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Modeling a dependence system with specified conditions

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The article describes the formation of a multidimensional geometric object that reflects a system of dependencies of many variables based on geometric modeling. It is noted that in the process of creating geometric objects, the tasks of fulfilling the specified conditions and the need to model dependencies between all or some parts of the variables of a given system arise. The article develops a technique for constructing multidimensional geometric objects formed by other manifolds of different sizes and weights. These types must satisfy the given conditions. The structure of a multidimensional figure is applicable for solving complex optimization problems with many criteria in computer-aided design environments.

Keywords: geometric modeling, manifolds, dependencies of many variables

Моделювання системи залежностей із заданими умовами

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В статті описано утворення багатовимірного геометричного об'єкта, що відображає систему залежностей багатьох змінних на основі геометричного моделювання. Наочне моделювання багато параметричних залежностей використовує множини ліній як багатовидів, що мають один вимір. У статті опрацьовано утворення багатовимірних геометричних об'єктів із врахуванням заданих умов. Зазначено, що у процесі створення геометричних об'єктів виникають задачі виконання заданих умов та необхідність моделювання залежностей між усіма або деякою частиною змінних заданої системи. Процес розв'язання задачі потребує побудову багатовиду, який об'єднує деякі особливості як свої складові частини за умов їх важливості. Також у побудові застосовуються задані багатовиди меншої розмірності. У роботі опрацьовується методика побудови багатовимірних геометричних об'єктів, що утворюються іншими багатовидами різної розмірності та ваги. Ці багато види мають відповідати заданим умовам шляхом поступової зміни розмірності та числа змінних у його аналітичному виразі. Залежність коефіцієнтів рівнянь, що утворюють геометричні фігури меншої розмірності від змінних величин використовується у геометричній моделі багатовимірної функції. У публікації відображено особливості структури геометричного багатовимірного об'єкта із використанням множини інших фігур меншої розмірності. Різне число параметрів у аналітичному виразі, що описує багатовимірний геометричний об'єкт відповідає множинам інших багатовидів меншої розмірності у паралельних просторах заданої координатної системи. Показано визначення множин ліній, що належать багатовимірним фігурам у різних просторах рівня. Структура багатовимірної фігури може бути застосована у вирішенні складних задач оптимізації з багатьма критеріями у середовищах систем автоматизованого проектування. Графічне зображення деякого багатовиду в n-вимірному просторі представляється перетином множини гіперплощин рівня, які паралельні координатним просторам. Цей процес виконується для отримання набору одновимірних багатовидів, наприклад, кривих ліній.

Ключові слова: геометричне моделювання, багатовиди, залежності багатьох змінних

Introduction

Geometric modeling of the dependencies of many variables has always been and remains relevant because in nature everything is interconnected and the various processes are always influenced by many factors. The geometric model of such dependencies has many advantages: it is simple, accessible, and visual, allowing you to fully formalize and successfully solve a wide range of different applied problems, including such complex ones as multicriteria. The geometric model of the study allows you to fully formalize the study and translate the physical actions on the system to the corresponding geometric operations of the study. Each technical problem and its solution can be formulated from geometric positions. Geometric problems are solved by means of visual operations on many according to the rules of multidimensional geometry. The obtained geometric solution is then interpreted on the physical content of the formulated problem and the necessary recommendations for their use. The study of relationships determined by various factors between the variables of different complex systems is quite relevant [1]. These relationships are represented by geometric models of multidimensional objects.

The geometric model conveys the relationships of all variables and visually reveals them in the image. The application of the model allows solving practical problems not only analytically, but also through formalized graphic operations. Visualization of the complex dependence between many variables is a tool for obtaining maximum visibility of all operations in the model so that the method of its research is achievable and understandable. The relationship between variables is embodied in the form of a multidimensional figure of an n-dimensional space of n variables, where $1 =< k =< n-1$. If $k=1$, then the manifold is in some curve of n-dimensional space, if $k=n-1$, then the manifold is a hypersurface of n-space. Of the n variables included in the studied multidimensional model, p functions and k arguments are distinguished, where $p+k=n$. Thus, the manifold's dimension as a geometric model of dependence is equal to the number of independent variables (arguments), i.e. k [1].

