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VERIFICATION OF THE PIPELINE DEFORMATIONAL MODEL IN NON-STANDARD SOIL CONDITIONS

This paper discusses existing methods for evaluating longitudinal stresses in the pipeline caused by uneven deformations of soil base; which is composed by soils with special properties. Verification of the pipeline design scheme have been performed. In research we based on the results of the previous experimental and theoretical investigations, data of the engineering inspections of the existing main pipeline above ground crossing. Comparison of the longitudinal stresses calculated by different methods have been done. Hypothesis that pipeline deformations are equal to the deformations of the soil base is confirmed. With help of numerical modeling by finite element method we obtained differential settlements of the loessial collapsible strata; which occur during the soaking of the soil local area. Respective longitudinal stresses have been calculated.

***Keywords**: model verification, pipeline above ground crossing, karst cavities, loessial collapsible soil basis, pipeline stiffness, external loads and influences, longitudinal stresses.*

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ВЕРИФІКАЦІЯ МОДЕЛІ ДЕФОРМАЦІЇ ТРУБОПРОВОДУ В ОСОБЛИВИХ ҐРУНТОВИХ УМОВАХ

Проаналізовано існуючі методи розрахунку поздовжніх напружень у трубопроводі, які виникають у результаті впливу нерівномірних деформацій основи, складеної ґрунтами з особливими властивостями. Проведено верифікацію розрахункової моделі, відштовхуючись від результатів попередніх експериментальних і теоретичних досліджень, даних інженерних обстежень діючих конструкцій магістрального нафтопроводу. Порівняно поздовжні напруження у трубопроводі обчислені різними методами. Доведено коректність гіпотези, що деформації трубопроводу еквівалентні деформаціям ґрунту під трубою. Шляхом чисельного моделювання методом скінченних елементів отримано нерівномірні деформації лесової просадочної товщі при замоканні локальної ділянки та відповідні напруження у стінках трубопроводу.

***Ключові слова:** верифікація моделі, надземний перехід, карстова воронка, лесова просадочна основа, жорсткість трубопроводу, зовнішні навантаження і впливи, поздовжні напруження.*

Introduction. Hoop σ_h, longitudinal σ_l and radial σ_{rad} stresses make impact in the main pipeline linear part (MPLP). Radial stresses have relatively small values in the thin-walled high-pressure pipelines, so it used do not take into account [1 – 4]. Hoop stress are calculated as follow

$$\sigma_h = \frac{nPD_{in}}{2t}, \tag{1}$$

where P – internal operating pressure in the pipeline;

n – the design (usage) factor for operating pressure [2 – 4];

D_{in} – pipeline internal diameter;

t – pipeline wall thickness.

Calculation of the pipeline wall thickness is almost the same for different codes. The hoop stress σ_h criterion limits the characteristic tensile hoop stress, according to the pipeline steel Specified Minimum Yield Strength (*SMYS*) with accounting of the design (usage) factors, which values are specific for each code. Should be noted, that *SMYS* is equal to the steel yield resistance R_y.

$$\sigma_h \le \gamma_i \cdot SMYS, \tag{2}$$

where γ_i – the design (usage) factor specific for each code [2 – 4].

Longitudinal stress σ_l value in the MPLP is determined by three main factors: operating pressure P, influence of the temperature deformations and stresses, which caused by MPLP curvature

$$\sigma_l = \mu\sigma_h \pm \alpha E_p \Delta t \pm \sigma_{bend}, \tag{3}$$

where μ – Poisson's ratio of the pipe steel;

α – linear expansion factor of metal pipes;

E_p – pipe steel Young's modulus;

Δt – calculating temperature difference, which is extremal difference between MPLP wall temperature during the exploitation and in the moment when pipeline design scheme fixing;

σ_{bend} – bending stress in the MPLP.

Bending stress in the MPLP σ_{bend} is composed of stresses caused by elastic bend of the pipeline sections (MPLP follows to the terrain relief) and by stresses caused by differential settlements of the MPLP soil base σ_{dif}

$$\sigma_{bend} = \pm\frac{E_p D_{ex}}{2\rho} \pm \sigma_{dif}, \tag{4}$$

where D_{ex} – pipeline external diameter;

ρ – pipeline axis curvature radius, which maximal values for each diameter are substantiate in the codes [4].

Analysis of recent sources of research and publications. MPLP soil basis differential settlements lead to additional longitudinal stresses in the pipeline walls, destruction of anti-corrosion coating, which significantly reduces pipeline durability [5 – 9]. In addition, MPLP large deflection may cause violation in the operating condition, which again confirms necessity of the different settlements regulation.

Large values of the MPLP differential settlements is typical for pipeline laying in non-standard soil conditions. Non-standard soil conditions it is when pipeline layer designed in areas with the following characteristic features [7, 8]: swamp or flooded areas, areas with underground cavities of various nature (mining and mine construction zones, areas with karst cavities, etc.), thawing permafrost areas, landslide territories, seismic zones.

For the Ukraine loessial collapsible soils is one of the most common problem, because such soil occupy 65 – 70% of the territory. Such problem is especially urgent for the southern region, where loessial layer reaches 45…50 m, and the value of the soil collapse from its own weight may occur 1…2 m [10].

