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## Proposals of diagrid structural systems

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The article describes various types of single-layer and double-layer diagrid structures. Types variants of diagrid structures that are used more for public buildings are given. The diagrid systems structural forms of existing types analysis, identified disadvantages and advantages of such structures. The calculating diagrid structural systems features using modern software systems are revealed. A new type of flat diagrid structures in the form of regular hexagons made of bent profiles is proposed. The variants of this structure type manufacturing and joining elements are considered. The bent channels and bolted dimensions or self-tapping screw systems are determined by calculation

**Keywords:** diagrid structural systems, structures, hexagon, bent elements, design model, bent channel

## Пропозиції ґратчастих структурних систем

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Описано різні види ґратчастих одно- і двошарових конструкцій. Наведено різні варіанти типів ґратчастих конструкцій, що застосовуються більше для громадських будівель. Виконано аналіз існуючих типів конструктивних форм ґратчастих структурних систем, виявлено недоліки та переваги таких конструкцій. Представлено типові приклади споруд з використанням ґратчастих структурних систем, зокрема вежа Шухова в Україні, хмарочос «Геркін» в Лондоні, оперний театр в Пекіні, Британський музей у Лондоні, купол «Кліматрон» у США й ін. Виявлено особливості розрахунку сітчастих оболонок за допомогою сучасних програмних комплексів. Запропоновано новий тип плоских ґратчастих конструкцій у вигляді правильних шестикутників із гнутих профілів. Розглянуто варіанти виготовлення і з'єднання елементів такого типу конструкцій. Розміри гнутих швелерів та систем кріплення на болтах або самонарізних гвинтах визначаються розрахунком. Запропоновано конструкцію металевого одношарового покриття з елементами із гнутих швелерів. Така конструкція складається з типового монтажного шестикутного елемента, що вирізняється тим, що всі елементи виконані з однакового перетину швелерів, монтажні стики – на самонарізних гвинтах, що дозволяє зменшити металоемність з'єднання і прискорити монтаж, а також спростити транспортування. Конструкція покриття заводського виготовлення з дрібних типових чи великорозмірних плоских елементів застосовується для сталевого покриття споруд з профільованим настилом прогонами 12 м і більше. Для більших прогонів можна застосовувати гнуті елементи товщиною 4 – 6 мм, тоді допускається нарізання однакових елементів зі скошеними краями і зварювання їх між собою. Розміри елементів визначаються розрахунком відповідно обумовлених замовником прогонів. Розрахункові схеми цього типу конструкцій, побудовані в програмному комплексі SCAD, мають конфігурацію прямокутника або квадрата, а спирання відбувається по контуру чи з двох сторін; можливе застосування проміжних опор (колон)

**Ключові слова:** ґратчаста структурна система, конструкція, шестикутник, гнуті елементи, розрахункова модель, гнутий швелер



## Introduction

The use of space-rod structures is not a new technology: construction with their use has more than half a history century. Diagrid structural systems were most often used in civil construction, where it was required to cover spans of more than 30-40 m with minimal metal costs. At the beginning of the industry development, diagrid structural systems were used in their simplest geometric forms - domes.

This was due to the relative simplicity in calculating individual structural elements. The first domes were designed by Richard Fuller in the 40s of the last century. Fuller decomposed the dome structure into triangles, the sides of which are located on lines connecting two points on a curved surface. This design made it possible to cover the maximum possible space using the least amount of building materials. The works of R. Fuller brought their results: the world community drew attention to a new promising type of structures, which made it possible to create several interesting projects in the following decades [1].

A new vector of industry development was set relatively recently, due to the improvement and widespread computer technology introduction. The emergence of new computer-aided design systems and programs allowed us to go beyond the simplest diagrid structural systems configurations, to build not only domes, but also to give objects a shapes variety. Nowadays, huge data amounts can be processed automatically, and a large number of similar structural elements that differ in a small range of parameters can be created in semi-automatic mode with minimal human input. For this reason, with the diagrid structures various configurations appearing, the proposed topic is gaining more and more relevance.

## Review of research sources and publications

Diagrids are advanced framing systems composed of diagonal steel grids [1]. Developed a decade ago for spectacular buildings like the Swiss Re (the gherkin) in London, diagrid structures connect a breathtaking appearance with tangible advantages: a massive reduction in material use, a gain of available floor surface area and more flexibility. For the first time, this book gives a comprehensive account of the key aspects of this structural system. Diagonalized grid structures have emerged as one of the most innovative and adaptable approaches to structuring buildings in this millennium. The diagrid system variations have evolved to the point of making its use non exclusive to the tall building. Diagrid construction is also to be found in a range of innovative midrise steel projects. The paper [2] will examine developments in the recent history of diagrid buildings to include the design and detailing. Images by author unless otherwise noted in [3].