Review of the research sources and publications

In geometric modeling of systems with many parameters there is always a need for their analytical and graphical display. They can be approximated by sets of other geometric objects, or their parts with joining on common borders [1]. Scientific publications investigate non-uniform rational B-splines [2], which in the geometric representation can be reflected in the form of curved lines. Various computer-aided design systems use NURBS splines to exchange information [3]. Rational B-splines define geometric objects in space in the same way. [4].

Definition of unsolved aspects of the problem

Considers the structure of the discrete frame of the manifold B^3, given in the rectangular Cartesian coordinate system of the 4-dimensional space Π^4. For this purpose, we will use the intersections of B^3 with spaces C^3, which are parallel to one of the coordinate spaces Π^3. According to Grassmann's theorem, in the 4-dimensional space Π^4, these spaces intersect the manifold B^3 by manifolds of lower dimension B^2, which are surfaces in their 3-dimensional spaces Π^3: $B^3 \cap C_i^3 = B_i^2$.

Surface B_1^2 is located in space C_1^3: ($x^3 = x_3^1$); surface B_2^2 – in the space of level C_2^3: $x_3 = x_3^2$,..., B_1^2 – in the space of level C_{m1}^3: $x_3 = x_3^{m1}$, $B_1^2 \in C_1^3$, $B_2^2 \in C_2^3$,..., $B_r^2 \in C_r^3$.

The set of surfaces forms a discrete frame of manifold: $B^3 = B_1^2 \cup B_2^2 \cup \ldots B_{m1}^2$, where m_1 is the number of surfaces in this frame. The larger the number m_1, the more accurate the B^3 manifold will be.

Each of the obtained manifolds Bi2, which is located in its intersecting space C_i^3, is similarly defined by a discrete frame of lines B_j^1 in intersecting planes C_j^2: $B_i^2 = B_{1j}^1 \cup B_{2j}^1 \ldots B_{m2j}^1$, where m^2 is the number of lines in the frame of the surface B_i^2. For example, these can be planes of level C_1^2, $C_2^2 \ldots C_{m2}^2$: $x_2 = x_2^1$; $x_2 = x_2^2$;... $x_2 = x_2^{m2}$. They are parallel to the coordinate plane $\Pi_3^2 = 0x^1 \cup 0x_4$. The surface B_1^2 intersects the plane $C_1^2 = x_2 = x_2^1$ along the curve B_{11}^1. The plane $C_2^2 = x_2 = x_2^2$ intersects the surface B_1^2 along the curve B_{12}^1, etc. And finally, $C_2 = x_2 = x_2^{m2}$ intersects the surface B_1^2 along the curve $B_{1m2}^{1(1m2)}$. $B_1^2 \cap C_1^2 = B_1^1$, $B_1^2 \cap C_2^2 = B_2^1$, $\Pi_1^2 = 0x_1 \cup 0x_2$.

Similarly, frameworks are created for other 2-dimensional multispecies: B_2^2, ..., B_{m1}^2. To build their frames, intersecting planes of level C_i^2 are used relative to the same coordinate plane as for the first surface, i.e. hyperplane of level $x_2 = x_2^1$; $x_2 = x_2^2 \ldots x^2 = x_2 = x_2^{m2}$. The intersection of these level planes with the corresponding surfaces gives the curves: for B_2^2-21; 22 ... 2_{m2}, etc. And, finally, for B^2_{m1} - m_1^1; $m_1^2 \ldots m_1 m_2$. In the designation of these lines, the first digit indicates the line belongs to the corresponding hyperplane of the level or to the 2-dimensional surface obtained in this hyperplane. The second digit indicates belonging to the secant 2-dimensional plane of the level.

