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Transformation of the retaining wall external geometry with rationalizing of system parameters

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The article presents the variable task formulation and implementation of finding a rational outline of the retaining wall back face. In the Coulomb theory framework, an analysis is made of a system consisting of a retaining structure and soil pressing on it for the formulating a rational design problem possibility. The possibility of formulating the problem of finding the rational rear face geometry of the retaining wall within a given horizontal projection is shown. The substantiation of the energy rationalization method operation in solving the problem under consideration is given. The proposed approach allows a variable method to determine the surface configuration of the retaining wall, rational from the standpoint of the accepted criterion. The example given in the work clearly proves the correctness of the problem and its solution statement.

Keywords: retaining wall, curved surface, approximation, algorithm construction, variation approach

Трансформація зовнішньої геометрії конструкції підпірної стіни при раціоналізації параметрів системи

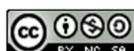
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Подано постановку і реалізацію варіативної задачі пошуку оптимального обрису задньої грані підпірної стіни. У рамках теорії Кулона проаналізовано систему, що складається з підпірної конструкції й ґрунту, що давить на неї, щодо можливості постановки завдання раціонального проектування. На простому прикладі показана можливість постановки задачі пошуку раціональної геометрії задньої грані підпірної стіни в рамках заданої горизонтальної проекції. Наведено обґрунтування експлуатації енергетичного методу раціоналізації при розв'язанні наведеної задачі. Суть пропонованого методу пошуку раціональної геометрії задньої грані підпірної стіни полягає в апроксимації поверхні підпірної стіни ламаною лінією. Для кожної ділянки ламаної лінії виведені ключові залежності по впливу на характер напружено-деформованого стану конструкції, зокрема в цій постановці, на величину згинального моменту в затисканні. Виведено ключові залежності й описано алгоритм розв'язання задачі. Показано, що при заданих характеристиках засипки величина моменту в затисканні фактично може бути описана через комбінацію кутів нахилу кожної з ділянок, а в загальному вигляді таких комбінацій безліч. Задача зведена до пошуку такої комбінації кута α_i , при якій введений критерій (у розглянутій постановці момент у затисканні) займе своє значення внизу. Реалізація підходу продемонстрована на чисельному прикладі. Запропонований підхід дозволяє варіативним методом визначати конфігурацію поверхні підпірної стіни, раціональну з позиції прийнятого критерію. Наведений у роботі приклад наочно доводить коректність постановки задачі та її розв'язання. Використання цього методу доцільне в інформаційному середовищі обчислення. Зокрема, практичне застосування наведеного підходу можливе шляхом постановки і розв'язання завдання лінійного програмування симплекс-методом.

Ключові слова: підпірна стіна, криволінійна поверхня, апроксимація, побудова алгоритму, варіаційний підхід



Introduction

Considering the search for optimal forms of load-bearing elements, it can be recognized as useful "excursions" for specialists who solve complex problems of generating constructive and architectural forms to botany, biology and even physiology. Technical decisions can be effectively parallel with the evolutionary nature of "experience" for a period of tens of millions of years. Construction of optimal forms that are fundamentally close in their constructive geometry to natural bearing (self-bearing) objects of living nature. At the same time, faced with significant cumbersomeness and complexity of mathematical operations generated by the nonlinearity of physical and geometric relationships between the parameters that determine the solution. Considering the rapid development of the IT sphere in recent years, in particular, in the field of building structures design, there is a wide opportunity for creating algorithmic, software methods for finding the optimal forms of load-bearing elements.

Review of research sources and publications

Structural elements of buildings and structures that perceive the sideload from a bulk material refer to systems at which the magnitude and nature of the load directly depend on the configuration of the element that receives this load. The generally accepted theory of the pressure of an incoherent loose material on a side surface, in particular, soil on retaining walls, is the Coulomb theory. [1]. According to this theory, the pressure of the bulk material on the lateral surface depends on the lateral pressure coefficient λ , which is trigonometric depending on the angular parameters of the system (the internal bulk material angle, the angle of wall inclination to the vertical, the angle of filling inclination) [8-12].

In the field of structures research that perceive the side pressure of a bulk material, an exact solution is known in the form of the 4th-degree equation [4, 5]. This equation describes the mutual configuration influence of the bulk material side pressure plot and the curvature of the surface receiving this pressure. This solution is obtained for a static statement. According to the solution, the slope angles of the tangent curve are set, which provides a given pressure profile. Thus, the magnitude and nature of the pressure depend on the surface curvature, and vice versa.