Estimation of the stresses caused by differential settlements is quite controversial tasks. Some researchers deny importance of stresses caused by soil deformations in the vertical plane down. Such stresses do not make impact on the pipeline strength according to that works [11, 12]. Different sources give widely various range of the pipeline stresses in the quite similar soil conditions. It is because one problem solving even in the different design scheme. According to the existing engineering experience it is possible to distinguish most common models:

– pipeline as beam on the elastic Winkler's base [5, 6, 11, 12]. Its advantage is relative simplicity.

– analyzing of the boundary cases. Soil is completely absent under the pipeline [7].

– analyzing of the pipeline Stress Strain State (SSS) with including of whole range of factors which are impact on the pipeline, accounting of the physic-mechanical and geometry system features [13, 14].

Estimation of the MPLP longitudinal stresses σ_{dif}, which are caused exactly from loessial soil collapsible deformations, is almost unexplored. But a lot of works dedicated to problems of the pipeline in the karst cavities areas [7] and thawing permafrost have [8].

Pipeline in the area of the karst cavity problem solving shows that most important factors are: cavity dimensions and stiffness (deformation modulus E_s) of the adjacent soil. For example, calculation of the follow system had been performed: pipeline 1420×16,5 mm, pipe deformation modulus $E_p = 2.1 \cdot 10^5$ MPa, Poisson's ratio $\mu = 0{,}3$, yield and ultimate resistance $R_y = 470$ MPa, $R_u = 600$ MPa. Increasing of the linear load caused by soil water saturation. Analytical solving results [7] are given in Table. 1.

Thus, longitudinal stresses σ_{dif} is quite valuable, and it is even comparable with hoop stresses σ_h, which are caused from operating pressure. Main disadvantage of approach presented in [7] is complexity of analytical equations, which are extremely difficult to use to conventional engineer. Obtained stresses and pipeline deflection we can consider as a benchmark, boundary case for the pipeline base differential settlements problem.

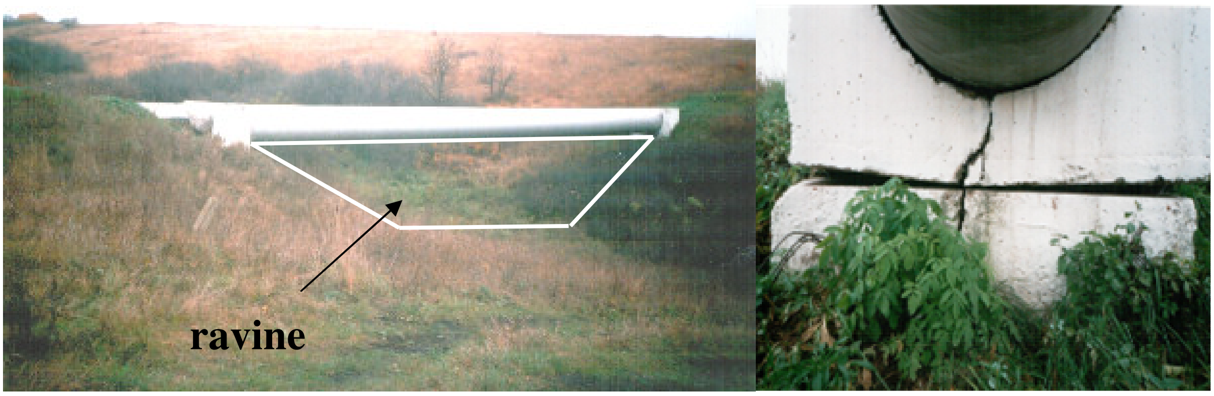
Highlight unsolved parts of the general problem. Hoop, temperature and stresses caused by elastic bending are sufficiently analyzed in the Ukrainian and international codes. It has analytical equations and simplified expressions for determination of their values. Stresses caused by differential settlements of the MPLP soil base σ_{dif} haven't such equations. In addition, existing normative documents haven't any specific recommendations for creation of the pipeline deformational deterministic models in the non-standard soil conditions, including collapsible loessial soils.

Limit values of the MPLP soil basis different settlements are also not regulated. Instead of Ukrainian and USA, Europe codes [3] recommend limit value of the soil basis different settlement that is 10 cm on the wavelength 40 m.

Formulation of the problem. With help of numerical modelling of the pipeline in the different soil conditions estimate longitudinal stresses that occur during the soil base local area soaking.

Model verification. Application of the modern software is advisable for correct calculation of stresses caused by pipeline soil base differential settlements. However, very often, obtained results are very complicated for estimation. We conducted a verification of our design scheme. In research we based on the results of the previous experimental and theoretical investigations [7], data of the engineering inspections of the existing main pipeline above ground crossing.