For example, the curve 21, denoted as B_{21}^1, belongs to the second 2-dimensional manifold B_2^2, which is formed at the intersection of the manifold B^3 with the hyperplane C_2^3: $x_3 = x_3^2$ and the first cutting plane of the level C_1^2: $x_2 = x_2^1$. On the other hand, the numbers in the notation correspond to the indices of successive intersecting spaces C_i^j, which are used to obtain the curves of the framework of the hypersurface B^3.

Thus, the manifold B^3 is defined by a discrete framework of 2-dimensional manifolds B_{l1}^2, where $l_1 = 1, 2, ..., m_1$, and each of these manifolds, in turn, is defined by a framework of lines: $B^1(l_2)$, where $l_2 = 1, 2, ..., m^2$. As a result, we obtain the expression of the manifold B^3 as a discrete framework of flat curved lines. The number of these lines is equal to the product m_1, m_2.

In the process of studying systems with many parameters, it is necessary to improve their geometric models to solve various optimization problems that occur in different fields of science and technology.

Problem statement

The aim of the work is to study the structural structure of a multidimensional geometric object that models the dependences of many variables using *B*-splines.

Basic material and results

The literature describes the analytical methods for solving multiparameter problems used in various calculations, in particular, the structural reliability of the system [1, 6-12, 14]. The model is based on a geometric understanding of structural modeling. Due to the uniqueness of the problem, some non-geometric components of the model are used.

The geometric model conveys the relationships of all variables and visually reveals them in the image. The application of the model allows to solve practical problems not only analytically, but also by formalized graphical operations. Visualization of complex relationships between many variables is a tool to obtain maximum visibility of all operations in the model, so that the method of its study was achievable and understandable.

The relationship between the variables is embodied in the form of a multidimensional figure of *n*-dimensional space *n* variables, where 1 =< *k* =< *n*-1. If *k* = 1, then the variety is in some curve of n-dimensional space, if *k=n-1* − then the variety is a hypersurface of *n*-space. From n variables included in the studied multidimensional model, p functions and k arguments are distinguished, where $p+k = n$. Thus, the dimension of the manifold as a geometric model of dependence is equal to the number of independent variables (arguments), ie *k*.

Construction of multidimensional space objects is possible by successively increasing their dimension. In method [6] the expression of the object B_b is known, and in this method the equation is finally formed at the end of the formation of the geometric model.

Let's build a model of some dependencies in an object or process that has initial data that are the result of theoretical or practical research. Consider the dependences in the form of manifolds B_1 (one-dimensional lines) in the local coordinate system of the corresponding space of level P_2.

One-dimensional objects B_1 are represented by *n-b* projections in the graphic image and are described by systems of *n-b* equations in the analytical description. Some function with variable argument u1 corresponds to an ordered set of objects B_1, describing 2-dimensional structural components (surfaces) of the manifold B_b.

The formed B_2 varieties can be represented by a function with variable argument u_2 and be part of 3-dimensional varieties. This formation occurs before the output of the final analytical expression of the desired multidimensional object B_k.

In method [6] the manifold B_k and its structural parts B_1, B_2, ..., $B_{b\text{-}1}$ are described in sequence, which gradually reduces the dimension of geometric objects. One-dimensional manifolds B_1 that are lines are described by a system of *n-b* equations:

$$x_{n-b+1} = f_s(N_{(i,j)_d}(u); x_1,\dots,x_{n-b+1}; u_1),\ \ d = 1,\dots,\prod_{i=1}^{b\text{-}1} m_i\ , \qquad (1)$$
$$x_{n-b+2} = const,\dots,x_{n-1} = const, x_n = const\ ,$$

where $u_1 = x_{n\text{-}b}$, $N_{i,\,j}\ (u)$ – parametric expressions of lines;

d – the number of expressions $N_{i,j}(u)$;

m_i – the number of varieties B_1.

