UDC 624.012.35:620.173/174

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STRENGTH OF REINFORCED CONCRETE IN BENDING ELEMENTS CALCULATIONS

The concept of the design strength of reinforced concrete in bending elements is proposed. This concept can be considered as generalized description of concrete. Such approach makes it possible to consider not only the separate strength of concrete and reinforcement, but also their interaction. The design strength of reinforced concrete is determined by ratio of force, which causes destruction of the standard reinforced concrete specimen, to the corresponding geometric characteristic. It was found that using the introduced concept reinforced concrete elements calculations can be reduced to well-known formulas of the strength of materials. It is based on generally accepted hypothesis and stress-strain diagrams of materials. The engineering method is developed, which allows to calculate the strength of the bending reinforced concrete elements of rectangular and circular cross-sections equally simple.

***Keywords:** design strength of reinforced concrete, bending, beam.*

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МІЦНІСТЬ ЗАЛІЗОБЕТОНУ В РОЗРАХУНКАХ ЗГИНАЛЬНИХ ЕЛЕМЕНТІВ

Запропоновано поняття розрахункового опору залізобетону в елементах, що зазнають згинання. Це поняття дозволяє розглядати узагальнену характеристику залізобетону. Такий підхід дає змогу враховувати не тільки окремо міцність бетону та арматури, а і їх взаємодію. Розрахунковий опір залізобетону визначається відношенням зусилля, при якому руйнується еталонний залізобетонний зразок, до відповідної геометричної характеристики. З'ясовано, що, використовуючи введене поняття, розрахунок залізобетонних елементів можна звести до загальновідомих формул опору матеріалів. У його основу покладено загальноприйняті гіпотези та діаграми деформування матеріалів. Розроблено інженерний метод, котрий дозволяє виконувати розрахунки міцності однаково просто згинальних залізобетонних елементів прямокутного та круглого поперечних перерізів.

***Ключові слова:** розрахунковий опір залізобетону, згинання, балка.*

Introduction. Modern methods of calculating of bending concrete elements involve the use of non-linear stress-strain diagrams. This allows to assess the strength of these elements more accurately not only with different reinforcement (single, double, distributed), but different in shape (rectangular, circular, tee, etc.) with common methodological aspects. But implementation of stress-strain diagrams [1, 2] into the practice of calculating of the building structures is needed in computer technology use, which often requires the results of computer testing verification by simple engineering techniques. One of these methods is proposed in this article.

Recent sources of research and publications analysis. Almost all existing approaches to create engineering design procedures of reinforced concrete elements are based on more accurate definition of design values of parameters used in the methods specified in the rules, particularly in the normative document [3]. Many scientists [4 – 8] devoted their works to the development of advanced engineering methods for calculating of the strength of bending reinforced concrete elements sections. Thus in their studies, they show that the use of non-linear stress-strain diagrams of materials in the calculation of the bearing capacity of reinforced concrete elements allows using unified methodology for calculation of almost all kinds of cross-sections and reinforcement. At the same time, these works suggest that for reinforced concrete elements with cross sections other than rectangular, solving many problems even with the use of computer technology is quite complex. Therefore there is still need for specific simplifications, including taking more simple stress-strain diagrams «$\sigma_c - \varepsilon_c$» and «$\sigma_s - \varepsilon_s$», or using specified ratios. This eliminates the majority of engineering techniques universality and the required accuracy of the calculation, which is inherent in a general method based on nonlinear deformation model, implemented with the help of computer technology.

Specifing still unsolved aspects of the problem. Designed for today engineering methods of calculation of the bearing capacity of bending reinforced concrete elements mainly is devoted to elements with rectangular and T cross-section with the single and in some cases double reinforcement. Practical recommendations for the strength calculations of elements of a circular cross-section are largely absent. But these elements can be successfully used in construction practices, and lack of proper engineering calculation methods inhibits their widespread adoption. There is a need for a practical engineering method of calculation of bending reinforced concrete elements with a round cross section because of this.