Oil pipeline above ground crossing over the ravine is considering. Ravine length is – 54 m, length between concrete supports – 33 m (Pic. 1). Concrete supports based on the humified clay loam. Crossing entered into operation in 1977, its physical and geometrical characteristics 1220×15,2 mm, pipeline steel 17Г1С, yield and ultimate resistance $R_y = 470$ MPa, $R_u = 600$ MPa. Operating pressure $P = 6{,}2$ MPa.



a b

Picture 1 – Oil pipeline above ground crossing of the ravine:
a – general view; b – destruction of concrete support

Design scheme of the pipeline above ground crossing – one-beam that doesn't have special devices for compensation of deformation elongation (contraction). Oil pipeline accepts load of its own weight and the weight of the transported product, the total linear load:

$$q = q_{pipe} + q_{prod} = 4{,}82 + 11{,}45 = 16{,}27 \text{ kN/m}.$$

Next values have been obtained in the result of the calculation according to the engineering methodic: pipeline deflection (5), longitudinal stresses from deflection (6), stresses from thrust (7), total longitudinal stresses (7):

$$f = \frac{l^2}{4} \cdot \sqrt{(a\gamma)/2E_p D_{in}} = \frac{33^2}{4} \cdot \sqrt{(3{,}38 \cdot 7{,}85)/2 \cdot 2{,}1 \cdot 10^7 \cdot 1{,}22} = 0{,}198 \text{ m}, \quad (5)$$

where l – above ground crossing length;
a – ratio of the additional to the pipeline own weight;
γ – steel unit weight.
Stresses from deflection:

$$\sigma_{bend} = \frac{ql^2}{12 \cdot W} = \frac{16{,}27 \cdot 33^2}{12 \cdot 16920 \cdot 10^{-6}} = 87{,}2 \text{ MPa}, \quad (6)$$

where W – pipeline cross-section resistance moment.
Stresses from thrust:

$$\sigma_{thrust} = \frac{ql^2}{8 \cdot f \cdot F} = \frac{16{,}27 \cdot 33^2}{8 \cdot 0{,}198 \cdot 568 \cdot 10^{-4}} = 196{,}9 \text{ MPa}, \quad (7)$$

where F – pipeline cross-section area.

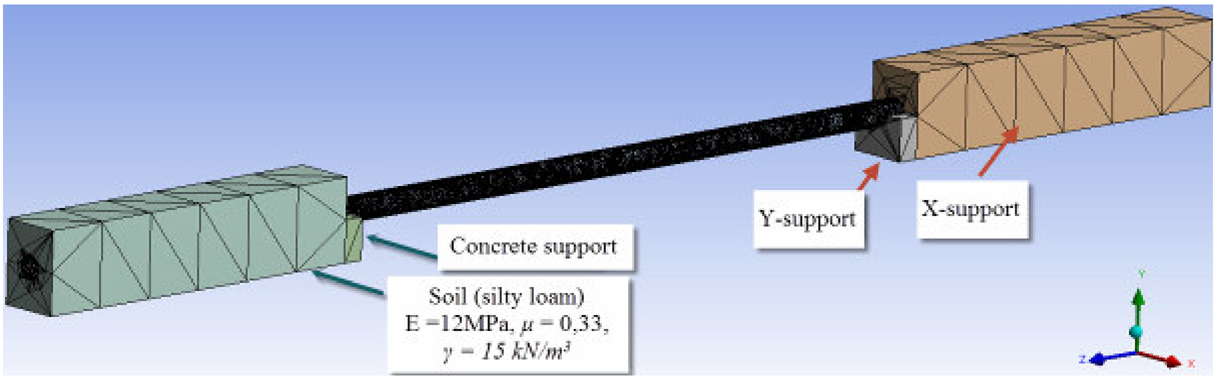
$$\sigma_{tot} = \sigma_{bend} + \sigma_{thrust} = 87{,}2 + 196{,}9 = 284{,}1 \text{ MPa}, \quad (8)$$

The actual measured deflection was f = 0,084 m. Therefore, calculation of the thrust stresses is looking incorrect.

We propose to consider following design scheme (Pic. 2). The length of the calculation area – 71 m, the length of the free span – 33 m, the width of the concrete support –1 m. Pipeline in the soil area length is 18 m. Soil massive width – 4 m. Materials linear models were used in the calculation, because in the all elements of the model stresses don't exceed yield limit. It is possible to calculate in the elastic phase. Last principle allows ignoring soil mechanical strength characteristics, such as cohesion and friction angle.

Soil base characteristics are follow unit weight γ = 15 kN/m^3, deformation modulus E_s = 12 MPa, Poisson's ratio μ = 0,33. The load is represented by earth gravity (pipeline own weight), weight of the transported product, total load

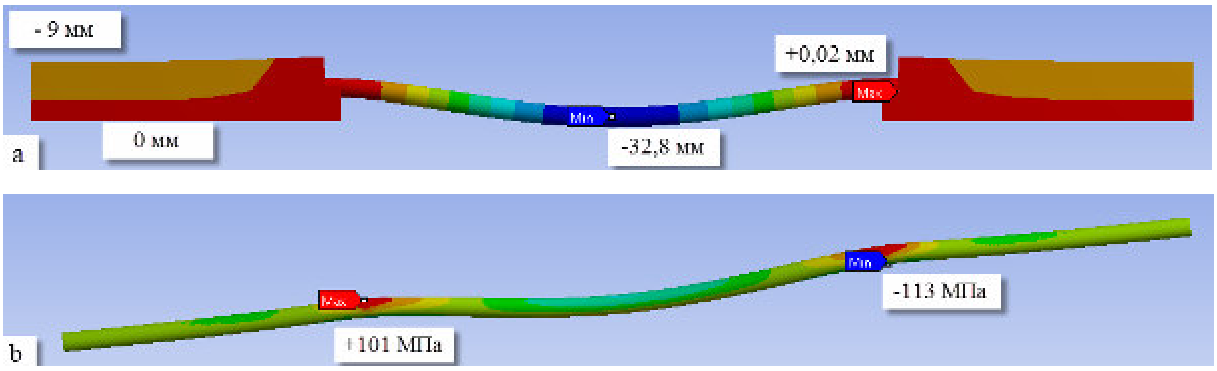
$$p_{prod} = \frac{q_{prod}}{\pi \cdot D_{ex}} \cdot D_{ex} = \frac{11,45}{3,14} = 3,65 \ kN/m^2 .$$