Definition of unsolved aspects of the problem

The described solution demonstrates the mathematical relationship between the curvature of the surface that perceives lateral pressure from a granular medium and the distribution nature of this pressure. At the same time, this solution assumes only a numerical relationship between the parameters mentioned. There is no analytical relationship between the functional describing the configuration of the retaining wall and the lateral pressure plot. At the same time, each individual retaining wall configuration has a unique lateral pressure distribution and hence a unique distribution of internal forces. If the system "retaining wall - backfill soil" is

constrained, for example, in the form of the wall horizontal projection constancy or maximum displacements limitation, it is possible to find its outline, which will predetermine the minimum of the accepted criterion [6,7]. The criterion here can be the volume of material, cost, the maximum principal stresses value, the value of the system deformation potential energy, and others.

Problem statement

Taking into account the presented information, proposed a variable method for finding a rational surface of a retaining wall. The method essence consists etc.

A cantilever retaining wall of arbitrary shape has predetermined external dimensions – horizontal B and vertical H projection of the system. The wall is divided into n linear sections (Fig. 1). Each section has an inherent only slope angle α_i , which lies in the range $\alpha_i \in [\varphi; 90^\circ]$. The horizontal projection of the system defined as:

$$B = \sum_1^n \frac{h}{\operatorname{tg}(\alpha_i)}. \quad (1)$$

The distributed pressure of the bulk material at the base and top of each of the sections $q_{1,2}$ is respectively determined by the expressions:

$$q_{i,1} = (H - h \times i) \gamma \lambda_i; \quad (2)$$

$$q_{i,2} = (H - h(i - 1) \gamma \lambda_i; \quad (2)$$

where: γ – the bulk density;

λ_i – the lateral pressure coefficient of the bulk material.

The distributed pressure of the bulk in the area of the site is reduced to a concentrated force acting normal to the site (Fig. 2), which is defined as

$$Q_i = \frac{q_{i,1} + q_{i,2}}{2} \times \sin(\alpha_i) \times h. \quad (4)$$

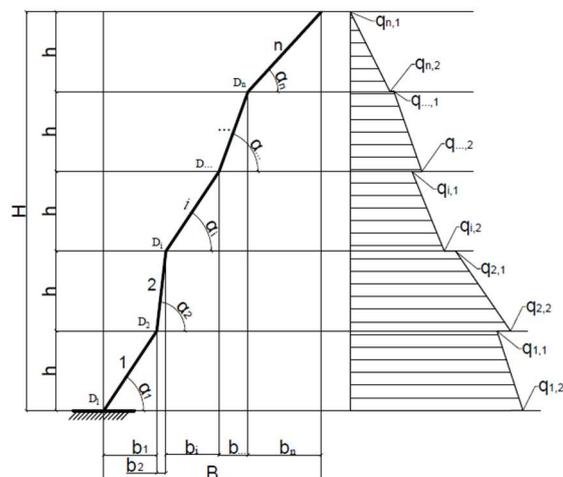


Figure 1 – Curvilinear generatrix approximation

The point coordinates of the concentrated force application relative to the section base are determined by the expressions:

$$h_{0,i} = h \times m ; \quad (5)$$

$$b_{0,i} = \frac{h_{0,i}}{\operatorname{tg}(\alpha_i)} ; \quad (6)$$

$$l_{0,i} = \frac{h_{0,i}}{\sin(\alpha_i)} ; \quad (7)$$

where:

$$m = \frac{2q_{i,1} + q_{i,2}}{2(q_{i,1} + q_{i,2})} . \quad (8)$$

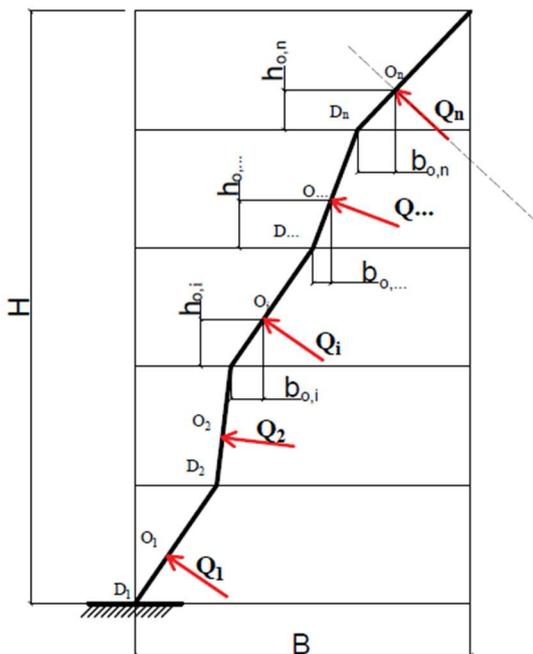


Figure 2 – The pressure of the bulk in the form of concentrated forces

The diagram of the bending moment from the action of the j -th force will have the form shown in Fig. 3. The bending moment magnitude $M_{i,j}$ at the base of the i -th section (point D_i) from the action of the j -th force is determined as