In recent time diagrid structures are often used as the main framework for high-rise buildings due to structural efficiency and unique geometry. System structural features allow to design the most innovative architectural solutions in the form and layouts of the building. Author of the article [4] considered the

possibility of using a diagrid system for construction of low-rise public buildings and the effectiveness of such a solution. The article [4] describes the shell frame design and the impact loads calculation. The calculation results were compared with a structure with a regular grid of columns. Conclusions about the effectiveness of the diagrid structures use are described.

The diagrid structural system has been widely used for recent tall buildings due to the structural efficiency and aesthetic potential provided by the unique geometric system configuration. The paper [5] presents a stiffness-based design methodology for determining preliminary member sizes of steel diagrid structures for tall buildings. The methodology is applied to diagrids of various heights and grid geometries to determine the optimal grid configuration of the diagrid structure within a certain height range. Constructability is a serious issue in diagrid structures because the nodes of diagrids are more complicated than those of conventional orthogonal structures. The paper [5] also presents various strategies to improve constructability of diagrids through prefabrication of the nodes.

Diagrid structural systems are emerging as structurally efficient as well as architecturally significant assemblies for tall buildings. The paper [6] presents a simple methodology for determining preliminary member sizes. The methodology is applied to a set of building heights ranging from 20 to 60 stories, and parameters for the optimal values of the grid geometry are generated for representative design loadings. These values are shown to be useful for architects and engineers as guidelines for preliminary design.

Diagrid structures are prevalently used for today's tall buildings due to their structural efficiency and architectural aesthetic potentials. The paper [7] studies structural performance of diagrid systems employed for complex-shaped tall buildings such as twisted, tilted and freeform towers. For each complex form category, tall buildings are designed with diagrid systems, and their structural efficiency is studied in conjunction with building forms. In order to investigate the impacts of variation of important geometric configurations of complex-shaped tall buildings, such as the rate of twisting and angle of tilting, parametric structural models are used for this study. Based on the study results, design considerations are discussed for the efficient use of diagrid structures for complex-shaped tall buildings.

Characteristics and stiffness-based preliminary design methodology of diagrid structures are discussed in [8]. The design methodology is applied to a set of diagrid structures, 40, 50, 60, 70, and 80 stories tall. The diagrid structure of each storey height is designed with diagonals placed at various uniform angles as well as gradually changing angles along the building height in order to determine the optimal uniform angle for each structure with a different height and to investigate the structural potential of diagrids with changing angles. Based on these design studies, design guidelines are provided for the optimal configuration of the diagrid structure grid geometry within a certain height range.

Diagrid systems have stronger structural efficiency than other systems like braced systems. Diagrid systems have unique geometric configuration used for various heights of tall structure. Geometric configurations and grid geometries of diagrid structure depends on the heights and angle of the diagrids. Joints of diagrid structure are more complicated than conventional structure therefore Construction of diagrid structure is complicated than conventional structure. In early days tall buildings have importance due to their architectural view and diagrid structure have better architectural view as well as good structural stability. Recent forms in tall structures have complex shapes like tapered, twisted and tilted. The paper [9] includes required data, model, Earthquake and Wind analysis of Braced Tube Structure and diagrid structure with Circular, Square and Rectangular plan. Then by keeping the same plan area and structural data for circular, square and rectangular plan, Earthquake and Wind analysis result of both Braced Tube and Diagrid Structures is carried out and by comparing the braced tube structures results and diagrid structures results conclusions drawn from the present investigation.

In the paper [10], an overview on application of such typology to high-rise buildings is carried out; in particular, in the first part of the paper, the diagrid systems peculiarities are described: starting from the analysis of the internal forces arising in the single diagrid module due to vertical and horizontal loads, the resisting mechanism of diagrid buildings under gravity and wind loads is described, and recent researches and studies dealing with the effect of geometry on the structural behavior are discussed. In the second part of the paper [10] a comparative analysis of the structural performance of some recent diagrid tall buildings, characterized by different number of stories and different geometries, namely the Swiss Re building in London, the Hearst Headquarters in New York and the West Tower in Guangzhou, is carried out, and some general design remarks are derived.