Picture 2 – Oil pipeline above ground crossing of the ravine design scheme

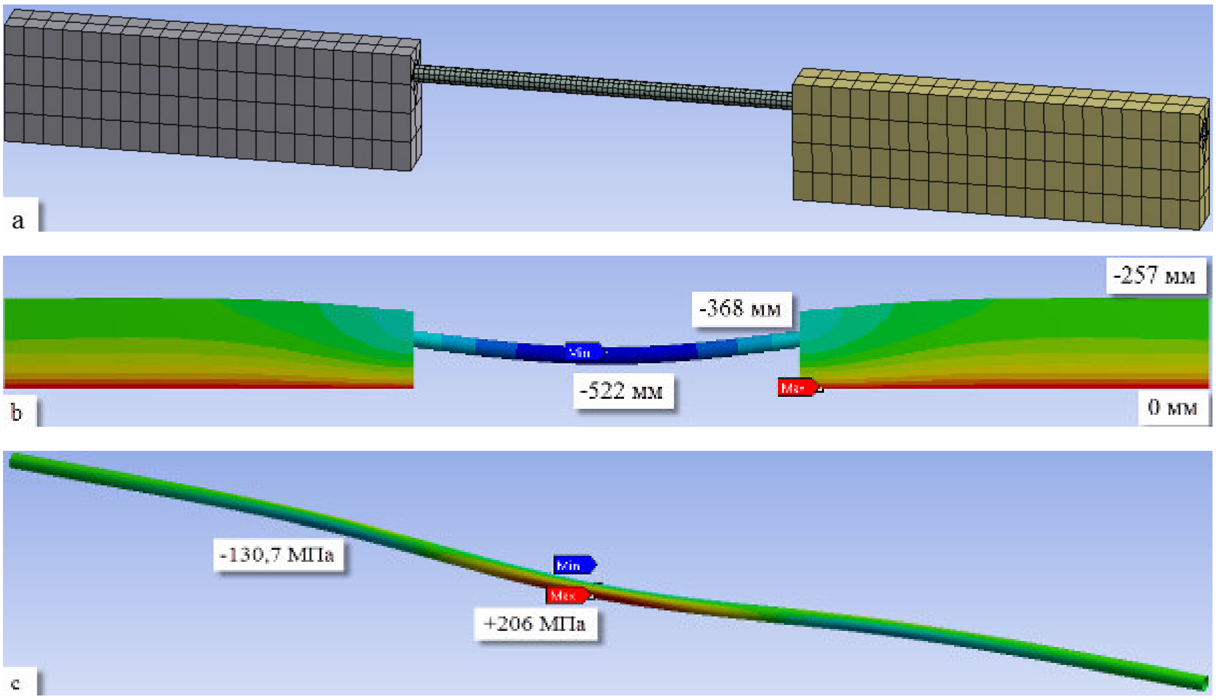
It should be noted, that concrete supports are rigid, its deformation modulus is E_c = 30 GPa, therefore very important that FE mesh generated that nodes of all elements coincided in the same points. Last principle is important to correct estimation of the contact stresses on the border of the pipeline with concrete supports (Pic. 3, b).

Modeling results are follow pipeline deflection f = 0,0328 m. A significant difference in the values obtained during modelling and real measured deflection may be explained by the 40 years of operation, unknown pipeline deflection during its construction, but modelling results looks much more correct than calculated according (5). Because modelling results lower then real measured deflection, conclusion could be done that pipeline has some overstated stiffness. Pipeline longitudinal stresses on the support -113 MPa (Pic. 3, b).



Picture 3 – Numerical modelling results:
a – pipeline deflection;
b – longitudinal stresses from deflection

Pipeline modeling in the area of the karst cavity presented in [7]. Soil massive height – 10 m, pipeline area in the soil – 34 m, pipeline free span (over the cavity) – 32 m. Fixing conditions are similar to the scheme (Pic. 2), but with adding support in Z-direction on the border of soil and cavity. Geometrical dimensions and physic-mechanical material properties, load values are similar to the example from [7].



Picture 4 – Determination of the stresses and deflection in the pipeline in the karst cavity area (cavity length l = 32 m): a – design scheme; b – settlements for case (E_s = 2 MPa); c – longitudinal stresses for case (E_s = 2 МПа)

Numerical modelling and analytical solution results are recorded in the Table. 1. Result analyze shows that stresses extremes and its distribution character are close enough for both variants. The tendency could be seen, that results closing with reducing of the surrounding soil stiffness. Analytical results were accepted for absolute values.

