^2-$$
$$-[(m_3+m)\cdot\ell+m_6\cdot\ell_6]\cdot\dot x_C\cdot\dot\alpha\cdot\cos\alpha-m\cdot e\cdot\dot x_C\cdot\dot\alpha\cdot\cos\alpha\cdot\cos\phi-$$
$$-m\cdot e\cdot\dot x_C\cdot\dot\psi\cdot\cos\psi\cdot\sin\phi+[(m_3+m)\cdot\ell-m_6\cdot\ell_6]\cdot\dot y_C\cdot\dot\beta\cdot\cos\beta+$$
$$+m\cdot e\cdot\dot y_C\cdot\dot\beta\cdot\cos\varphi-m\cdot e\cdot\dot y_C\cdot\dot\psi\cdot\sin\psi\cdot\sin\varphi+$$
$$+m\cdot e\cdot\dot y_C\cdot\dot\phi\cdot\cos\phi+[(m_3+m)\cdot\ell+m_6\cdot\ell_6]\cdot\dot z_C\cdot\dot\alpha\cdot\sin\alpha+$$
$$+m\cdot e\cdot\dot z_C\cdot\dot\alpha\cdot\sin\alpha\cdot\cos\phi+[(m_3+m)\cdot\ell-m_6\cdot\ell_6]\cdot\dot z_C\cdot\dot\beta\cdot\sin\beta+$$
$$+m\cdot e\cdot\dot z_C\cdot\dot\beta\cdot\sin\phi+m\cdot e\cdot\dot z_C\cdot\dot\phi\cdot\sin\phi+\frac{m_3\cdot\ell^2+m_6\cdot\ell_6^2}{2}\cdot\dot\alpha^2-$$
$$-\frac{m\cdot\ell^2}{2}\cdot\dot\alpha^2\cdot\cos2\alpha+m\cdot\ell\cdot e\cdot\dot\alpha^2\cdot\cos\phi-\frac{m\cdot e^2}{2}\cdot\dot\alpha^2\cdot\cos^2\phi\cdot\cos2\alpha+$$
$$+\frac{(m_3+m)\cdot\ell^2+m_6\cdot\ell_6^2+m\cdot e^2}{2}\cdot\dot\beta^2+m\cdot\ell\cdot e\cdot\dot\beta^2\cdot\cos(\beta-\phi)-$$
$$-\frac{m\cdot e^2}{2}\cdot\dot\psi^2\cdot\sin^2\phi+[(m_3+m)\cdot\ell^2-m_6\cdot\ell_6^2]\cdot\dot\alpha\cdot\dot\beta\cdot\sin\alpha\cdot\sin\beta+$$
$$+m\cdot\ell\cdot e\cdot\dot\alpha\cdot\dot\beta\cdot\sin\alpha\cdot\sin\beta\cdot\cos\phi+$$
$$+m\cdot\ell\cdot e\cdot\dot\alpha\cdot\dot\beta\cdot\sin\alpha\cdot\sin\phi+m\cdot\ell\cdot e\cdot\dot\alpha\cdot\dot\psi\cdot\cos\alpha\cdot\cos\psi\cdot\sin\phi+$$
$$+m\cdot\ell\cdot e\cdot\dot\alpha\cdot\dot\phi\cdot\sin\alpha\cdot\sin\phi+\frac{m\cdot e^2}{2}\cdot\dot\alpha\cdot\dot\beta\cdot\sin\alpha\cdot\sin2\phi+$$
$$+\frac{m\cdot e^2}{2}\cdot\dot\alpha\cdot\dot\psi\cdot\cos\alpha\cdot\cos\psi\cdot\sin2\phi+\frac{m\cdot e^2}{2}\cdot\dot\alpha\cdot\dot\phi\cdot\sin\alpha\cdot\sin2\phi-$$
$$-m\cdot\ell\cdot e\cdot\dot\beta\cdot\dot\psi\cdot\cos\beta\cdot\sin\psi\cdot\sin\phi+m\cdot\ell\cdot e\cdot\dot\beta\cdot\dot\phi\cdot\cos(\beta-\phi)-$$
$$-\frac{m\cdot e^2}{2}\cdot\dot\psi\cdot\dot\varphi\cdot\sin\psi\cdot\sin2\varphi+\frac{I_y}{2}\cdot\dot\alpha^2+\frac{I_x}{2}\cdot\dot\beta^2+\frac{I_z}{2}\cdot\dot\psi^2+\frac{I_5}{2}\cdot\dot\varphi^2. \qquad (10)$$

where $M=m_1+m_3+m_6+m$ – the total weight of the vibrating table;

$I_x=I_{1x_1}+I_{3x_3}+I_{6x_6}$, $I_y=I_{1y_1}+I_{3y_3}+I_{6y_6}$ and $I_z=I_{1z_1}+I_{3z_3}+I_{6z_6}$ – reduced moving part inertia moments of the table relative to the corresponding axes.