The paper [12] aims at discovering the evolutionary process of diagrid structures and their progresses which leads to major breakthroughs in architectural, structural and sustainability concepts. Indeed, these recent advances are investigated and reported for architects and engineers. The results, based on case studies, show that these structures have been able to address most of the designing requirements. They have also been used in different projects with totally different heights, areas and functions, suggesting diamond modules can be applied not only for high-rises but for a wide range of projects.

Recently diagrid structural systems have been adopted in tall buildings due to its structural efficiency and flexibility in architectural planning [13]. Compared to closely spaced vertical columns in framed tubes, diagrid structure consists of inclined columns on the exterior surface of the building. Due to inclined columns lateral loads are resisted by axial action of the diagonal compared to bending of vertical columns in framed tube structure. Diagrid structures generally do

not require cores because lateral shear can be carried by the diagonals on the building periphery.

Recently a diagrid structural system has been adopted in tall buildings due to its structural efficiency and flexibility in architectural planning. Diagrid structures consist of inclined columns on the exterior surface of buildings compared to closely spaced vertical columns in framed tubes. The lateral loads are resisted by axial action of the diagonal. In the paper [14], the comparison study of 20-storey simple frame building and diagrid structural system building is presented in [14].

#### **Definition of unsolved aspects of the problem**

For a long period of light metal structures development, scientists around the world have been studying the problems of designing diagrid structural systems and selecting methods for calculating them. The issue of developing new effective design solutions remains relevant and requires further research and suggestions.

#### **Problem statement**

The purpose of this work is to conduct a thorough analysis of existing design solutions for diagrid structural systems, as well as to identify their advantages and disadvantages. One of the main objectives of this study is to develop an easy-to-install coating of spatial hexagonal elements made of bent channels with reduced metal consumption and labor costs, assembly on the ground and installation of the coating as a whole. The research also aims to find optimal calculation methods using existing software packages.

#### **Basic material and results**

In the last 10-20 years, the so-called diagrid structural systems have become an increasingly popular solution when choosing a structural scheme of buildings with different purposes, shapes, heights and spans. In such structures, the usual vertical load-bearing elements, columns, along the perimeter of the building are replaced by inclined elements that form the diagrid structures of the building.

In this case, the internal columns are completely or partially excluded, and the loads from the floors and roof are borne by the beam or truss system, which transmits them to the diagrid structures. Also, additional diagrid elements can be created inside the building, which form a core of rigidity. Complete replacement of columns is possible due to the fact that the diagrid structural systems equally well perceive both vertical and horizontal loads on the building, as well as reduce shear and bending deformations due to the nature of the work of inclined elements.

Other design diagrid structural systems advantages include:

- increased stability, thanks to triangular elements;
- providing multiple load distribution options and reducing the possibility of failure;
- reducing the own structures weight;
- reduced material consumption.

From an architectural point of view, these structures also have a number of advantages:

- building unique shape creation both in plan and in height;
- architectural expressiveness due to diagrid elements, even with a simple building shape;
- large variation in the parameters of the load-bearing elements grid: size, angle of inclination, shape and cells number;
- large sunlight amount due to panoramic windows;
- creating column-free interiors.

Diagrid structural systems are load-bearing building structures that are made of metals, composite materials or wood. Today, the structures are relevant for the world's progressive architecture in the "high-tech" style. The mesh structure's advantage is the ability to block large spans or achieve a unique shape and architectural expression of building elements-facades and roofs.

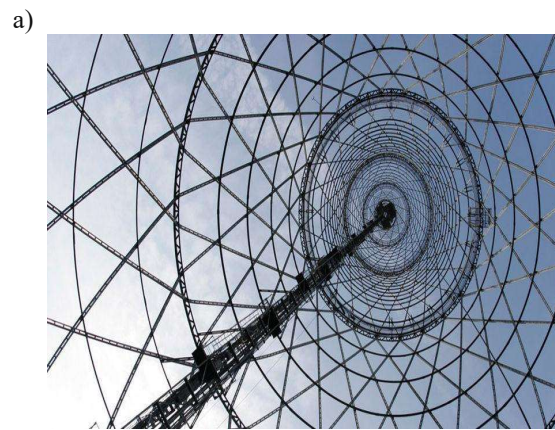
The world's first diagrid structural systems were used by engineer and inventor Vladimir Shukhov. He designed and built the 150-meter hyperboloid tower in 1922 (fig. 1, a). The inventor had a landmark-the Eiffel tower with a height of 325 m and a weight of 7300 tons. Shukhov managed to reduce the metal consumption per unit height of the structure by 14 times relative to the tower in Paris. He managed to increase the expressiveness of the building elements-facades and roofs.