Table 1 – Comparison of the results obtained from analytical solving and numerical modelling of the pipeline in the karst cavity area (cavity length l = 32 m)

Results / Soil		Sand	Clay loam	Water saturated clay loam
Deformational modulus of the adjacent soil, E_s, MPa		48	20	2
Linear load on the pipeline	Analytical, q, kN/m	25,7	25,7	35,5
	Modelling, p, kN/m	8,18	8,18	11,3
Deformation in the span midpoint	Analytical, S, m	-0,084	-0,116	-0,512
	Modelling, S, m	-0,073	-0,108	-0,522
	Difference, Δ, %	-13,1	-6,9	-1,9
Stresses in the span midpoint	Analytical, σ_{dif}, MPa	107,6	121,8	250,4
	Modelling, σ_{dif}, MPa	83,1	96,2	206,0
	Difference, Δ, %	-22,8	-20,8	-17,7
Deformation on the border of cavity and soil	Analytical, S, m	+0,028	+0,003	-0,361
	Modelling, S, m	-0,02	-0,05	-0,368
	Difference, Δ, %	-	-	-
Maximal stresses on the border of cavity and soil	Analytical, σ_{dif}, MPa	-84,5	-85,6	-121,7
	Modelling, σ_{dif}, MPa	-101,8	-86,3	-130,7
	Difference, Δ, %	+20,9	+0,08	+6,9

Difference in the settlements values in the soil-cavity border areas is explained by the difference between models and the method of load application. In the analytical solution soil make reaction to the active pipeline pressure, for the numerical modelling (under earth gravity) soil partially deformed by its own weight, but it also creates reaction for pipeline pressure.

Performed verification shoved that result of the numerical simulation FEM is quite close to the results of the previous experimental and theoretical investigations and data of the engineering inspections of the existing main pipeline above ground crossing. Obtained difference doesn't exceed 22,8%, therefore FEM are correctly modelling pipeline and soil stiffness.

Loessial collapsible soil local soaking modelling. Our main purpose is estimation of the pipeline longitudinal stresses from the local area water saturation. In the most of the theoretical works [7] and normative documents [2, 4] hypothesis is used that pipeline deformations are equal to the soil base deformations. There are questions to the pipeline stiffness, is it enough strong to keep its position under soil collapsible deformations. Therefore, it make sense through the gradual increasing complexity of the design scheme of the beam above ground crossing to estimate the relevance of this hypothesis. Pipeline is laid in the follow soil conditions (Pic. 5), physic-mechanical soil properties presented in the Table 2.

Table 2 – Soil properties of the of Kremenchug loess plateau

<table>
<tr><th colspan="2" rowspan="2">Soil properties</th><th colspan="3">Numerical values</th></tr>
<tr><th>Strata 1</th><th>Strata 2</th><th>Strata 3</th></tr>
<tr><td colspan="2">Strata thickness, h, m</td><td>1,5</td><td>9,0</td><td>2,0</td></tr>
<tr><td colspan="2">Soil density, ρ, kg/m³</td><td>1500</td><td>1495</td><td>1860</td></tr>
<tr><td colspan="2">Dry density, ρ_d, kg/m³</td><td>-</td><td>1410</td><td>-</td></tr>
<tr><td colspan="2">Saturated soil density, ρ_{sat}, kg/m³</td><td>1840</td><td>1840</td><td>-</td></tr>
<tr><td colspan="2">Void ratio, e</td><td>-</td><td>0,90</td><td>0,7</td></tr>
<tr><td rowspan="4">Relative collapsibility, ε_{sl}, %, for pressure, P, MPa</td><td>0,05</td><td>-</td><td>0,3</td><td>-</td></tr>
<tr><td>0,10</td><td>-</td><td>3,0</td><td>-</td></tr>
<tr><td>0,20</td><td>-</td><td>6,0</td><td>-</td></tr>
<tr><td>0,30</td><td>-</td><td>8,0</td><td>-</td></tr>
<tr><td rowspan="2">Deformation modulus, E_s, MPa</td><td>natural conditon</td><td rowspan="2">6</td><td>12</td><td rowspan="2">14</td></tr>
<tr><td>water saturated state</td><td>2</td></tr>
<tr><td rowspan="2">Poisson ratio soil, μ</td><td>natural conditon</td><td>0,31</td><td>0,33</td><td>0,36</td></tr>
<tr><td>water saturated state</td><td>-</td><td>0,35</td><td>-</td></tr>
</table>

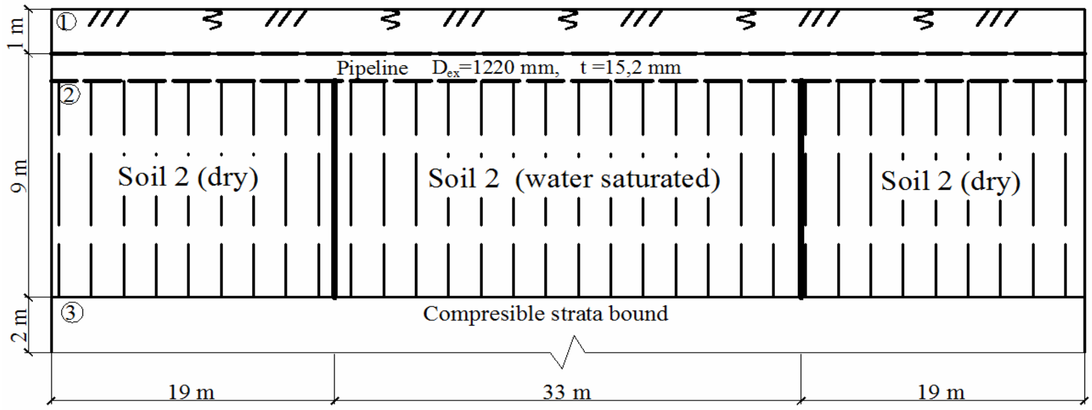
Very important observations were made in experimental work [15], limit value of the vertical load, that soil backfilling make on pipeline, depends from: pipeline diameter, trench width, and physic-mechanical soil properties. Presented feature most clearly expressed in clay soils, which are considered in our work. Thus linear load q on the pipeline is determined by the following formula [15]:

$$q = \gamma \cdot H \cdot \frac{B + D_{ex}}{2} = 18{,}4 \cdot 1 \cdot \frac{1{,}22 \cdot 1{,}5 + 1{,}22}{2} = 28 \text{ kN/m} \tag{9}$$

where γ – soil unit weight of the Strata 1 in water saturated state;

H – height of the back-filling over the pipe;

B – trench width, according [4] $B = 1{,}5 \cdot D_{ex}$.