$$M_{i,j} = Q_j \times D_i C_{i,j} ; \quad (9)$$

where $D_i C_{i,j}$ – normal from the base of the i -th section (D_i) to the vector of the j -th force (Fig. 3), determined by the expression:

$$D_i C_{i,j} = \left[\frac{b_{o,j} + \sum_{i=1}^{j-1} b_i}{\operatorname{tg}(\alpha_i)} + h \cdot (j-i) + h_{o,j} \right] \sin(\alpha_i) . \quad (10)$$

The general diagram of moments is the sum of all diagrams M_j , from the force Q_1 to the force Q_n .

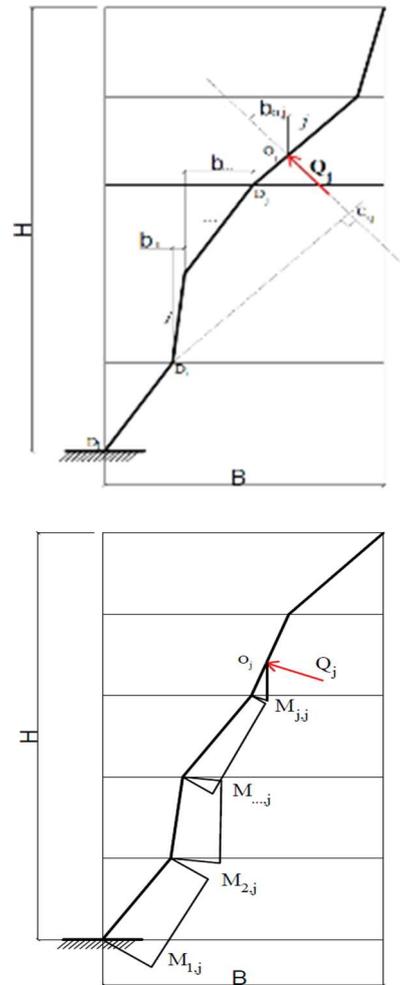


Figure 3 – Plotting bending moments

The energy criterion of rationalization is introduced into consideration. As applied to the problem, its interpretation is as follows: within the given constraints, it is necessary to find a retaining wall configuration, in which the potential energy of deformation (PED) will take a minimum value [2, 3].

Analytically, the potential energy of deformation can be represented as the sum of the partial (PED) $U_{j,tot}$ from the each of forces action in the section from 1 to n of the force

$$U = \sum_1^n U_{j,tot} . \quad (11)$$

The partial PEDs $U_{i,tot}$, from the action of the j -th force are the sum of the volumes of the truncated pyramids (Fig. 4) in the sections from $i = 1$ to $i = j$. On the sections from $i = 1$ to $i = j-1$ at the base of the truncated pyramid (the volume of the truncated pyramids - U_i) there is a square with sides $M_{i,j}$, at the top - a square with sides $M_{i+1,j}$, the height of the pyramid - l_i . The upper pyramid is a special case - a non-truncated pyramid (the volume of a non-truncated pyramid - U_j) of height $l_{o,j}$, at the base of which there is a square with an edge $M_{j,j}$. The particular PED values for each of the sections are determined by the expression:

$$U_{j,tot} = U_j + \sum_1^{j-1} U_i, \quad (12)$$

where:

$$U_i = \frac{M_{i,j}^2 + \sqrt{M_{i,j} \times M_{i+1,j}} + M_{i+1,j}^2}{3} l_i, \quad (13)$$

$$U_i = \frac{M_{j,j}^2 \times l_{o,j}}{3}. \quad (14)$$

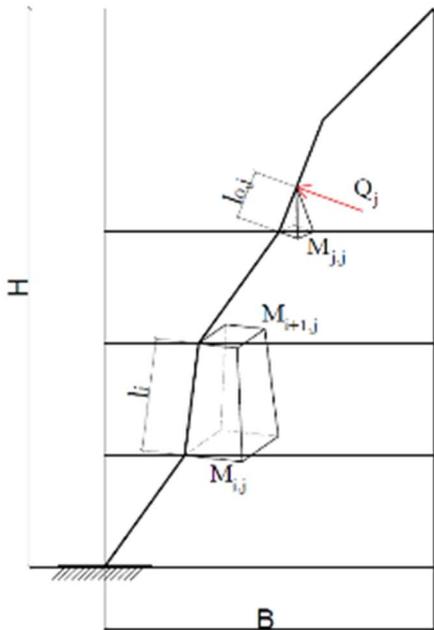


Figure 4 – To the definition of PED from the action of force in the *i*-th section