The set of lines B_1 in the local coordinate system of $P_{n\text{-}k+1}$ space spaces parallel to the coordinate space is described in the first line (2). Lines B_{12}, B_{22}, ... , B_{2m} in the spaces of level P_1, P_2, ... , P_1 are expressed:

$$\begin{matrix} S_{1,1}, S_{2,1},\dots, S_{m,1} \\ \dots\dots\dots\dots\dots\dots\dots \\ S_{1,3}, S_{2,3},\dots, S_{m,3} \end{matrix}\ . \qquad (2)$$

These expressions reflect the affiliation of the lines B_2 of the multidimensional object B_b to the corresponding spaces of the level $P_{n\text{-}k+1}$, passing through points with certain values of the coordinates x_1,… .x_n in the space P_n. Similarly, the sets of lines belonging to the varieties B_q in the spaces of the level $P_{n\text{-}b+q}$ are determined.

When the geometric objects B_q are lines: $q = 1$, then we have a special case (2) of expression:

$$r_i^{b-q}(u_1,\dots,u_q) = f(a_i^j, u_q, r_i^{q-1}, u_q^j),\ \ i = 0,\dots n\text{-}b\text{-}q, \qquad (3)$$
$$u_b = const,\dots,u_{q+1} = const,\quad q < b\ .$$

Equation (1) corresponds to different values of the parameter $u_2 = x_{n\text{-}b+2}$. Let us establish the dependences on this parameter in the explicit function

$$N_{i,j}(u)\ = f(u_2, K_g),\quad s = 1,\dots,\prod_{i=1}^{b\text{-}1} m_i\ , \qquad (4)$$

where K_g – coefficients of expression;

g − number of coefficients.

Let us replace the coefficients K_g of the dependence (4) in formula (2) and find the analytical expressions of the varieties B_2 with variables u_1 and u_2

$$\begin{matrix} u_1 = x_{n-k} . u_2 = x_{n-k+2} \\ x_{n-k+1} = f\ (x_1,\dots,x_{n-k-1}, u_1, u_2, K_g) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ x_n = const \end{matrix}\ , \qquad (5)$$

where K_g – coefficients of the formula, $u_2 = x_{n\text{-}k+2}$. Figures B_2 are described by expression (5) (which are surfaces) in the structure of the manifold B_b.

In comparison, the obtained varieties B_2 (5) with the corresponding values of the parameter u_3, there is a dependence of the expressions K_g of formula (5)

$$K_g = f_\mu(u_3, a_{vq})\ , \qquad (6)$$

where $u_3 = x_{n\text{-}k+3}$.

Increasing the number of variables and, accordingly, the dimension in the expression sets the analytical representation of the figure B_b

$$x_{n-k+1} = f(x_1,...,x_{n-k+1},u_1,...,u_k,w_i)\,, \tag{7}$$

where w_i − coefficients of the equation;
u_1, u_2, .., u_k − arguments of the function taken by the coordinates x_{n-b}, ... , x_n multidimensional space.

In the special case, when $b = n - 1$, the lines B_1 are incidental to the planes of the level P_2. In the construction of geometric objects of multidimensional space, there are problems with the fulfillment of given conditions.

In addition to the conditions set for the studied dependencies, it may be necessary to model the dependencies between all or some of the parameters of this system. Solving the problem requires the construction of a variety that integrates certain features as its components in terms of their importance. Or a variety is formed by given varieties of smaller dimensions.

The importance of the studied dependencies is used for objects of multidimensional space of smaller dimensions than the dimension of the studied variables. Thus, there is a statement of the geometric problem: for several given varieties B_0, B_1, … , B_{b-1} of different dimensions and weights to build a variety B_b, which best integrates and approaches the set of given.