Picture 5 – Pipeline in the loessial soil with collapsible property (central area in the water saturated state, side areas in the natural condition)

It make sense to compare result of the two design schemes: first – soil under pipeline is presence (Pic. 5); second – soil under pipeline is absent, design scheme is equal to the (Pic. 5), but without central water saturated part.

Linear model with deformational modulus of soil in the water saturated state can represent soil in the water saturated (Pic. 6, b). Modern material models, such as hyperelastic, are also appropriative to approximate diagram of the soil relative strain (Pic. 2,b) [16]. Last model advantage is nonlinear character of the deformations especially in the wide range of the pressure, which allow more accurately calculate soil deformations. But soil linear model allows to reduce calculating time compare with hyperelastic, without accuracy lost. According to the engineering methodic soil deformation value calculates as follow (10) [17]:

$$S_{s\ell g} = \sum_{n=1}^{n} \varepsilon_{s\ell.i} h_i k_{s\ell.i}, \tag{10}$$

where $\varepsilon_{s\ell.i}$ – relative strain of the Strata *i* element, from water saturated soil own weight average pressure;

h_i – thickness of the Strata *i* element;

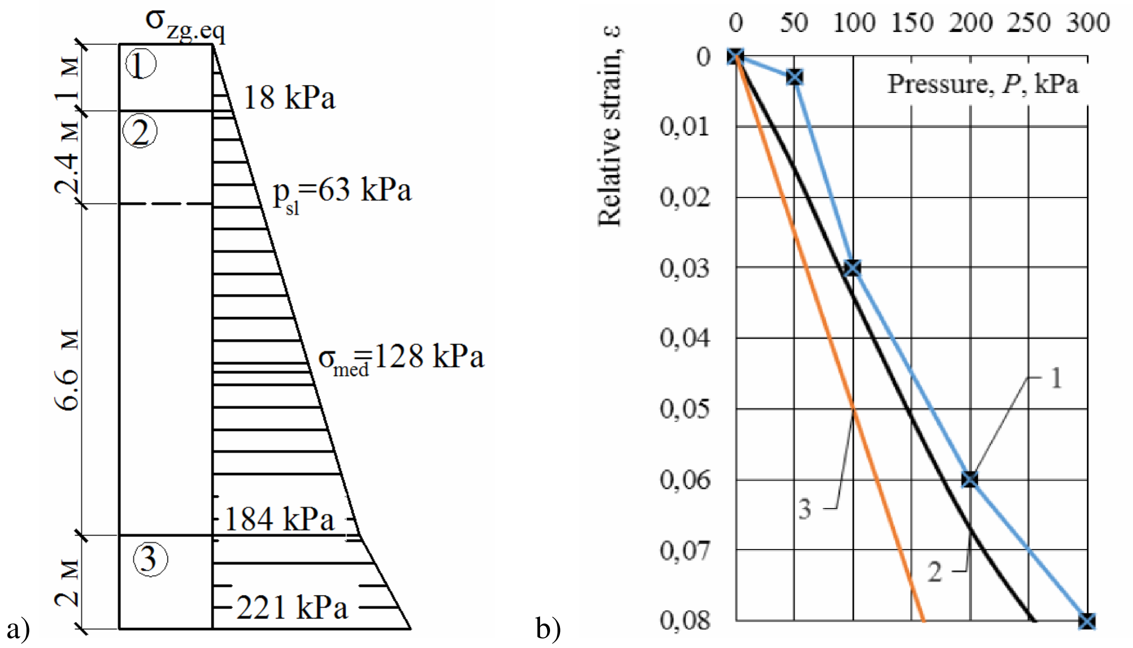
$k_{s\ell}$ – coefficient, which is for the loessial less than 15 m thickness is equal to 1.

Design scheme for estimation soil collapse value is on (Pic. 6, a): initial collapsible pressure $p_{s\ell}$= 63 kPa; pressure on the bound of the collapsible strata $\sigma_{zg.eq}$= 184 kPa, average pressure in the Strata 2 σ_{med}= 125 kPa, respective relative strain value calculates from the «pressure *P* – relative strain *ε*» diagram $\varepsilon_{s\ell}$ = 0,038. Collapsible strata thickness is h_{sl} = 6,6 m. According to the (10) absolute value of the soil collapse deformation from its own weigth is S_{slg} = 252 mm.

Both models of the loessial collapsible silty loam in results give appropriative result, but because linear model using allow to reduce calculation time and it is also more convenient for further probabilistic representation, therefore, we propose to use linear model.