The arbitrary curvature of the rear face curved generatrix has its unique combination of the inclination angles α_i of each of the sections, which it can be described by. With an increase in the number of sections n in the limit, the broken line tends to a curved outline. With the given characteristics of the bulk (φ, γ), the PED value, in fact, can be described through a combination of the inclination angles α_i of each of the sections, and in general, there is an infinite number of such combinations. Varying the configuration of the retaining wall lateral surface, it is possible to find such a geometry at which the PED value will take a lower value. Then the problem is reduced to the search for such a combination of α_i , for which the introduced criterion (in the considered formulation, PED) will take its lower value.

Basic material and results

The stated formulation of the problem presupposes an iterative method of finding the rational outline of the retaining wall. With an increase in the steps of dividing the system in the horizontal and vertical directions, the number of virtual variants that form structures increases significantly. To determine the number of variants of the structure generators, depending on the magnitude of the horizontal and vertical steps of the partition, we reduce the problem under consideration to the classical problem of combinatorics.

From each row, you need to select any one number m_i so that the sum of all m_i equals $n - 1$:

$$\sum_1^{i=m} m_i = n - 1.$$

In the table 1 n – the number of columns, m – rows. Each line looks like 0,1,2... i .

Table 1 - Determination of the number of variable options

		<i>n</i>				
		1	2	3	...	<i>j</i>
<i>m</i>	1	0	1	2	...	<i>i</i> - 1
	2	0	1	2	...	<i>i</i> - 1
	3	0	1	2	...	<i>i</i> - 1
	...	0	1	2	...	<i>i</i> - 1
	<i>i</i>	0	1	2	...	<i>i</i> - 1

One variation of C is a combination of m -values from 1 to m_i , which adds up to $n-1$. The problem is formed as follows: for a given n and m , how many variations of C can be constructed.

The solution is determined by dependency

$$C_{n+m-1}^{m-1} = \frac{(n+m-2)!}{(m-1)!(n-1)!}.$$

Using the obtained expression, it is possible to determine the number of variants of generators depending on the number of vertical and horizontal sections of the system partition (Table 2). From the given data it follows that when the system is divided into 10 vertical and 10 horizontal sections, the number of variants forming the system under study is close to 100 thousand. The foregoing leads to the need to search for informational ways to solve the problem.

To implement the solution, a script was written in the Dynamo visual programming environment (Fig. 5).

For a clear demonstration of the operation of the proposed approach, we present the simplest example of its applications. Let us set the initial data of the retaining wall and the soil acting on it: vertical projection $H = 10\text{m}$; horizontal projection $B = 5\text{m}$; the volumetric weight of soil $\gamma = 1.5\text{ t/m}^3$; angle of internal friction of the soil $\varphi = 30^\circ$; the number of subdivisions $n = 10$.

In accordance with the formulation of the problem, such a combination of the angles of inclination of each of the sections is determined, which will predetermine the minimum PED of the system. To generate the assigned angle values, the Refinery add-in was used, which is aimed at solving such problems. Here you set such parameters as a limitation for the magnitude of the angles, the magnitude of the horizontal projection, the criterion for finding a rational solution, the accuracy of the solution and the number of options under consideration. Out of the calculated 1000 variants, such a configuration was found in which the PED of the system took the minimum value (Fig. 6).

Table 2 – The number of variants of the generating structure

		Number of sections horizontally, n									
		1	2	3	4	5	6	7	8	9	10
Number of sections vertically, m	2	2	3	4	5	6	7	8	9	10	11
	3	3	6	10	15	21	28	36	45	55	66
	4	4	10	20	35	56	84	120	165	220	286
	5	5	15	35	70	126	210	330	495	715	1001
	6	6	21	56	126	252	462	792	1287	2002	3003
	7	7	28	84	210	462	924	1716	3003	5005	8008
	8	8	36	120	330	792	1716	3432	6435	11440	19448
	9	9	45	165	495	1287	3003	6435	12870	24310	43758
	10	10	55	220	715	2002	5005	11440	24310	48620	92378