We assume that j objects B^h of multidimensional space, $j \in N$ is described by the system

$$x_{i_s} = \varphi_{i_s}(u_1^{(s)},...,u_{\rho_s}^{(s)}),\; i = 1,...,n\,,\;\; s = 1,..., j\,, \tag{8}$$

where $u_j^{(s)}$ − variables;
n – the number of variables;
j − the number of figures of multidimensional space;
s − the dimensional index of the object B_h.

These objects have dimensions h_s, $s = 1, 2, … , j$, $1 \le h_s \le n - 1$, h_s, n− the number of related variables with the weight w_s, $s = 1, 2, …, j$, $1 \le w_s \le n - 1$. Generalized multidimensional object B_b, $h_{s_{\max}} \le b \le n-1$ given by the system

$$\begin{aligned}
&f_1(x_1,x_2,...,x_{n-1},x_n;K_{1,1},K_{1,2},...,K_{1,m-1},K_{1,m}=0,\\
&f_2(x_1,x_2,...,x_{n-1},x_n;K_{2,1},K_{2,2},...,K_{2,m-1},K_{2,m}=0,\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\
&f_{n-b}(x_1,x_2,...,x_{n-1},x_n;K_{n-b,1},K_{n-b,2},...,K_{n-b,m-1},K_{n-b,m}=0,
\end{aligned} \tag{9}$$

where x_i are the coordinates of the object in multidimensional space, $K_{i,j}$ are the unknown coefficients of the equations.

The value of the weight w_s of multidimensional objects B_m in fractional form is rounded to the nearest whole number. The solution of the problem is a way to determine approximations, in particular, for example, the method of least squares [5]. Let's make an expression

$$\sum_{s=1}^{h}\delta_{i_s}^2 = \sum_{s=1}^{h}\int\limits_{M_{h_s}}\dots\int \mu w_s \Delta^2 du_1^{(s)}...du_{h_s}^{(s)},\;\; i = 1,...,n \tag{10}$$

where Δ is the difference between expressions (9) and

$$\begin{aligned}
&f_1(\varphi_{1_s}(u_1^{(s)},..,u_{h_s}^{(s)}),..,\varphi_{n_s}(u_1^{(s)},..,u_{h_s}^{(s)},K_{11},..,K_{1_m}) = 0,\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\
&f_{n-k}(\varphi_{1_s}(u_1^{(s)},..,t_{h_s}^{(s)}),..,\varphi_{n_s}(u_1^{(s)},..,u_{h_s}^{(s)},K_{n-k,1},..,K_{n-k,m}) = 0.
\end{aligned} \tag{11}$$

The coefficients $K_{i,j}$ of the analytical description of the object B_b (9) are determined from the condition

$$\sum_{s=1}^{h}\delta_{i_s}^2 = min, i = 1,...,n\;\; . \tag{12}$$

The manifold B_k is given by its discrete framework from manifolds of lower dimension to 1-dimensional lines: $K_b = B_{k-1}, B_{k-2}, ..., B_1$. The graphical representation of some manifold B_k of general position in the n-dimensional space P_n is represented by the intersection of B_k by the set m of parallel hyperplanes of the level S_k, which are parallel to the coordinate space P_{n-1}.

This is a set of varieties of smaller dimensions $B_{1k-1}, B_{2k-1},…, B_{dk-1}$. Each of the latter, in turn, intersects a system of level hypersurfaces that are parallel to the n-2 subspaces of the P_{n-2} level. The cross-sections obtained at this stage are intersected by a unit of smaller dimension by parallel subspaces of S_{k-1} level, etc. This process is performed to obtain a set of manifolds B_1, ie lines (in the general case of curves) given surface.

Since with each operation the dimension of the submanifold is reduced by one, to obtain a q-dimensional section C_q it is necessary to perform: k-q sequential operations to reduce the dimension (k-q consecutive sections C-q). And q is the dimension of the cross-section, which determines the variety in the space of dimension n-k+q. To obtain 1-dimensional sections that are lines B_1 requires k-1 operations to reduce the dimension. Each of the cross-section lines is in n-k+1-dimensional space.