According to numerical modeling soil collapse deformation is S_{slg} = 280 mm, which is quite close to the obtained result from engineering methodic S_{slg} = 252 mm. Therefore, with acceptable accuracy, soil is modeled with help of linear deformation diagram.

Should be noted, that for case with soil under the pipeline we consider the limiting case – design value of soil deformation modulus, we consider that water saturation value reach the ultimate value. Calculated values of the pipeline differential settlements for case without soil are lower then with soil (Pic. 7, a, c), but respective longitudinal stresses higher, which is indicating about larger correctness for scheme with soil under the pipe and the correctness of the hypothesis that pipeline deformations are equal to the soil base deformations.



Picture 6 – Calculation of soil collapsible deformations from its own weight:
a – design scheme for current soil conditions;
b – deformation properties of the water saturated Strata 2:
1 – dependence diagram pressure *P* – relative strain *ε*;
2 – Yeoh hyperelastic model diagram approximation;
3 – Strata 2 deformation modulus

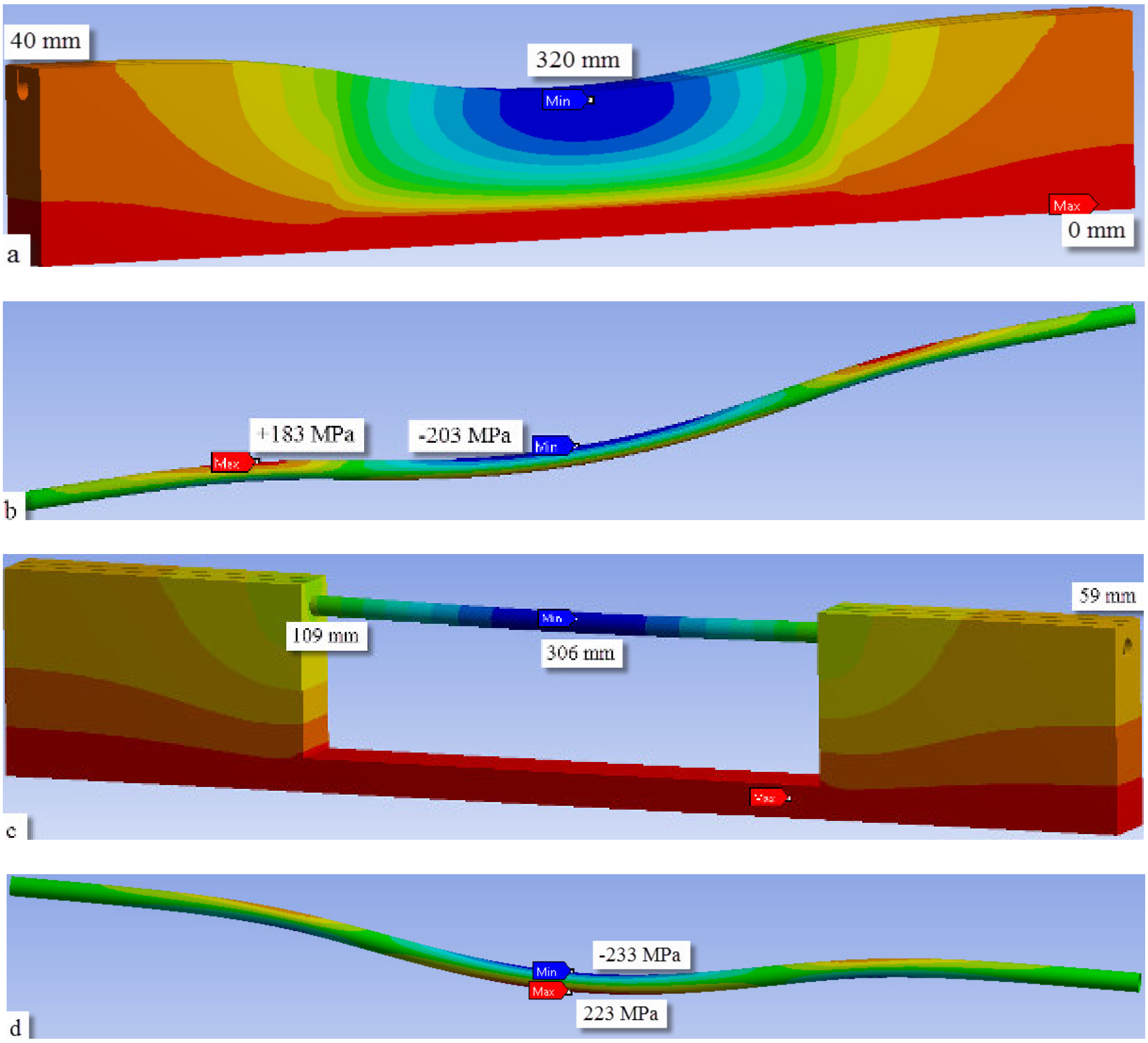
In the end should be noted, that obtained results should be seen as a single point in the space of possible pipeline stress-strain states. Much more comprehensive figuration may be obtained with help of probabilistic approach, which allow to estimate pipeline possible states under the action of various combinations of loads and influences.

Conclusions. Numerical FEM simulation results more correct represent stresses and deflection of the oil pipeline above ground crossing compare to the engineering methodic. Pipeline deflection from modeling f = 0,0328 m, which is lower than measured f = 0,084 m. Pipeline has some overstated stiffness in the software.