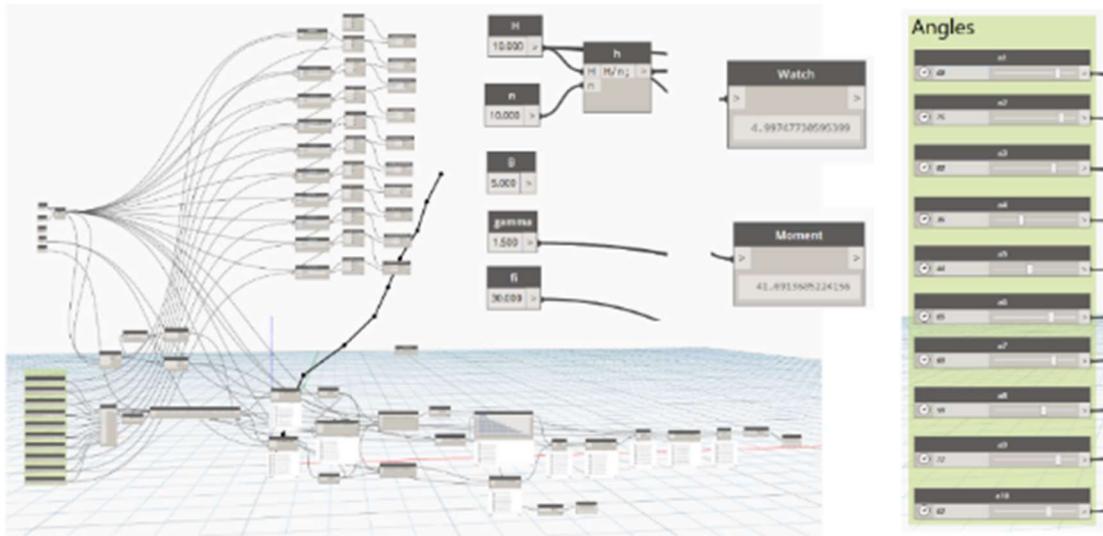


Figure 5 – General view of the Dynamo script

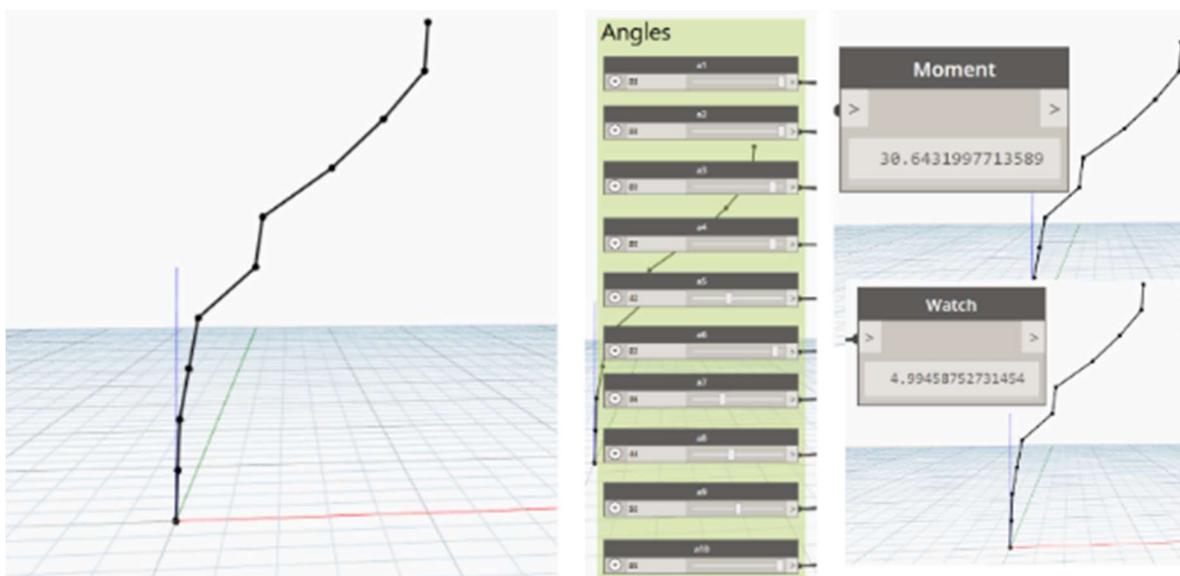


Figure 6 – Solution result

Conclusions

The proposed approach allows using the variational method to determine the configuration of the retaining wall surface that is rational from the standpoint of the accepted criterion. The example given in the work clearly proves the correctness of the problem statement and its solution. In general, with a much larger number of sections n , the broken generatrix of the retaining wall rear face is smoothed and tends to a curvilinear outline. In further research, it is of interest to formulate and solve this problem using linear programming methods.

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