Today, diagrid structural systems are used quite often. Especially in Europe, America and Asia, buildings and structures for various purposes are being actively built using mesh structures. These are the skyscraper in London "the Gherkin" (fig. 1, b), the Opera house in Beijing (fig. 1, c), the British Museum (fig. 1, d), diagrid structures of research centers in Canada, etc.

One of the reasons why diagrid structures are not often designed by our specialists is the complex modeling of structures in calculation complexes. It is important to take into account the geometric and physical nonlinearity in the work of mesh structures under the loads influence in order to obtain the most reliable picture of the spatial structures stress-strain state and stability [15].

Today, the studying methods issue for modeling diagrid structures using software and computer systems is relevant. To do this, it is necessary to consider the main modern modeling structures methods – discrete and continuous.

In a discrete model, edges are represented as diagrid, beam or three-dimensional finite elements. Today, the model can be used to successfully solve problems where restrictions are imposed on critical forces or moments. The reason is the modern computers computational capabilities. Software packages for finite element modeling, such as SCAD and others, allow you to use programming to generate diagrid structures models. These programs help you determine the degree of the diagrid structures parameters influence on the critical force or moment in a short time.



**Figure 1 – Examples of the diagrid structural systems:**

- a – the Shukhov Tower in Ukraine;
- b – "the Gherkin" in London;
- c – the Opera house in Beijing;
- d) the British Museum

The continuum diagrid structure model is characterized by frequently positioned edges. This system is conditionally replaced by a solid shell, the stiffness of which takes an average value. It depends on the layout and stiffness of the forming edges. When using the continuum model, the diagrid structural systems are described by the traditional orthotropic shells equations. In this case, the issue of a correct mathematical model constructing a continuous construction with the most accurate geometric and physical properties of the real construction is relevant.

A new industry development vector was set relatively recently, due to the improvement and widespread introduction of computer technology.

The emergence of new computer-aided design systems and programmable machines allowed us to go beyond the simplest mesh structures configurations, to build not only domes, but also to give objects a shapes variety. Nowadays huge data amounts can be processed automatically, and a large number of similar structural elements that differ in a small parameters range can be created in semi-automatic mode using programs with minimal human involvement.

A characteristic diagrid structures feature is the load-bearing structures absence in the form of various columns, beams. The structure is self-supporting and in most cases has higher load-bearing properties in comparison with other structures types. This is due to the uniform load distribution on all the structure rods, which virtually eliminates brittle destruction. The dome-based structure has good aerodynamic characteristics in addition to its high load-bearing properties, which expands the applications range.

The diagrid structures assembly is carried out in a faster time and requires a magnitude order less labor compared to traditional structures. Installation does not require special construction equipment, equipment and accessories, the main working tool is a wrench.

Membrane materials are often used to cover diagrid structures. The membrane is a high-tech, universal coating. These coatings are easy to transport and install, compact and non-flammable. In a harsh climate, it is possible to use insulated membranes. In addition to the membranes, steel sheet materials, sandwich panels, etc., cut in the form of triangles can be used for coating. They are attached to each other by bolted and riveted connections.

Structures glazing is widely used. Such a coating is most attractive from an architectural and aesthetic point of view, but the glass use as an enclosing structure always leads to higher prices and an increase in metal consumption due to a decrease in the movement tolerance of structural elements and precipitation.

The designing mesh structures process is carried out in specialized computer-aided design systems software complexes. These calculation programs meet all requirements and standards.

The creating a model process and calculating strength includes several stages:

1. The surface shape and size are determined depending on the building or structure purpose, the architectural and design concept, and the customer's

wishes. There are an infinite number of surfaces used in construction.