Qualitative and quantitative results of pipeline analytical calculation as beam on the elastic foundation and numerical modelling are quite close. Relative difference in the calculation doesn't exceed 22,8%. It is caused by differences in the methods of load application. Tendency could be seen, that results closing with reducing of the surrounding soil stiffness. Therefore, general approach and the chosen design scheme are correct to calculate the strength and deformability of the pipeline in non-standard soil conditions.

Loessial collapsible silty loam in the water saturated state is well simulated with help of deformational linear model, which allows to calculate settlements close to the engineering methodic. Relative difference doesn't exceed 11%. It also allows to simplify calculations compare with modern material hyperelastic models.

Hypothesis relevance have been confirmed, that pipeline deformations are equal to the deformations of the soil base. Calculated longitudinal stresses are σ_{dif} = -203 MPa, which is comparable with hoop stresses.



Picture 7 – Results of the hypothesis testing that pipeline deformation equivalent to soil deformations: a – uneven deformations for model with soil under pipe; b – longitudinal stresses for model with soil under pipe; c – pipeline deflection for model without soil; d – longitudinal stresses for model without soil

References

1. Yong Bai Pipelines and risers / Yong Bai. – USA, Oxford: Elsevier, 2001. – 495 p.

2. ASME B31.8-2003. Gas transmission and distribution piping system New York: American Society of Mechanical Engineers. 2003. – 168 p.

3. EN 1993-4-3 (2007) (English): Eurocode 3: Design of steel structures - Part 4-3: Pipelines [Authority: The European Union Per Regulation 305/2011, Directive 98/34/EC, Directive 2004/18/EC]. – 44 p.

4. СНиП 2.05.06-85. Магистральные трубопроводы. – М. : ЦИТП Госстроя СССР, 1988. – 52 с.

5. Болотин В. В. Методы теории вероятностей и теории надежности в расчетах сооружений / В. В. Болотин. – М. : Стройиздат, 1982. – 351 с.

6. Pichugin S. F. Calculation of the reliability of steel underground pipelines / S. F. Pichugin, A. V. Makhin'ko // Strength of Materials. – Springer Science, 2009. – Vol. 41. – Number 5. – P. 541 – 547.
7. Расчет и обеспечение прочности трубопроводов в сложных инженерно-геологических условиях. Т. 1 / А. М. Шаммазов, Р. М. Зарипов, В. А. Чичелов и др. – М. : Интер, 2005. – 706 с..
8. Котляревский В. А. Расчет деформаций трубопроводов в ореолах оттаивания адаптацией нормативных требований [Электронный ресурс] / В. А. Котляревский // Электронный научный журнал «Нефтегазовое дело». – 2013. – № 3. – С. 206 – 216. – Режим доступа: http://ogbus.ru/authors/KotlyarevskyVA/KotlyarevskyVA_1.pdf.
9. Faeli Z. Allowable Differential Settlement of Oil Pipelines / Z. Faeli, A. Fakher, R. Maddah // International Journal of Engineering (IJE). – 2010. – Issue 4, Vol. 4. – P. 308 – 320
10. Interaction of the artificial bases with Collapsing Soils / V. Shokarev, V. Shapoval, A. Tregub, V. Grechko, A. Shokarev, A. Serdyuk, G. Rozenvasser, M. Kornienko, E. Petrenko, N. Zotsenko, Y. Vynnykov // Geotechnical Engineering in Urban Environments. – Proc. of the 14th European Conf. on Soil Mechanics and Geotechnical Engineering (Madrid, 24 – 27 September 2007). – Millpress Science Publishers Rotterdam, 2007. – P. 481 – 486.
11. Айнбиндер А. Б. Расчет магистральных и промысловых трубопроводов на прочность и устойчивость / А. Б. Айнбиндер. – М. : Недра, 1991. – 284 с.
12. Орыняк И. В. Проблема больших перемещений подземных трубопроводов. Сообщение 1. Разработка численной процедуры / И. В. Орыняк, А. В. Богдан // Сб. науч. тр. Института проблем прочности им. Г.С. Писаренко НАН Украины, Проблемы прочности. – 2007. – №3. – С. 51 – 74.
13. Physical modeling of a pipeline subjected to an embankment load / J.R.M.S. Oliveira, K.I.Rammah, M.S.S.Almeida, M.C.F.Almeida1// Proc. of the XVI ECSMGE Geotechnical Engineering for Infrastructure and Development. – Edinburg. – 2015. – P. 4149 – 4154.
14. Селезнев В. Е. Основы численного моделирования магистральных трубопроводов / В. Е. Селезнев, В. В. Алешин, С. Н. Прялов. – М. : МАКС Пресс, 2009. – 436 с.
15. Трегуб А. С. Экспериментальные исследования подземных трубопроводов на подрабатываемых территориях / А. С. Трегуб // Світ Геотехніки. – 2004 р. – № 2. – С. 15 – 20.
16. Бруяка В. А. Инженерный анализ в Ansys Workbench. Часть 2 / В .А. Бруяка, В. Г. Фокин, Я. В. Кураева. – Самара : Самар. гос. техн. ун-т, 2013. – 149 с.
17. Крутов В. И. Проектирование и устройство оснований и фундаментов на просадочных грунтах / В. И. Крутов, А. С. Ковалев, В. А. Ковалев. – М. : Изд-во АСВ, 2013. – 544 с.

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