Thus, the surface B_k (1 = <k = <n-1) is given by the framework of q-dimensional cross-sections, the number of which is equal to: m_{k-q}. Where m is the accepted number of cross-section operations at each stage of reducing the dimensionality of the manifold. If we take the number of cross-section operations in each dimension reduction operation different, for example, m_1, m_2, ..., m_{k-q}, then, obviously, the variety B_k will be determined in this case: m_1, m_2 ... m_{k-q} q-dimensional sections n+q-k -dimensional level spaces.

For a framework of one-dimensional cross-sections, m_{k-1} lines in n-k+1 dimensional sectional subspaces of the C^i level are required. In this case, each of these lines will be given by n-k its projections on two-dimensional projections of the subspace, including these curves. Having designed the section in conjunction with the section spaces on the two-dimensional plane of projections, we turn to the epure of Monge. In this case, on one of the n-k+1-dimensional spaces of projections, which parallel the intersecting spaces of the level, including the lines of the frame, the cross-sectional curves are projected without distortion. In their other projections, they coincide with the traces of the intersecting spaces *C*.

The spatial curved line is a special case of the manifold B_k, when k=1. For the graphic and analytical task of this manifold, the methods of defining the manifold B_k can be used.

On the one hand, the curve can be set by its two (or more) surfaces (carriers) to which it is incidental. Then the analytical description of the curve is a system of equations for these surfaces. Graphically, the curve is also determined by the mutual intersection of surfaces.

On the other hand, the spatial curve can be set by a discrete framework of points defined by the cross-section C of its level hypersurfaces, which are parallel to some hypersurface of projections (n-1) + 1-n = 0).

Thus, the value of one variable determines the value of the rest. Accordingly, the equation of the curve is a one-parameter relationship between variables. On the diagram, the curve, which is a 1-dimensional manifold, is given by (n-1) projections on the coordinate plane P_2.

Analytically, the n-spatial curve is defined as the geometric location of points belonging simultaneously to the projecting hypercylinders, the number of which is equal to the number of flat projections of the curve that define it. The equations of these hypercylinders in n-space are the equations of curve projections on the corresponding planes of projections. Thus, the curve is analytically expressed by the system of equations of these hypercylinders lying on them.

In the geometric modeling of the Π^3 space, the kinematic method of surface formation is often used. Its main disadvantages can be considered: the high order of the formed surface and the peculiarities of formation associated with the use of given guide lines.

In the space Π^n, a hypersurface is a special case of the manifold B^b with b=n-1, where n is the dimension of the space. Let us consider the formation of varieties B^b with the help of tangent spaces to the guiding manifold. Let the equation of the manifold B^b have the form

$$B^b = \frac{\sum\limits_{i_n=0}^{k} p_{i_n} \dots \sum a_{i_n} \dots i_1 r_{i_n} \dots i_1 p_{i_1}}{\sum\limits_{i_n=0}^{k} p_{i_n} \dots \sum\limits_{i_1=0}^{k} a_{i_n} \dots i_1 p_{i_1}}, \qquad (13)$$

where: k is the dimension of the directing object B^b, n is the order of the manifold, p_0, ... , p_b are variable parameters, $r_{in\dots i1}$ are the radius-vectors of the points of the manifold, $a_{in\dots i1}$ are the coefficients of the equation of the manifold B1 (curved line) at the corresponding points; the order of the indices does not affect the value of the parameters; for example, a_{001}=a_{010} = a_{100}.