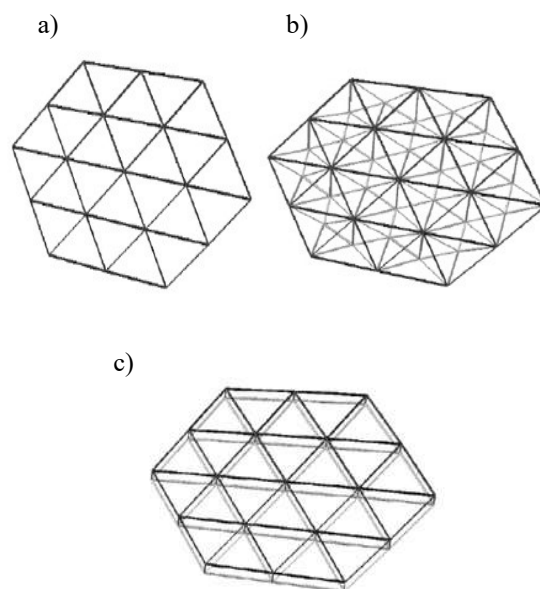
2. The pattern, shape, and size of the triangle grid cell can be different (Fig. 2). The most popular division into triangles, quadrilaterals, and hexagons is a structure based on the icosahedron vector division. A diamond-shaped partitioning system is also used, which is universal and suitable for any free-form mesh structures type. The mesh cells' frame consisting can be single-layer or multi-layer. The frame type is selected depending on the span size, seismic, climatic conditions, and other factors.

In diagrid structures with the less than 30 m diameter, a single-layer rod structure is used. For more than 30 m spans, as a rule, a two-layer truss of the first or second type is used, depending on the surface shape, external loads and the structural elements type choice.

3. When the construction model is ready, it is imported into the calculation program to determine the rod elements stiffness. Depending on the construction area, loads on the structure are selected in accordance with regulatory documents.

4. The last stage is the structure 3D model creation, design and working documentation calculation and development.

After creating the design model, it is imported into CAD, where all further design work will be performed. The construction and calculation of individual nodes and rods are carried out on the basis of the original model with fully integrated CAD tools and do not cause difficulties. However, for a structure containing tens of thousands of such elements, manual design will require a huge amount of time. In addition, the slightest change to the original design model will again require recalculation of all elements.



**Figure 2 – Structures:**

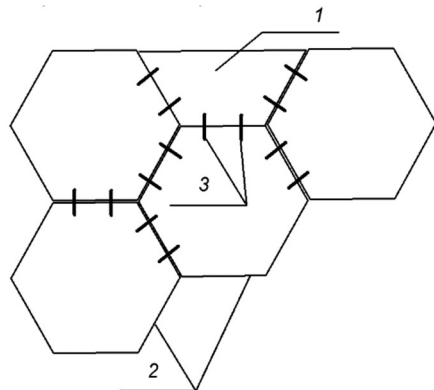
- a – single-layer structure;
- b – two-layer structure (diagonal);
- c – two-layer structure (with racks)

In the course of the study, the single-layer metal covering design with hexagonal elements made of bent channels was proposed (Fig. 3).

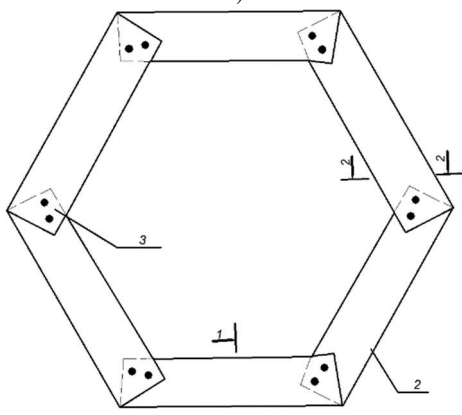
This diagrid design is used in flat and spatial cross systems.

This metal coatings construction type with triangular, quadrilateral elements is mainly used in diagrid domes, diagrid for various purposes. Such structures are common abroad and are effective for large spans and have the architectural expressiveness of buildings.

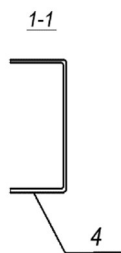
a)



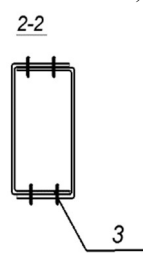
b)



c)



d)



**Figure 3 – Construction of a single-layer metal covering with hexagonal elements made of bent channels:**

- a – a fragment of the coating made of hexagonal standard elements;
- b – a typical hexagonal element made of bent channel is connected on self-tapping screws;
- c – cross-section 1-1 of the bent channel,
- d – cross-section 2-2 of the connection of channel belts on self-tapping screws, (bolts according to calculation) [16]

Notes: end elements (1); hexagonal elements (2) obtained from a straight channel, then bent (4) and connected on self-tapping screws (3) for rigidity.