As an example of the application of equation (10), we construct a graph of the change in kinetic energy T of vibrating table under study, which it is able to reproduce depending on the length of the lever $\ell_{\text{важ.}}$, to which the vibration alarm is fixed in the center under the vibrating plate. For construction, the mathematical program "Mathcad Prime" was used and taking into account the nominal numerical values of the greatness of the parameters of the vibrating table, which are included in the equation (10) (See. Table. 1).

Table 1 – Nominal parameters of the vibrating table.

№	Parameter name	Denomination	Unit of measure	Size
1.	Mass of vibrating plate	m_1	kg	170
2.	Mass of the excitor	m_3	kg	10
3.	Mass of imbalance	m	kg	2
4.	Mass of load with form	m_6	kg	50
5.	Imbalance eccentricity	e	m	0,03

A vibration excitor ИВ-99БУ2 was used (power N=0,5 Kw, frequency 50 Hz, rotational speed n = 3000 rpm, acting force P =2,5 KN), which was fixed to the vibrating plate in size $2a\times 2b$ = 1,6 m ×1,3 m, thru the lever. Lever length $\ell_{\text{важ.}}$ was accepted equal accordingly 0; 100; 200; 300 mm based on constructive images.

The plot is shown in Fig. 8. The dependence curve increases, which confirms the effectiveness of the lever fastening of the vibrating device to the vibrating table.

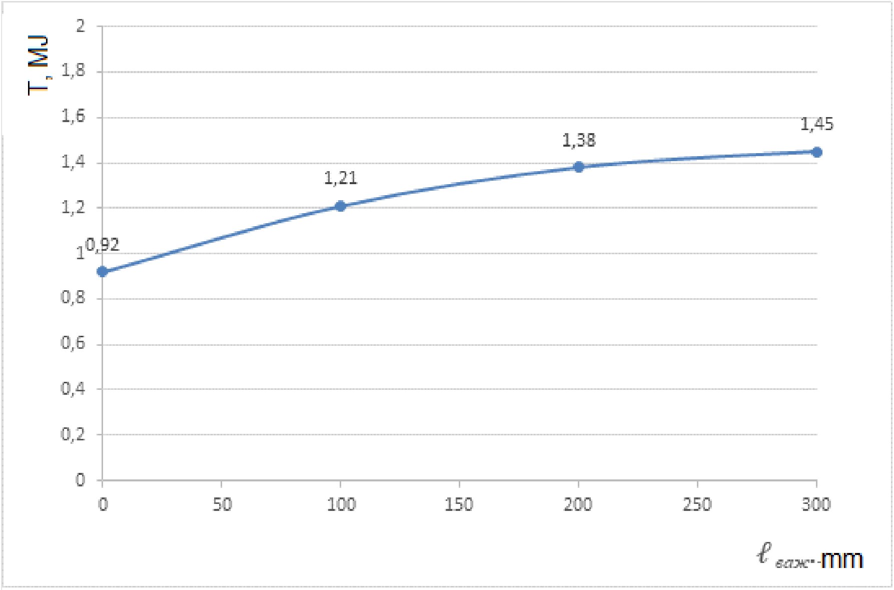

Fig 8. Graph of the change in the kinetic energy of the vibrating table T, which it is able to introduce when compacting concrete products, depending on the length of the lever $\ell_{важ.}$

Conclusions

To obtain a mathematical model of the developed design of the vibrating table, we proposed to use the Lagrange equation of the second kind. This method is the most common method used in solving problems regarding the motion of mechanical systems.

The considered vibrating table was modeled by a mechanical system, which consists of several material bodies - plate, vibration exciter body, imbalance and tank with concrete mix. To determine the position and description of the mechanical system above-mentioned material bodies free motions under consideration, an orthogonal vibration reference system from three coordinate systems was used.

The total kinetic energy of the vibrating table was determined, which is the sum of the kinetic energy of the four material bodies that make up it.

This functional dependence of the total kinetic energy will later be used to compile a mathematical model of the vibration table in the Lagrange equations of the second kind, with the help of which it will be possible to analyze the influence of its constituent parameters – geometric and kinematic – on the process of compacting the concrete mix to reduce energy consumption during vibration compaction of products.

As an example of the application of functional dependence, a graph of changes in the kinetic energy of the investigated vibration table depending on the length of the lever was constructed, on which the vibration excitor was fixed. Analysis of the obtained dependence indicates an increase in the kinematic energy of the vibrating table with increasing lever length. The lever made it possible to create conditions for successful power transfer from the vibration excitor to the table without unforeseen losses.

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