The equation of the tangent space Π^b at the point (p_0,..,p_k) to the multidimensional figure B^b is

$$B = \frac{\sum\limits_{i_n=0}^{k} p_{i_n} \sum\limits_{i_{n-1}=0}^{k} p_{i_{n-1}} \dots \sum\limits_{i_1=0}^{k} p_{i_n \dots i_1} r_{i_n} \dots_{i_1} p_{i_1}}{\sum\limits_{i_n=0}^{k} p_{i_n} \sum\limits_{i_{n-1}=0}^{k} p_{i_{n-1}} \dots \sum\limits_{i_1=0}^{k} p_{i_n \dots i_1} p_{i_1}}, \qquad (14)$$

where p_{in-1} are variable homogeneous parameters.
In the tangent space Π^b (14) we take the manifold B^m. His expression:

$$p_j = f_j(y_0, \dots, y_m), j = 0, \dots, k \qquad (15)$$

where: n<m<k-1.

By moving the space Π^b (14), which in all its positions touches the guiding manifold B^b (13), the manifold B^m (15) forms another manifold B^{b+m} with the order s+n-1, where s is the order of the manifold B^m (15) in spacious Π^{b+m+1}. The expression for the variety B^{k+m} is

$$B = \frac{\sum\limits_{i_{s+n-1}}^{k+m} p_{i_{s+n-1}} \dots \sum\limits_{i_1=0}^{k+m} a_{i_{s+n-1}} \dots_{i_1} r_{i_{s+n-1}} \dots_{i_1} p_{i_1}}{\sum\limits_{i_{s+n-1}=0}^{k+m} p_{i_{s+n-1}} \dots \sum\limits_{i_1=0}^{k+m} a_{i_{s+n-1}} \dots_{i_1} p_{i_1}} \qquad (16)$$

Let the initial (13) be a 2-dimensional manifold B^b, k=2, and the space (14) tangent to it - the hyperplane Π^b, k=2. In the multidimensional space Π^b, the manifold B^m, m=1 (line) of the 2nd order, n=2, is given as a generatrix (15). Expression (15) of the generatrix looks like

$$p_0 = t_0^2, \quad p_1 = t_0 t_1, \quad p_2 = t_1^2 . \qquad (17)$$

The equation of the resulting manifold B^{b+m}, b+m of the 3rd order s+n-1 = 3 in the space Π^4, b+m+1 = 4 in expanded form looks like this:

$$B = \frac{A_1 t_0^2 + A_2 t_0 t_1 + A_3 t_1^2}{A_4 t_0^2 + A_5 t_0 t_1 + A_6 t_1^2},$$

$$A_1 = a_{00} r_{00} p_0 + a_{01} r_{01} p_1 + a_{02} r_{02} p_2 ;$$

$$A_2 = a_{10} r_{10} p_0 + a_{11} r_{11} p_1 + a_{12} r_{12} p_{12} ;$$

$$A_3 = a_{20} r_{20} p_0 + a_{21} r_{21} p_1 + a_{22} r_{22} p_2 ; \qquad (18)$$

$$A_4 = a_{00} p_0 + a_{01} p_1 + a_{02} p_2 ;$$

$$A_5 = a_{10} p_0 + a_{11} p_1 + a_{12} p_{12} ;$$

$$A_6 = a_{20} p_0 + a_{21} p_1 + a_{22} p_2 .$$

Thus, this is another method of constructing multidimensional figures B^{b+m} using manifolds B^m in spaces Π^{b+m+1} tangent to the directional manifold B^b.

Conclusion

The method of geometric modeling allows realizing not only the dependence but also the sequence of all actions on the model, respectively, the developed algorithm for solving a specific practical problem.

Geometric modeling of multiparameter systems allows to fully formalize the solution of the problem in the form of geometric actions on a manifold according to the rules of multidimensional descriptive geometry, followed by interpretation of the result.

A technique is given for constructing multidimensional geometric figures formed by other polytypes of different dimensions and weights in accordance with given conditions. A method is proposed for constructing the manifold B_b by gradually changing the dimension and number of parameters in its analytical expression. The considered structure of the geometric model has its practical introduction to CAD in order to solve optimization problems in many parameters.

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