A special feature of the coating is the use of rectilinear bent channels, which can be cut and connected in the form of a hexagonal element separately (fig.4), or in the form of blocks with the best dimensions for transportation and subsequent installation. Assembly elements of these structures are manufactured in metal structure factories in the form of flat hexagonal elements to simplify delivery and are connected on the construction site using self-tapping screws or bolts according to the calculation.



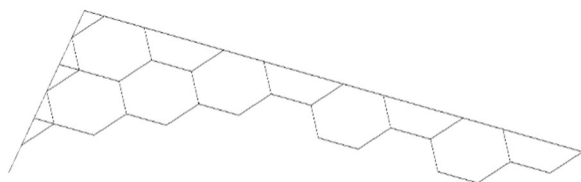
**Figure 4 – Diagram of a hexagon blank from a rectilinear bent channel**

The construction of a metal covering with spatial hexagonal elements from bent channels consists of: mounting end elements; mounting hexagonal elements obtained from a rectilinear channel marked on equal sections and at equal distances and a clipped shelf, then bent and connected on self-tapping screws for rigidity. Hexagons can be transported to the construction site separately, or connected in flat elements to simplify transportation and subsequent large-size installation. The structure is assembled on the ground and only then transferred to the design position on the walls or on the columns.

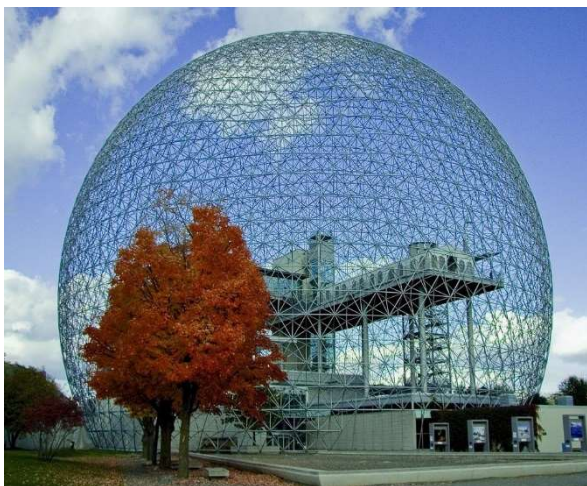
In general, all elements of the proposed design, performed according to the design calculation, are made of channels. A technological feature of the design of a metal covering with spatial hexagonal elements made of bent channels is the use of self-tapping screws on the construction site to obtain structures with minimal weight. Factory-made covering design made of small standard or large-sized flat elements. It is used for steel structures' covering with profiled flooring with spans of 12 m or more.

For larger spans, it is possible to use bent elements of a larger thickness of 4-6 mm, then we allow cutting identical elements with beveled edges and welding together. The element dimensions are determined by spans calculation, respectively, agreed by the customer. These type calculation schemes (fig.5) structures have a rectangle or square configuration. Support occurs along the contour or on both sides. It is possible to use intermediate supports (columns). In calculations, we accept the actual channel cross-sections. The criteria for load-bearing capacity are the ultimate strength and deformation.

Diagrid structural systems are used in the triangular and diamond-shaped grids form. An example of such an application is the dome "Climatron" of B.Fuller (fig. 6).



**Figure 5 – Fragment of the calculation scheme of the SCAD program**



**Figure 6 – The Dome "Climatron" (B. Fuller) [1]**

## Conclusions

The widespread diagrid structural systems use is constrained by the difficulties of designing, modeling, and calculating such structures. It is necessary to introduce new and atypical software complexes and related disciplines into the educational process in order to obtain in-depth knowledge in the programming and mathematical analysis field. The training of new highly specialized engineering personnel will allow us to put into practice the architects ideas.

Discrete and continuous models serve as complementary methods for modeling diagrid structures. The discrete model shows reliable results with a sparse grid step, or a small finite elements number. The continuum model is more suitable for a compacted grid and allows you to use the theory of differential equations to simplify the problem solution.

Diagrid structures are certainly one of the most promising areas in construction. They justify the name "structures of the XXI century", as they were dubbed by engineers. This is evidenced by the steadily growing structures number made using this technology and the customers interest. The technology potential is great, specialists predict special prospects for structures in the almost unlimited architectural forms possibilities, successful application in changing climatic conditions. But to realize this potential, many more developments in the materials science field, design automation, and building codes will need to be completed.

As the analysis result, the design of a single-layer metal covering with hexagonal bent channels elements was proposed.

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