Assignment formulation. The main objective is to develop practical method of strength calculation of normal sections of reinforced concrete elements with rectangular and round cross-section based on the entered for them design characteristic – strength of reinforced concrete.

Basic material and results. In general, the design strength of the material is determined by the ratio of force, under which the sample of the material is destroyed, to the corresponding geometrical characteristic. Area of the section of specimen is used to find this characteristic under compression. Resistance moment of the section is used in bending. This approach to determination materials characteristics regulates the testing of samples from a single material. As a result, the strength and deformation characteristics of concrete are set without considering the strength of materials, working with it, such as steel reinforcement. To eliminate this gap, this paper proposes to use generalized characteristic – the strength of reinforced concrete – instead independent design characteristics of concrete and steel. Thus, this characteristic is determined by testing samples of reinforced concret. This approach defines reinforced concrete given in standarts as a material formed of concrete and principal reinforcement [1; Б6]. Thus in the process of identifying synthetic material strength like reinforced concrete, the fact that it consists of two materials: concrete and steel is considered. Using this approach, the concept of reinforced concrete design strength as a composite material, in general, is presented in the following formula:

$$f_1(a_1,\ldots a_n) = F_{Ed} / f(b_1,\ldots b_n), \tag{1}$$

where $f_i(a_1,...a_n)$ – design strength of composite material (reinforced concrete) in the section of the element provided the destruction in i-composite (material), MPa;

F_{Ed} – external force factor, the value of which corresponds to the destruction of concrete elements in the considered section;

$f(b_1,...b_n)$ – corresponding geometrical characteristic;

$a_1,...a_n$ – physical and mechanical parameters of materials, cross section of element consists of;

$b_1,...b_n$ – geometric parameters of cross section of reinforced concrete elements.

It is known that the strength of sections of reinforced concrete elements depends not only on the strength of materials, but the location of reinforcement in which (single, double, distributed). Therefore, the design strength of reinforced concrete elements with different reinforcing schemes will have a different value.

Design strength of reinforced concrete is determined by the minimum value of strength provided the section collapse of the element in all materials it consists of,

$$f = \min\left(f_1(a_1,...a_n),...f_i(a_1,...a_n),...f_n(a_1,...a_n)\right), \tag{2}$$

where f – total material design strength (reinforced concrete) of the section.

The following conditions are accepted in order to determine the dependencies for f values:

1. Flat cross-section hypothesis are considered, ie,

$$\frac{1}{r} = \frac{\varepsilon_c}{x} \quad or \quad \varepsilon_c = \frac{1}{r} x, \tag{3}$$

where x – neutral axis depth;

$1/r$ – curvature.

Violation of the flat cross-section hypothesis occurs during the formation of cracks due to sharp reduction of force in the tension zone. That is the so-called «scrapping» of the section in compressed and tensioned zones. The lower the reinforcement ratio is, the greater «scrapped» one is. The section straightens again with further deformation of element. In general, the linear strain distribution in the section of element identifies the principle of minimum potential energy. Thus, the smaller the difference between the time point of cracking and the time point of destruction is, the more strains linearity is disturbed. But elements of this reinforcement ratio is rather limited in building practice, besides there are structural limitations of minimum reinforcement ratio.

2. The stresses in the concrete compressed zone are distributed according to the «stress – strain» diagram that meets the ultimate deformation criteria [4, 5]. This diagram can be obtained by testing standard concrete prisms in hard mode or by transformation diagrams «$\sigma_c - \varepsilon_c$», obtained by testing prisms according to norms.

3. Stresses in reinforcing steel are described by idealized bilinear diagram.

4. Work of tensioned zone of concrete is not included.

The equilibrium equation is worked out using the accepted prerequisites for any section of element shown in Figure 1:

$$N_c = \sum_{i=1}^{n_s} N_{si}\ , \qquad (4)$$

$$M_c = \sum_{i=1}^{n_s} M_{si} = M_{Ed}\ , \qquad (5)$$

where M_c, M_{si} – respectively internal bending moments in concrete and in the i-th reinforcing bar about the neutral axis;

n_s – number of reinforcing bars in the section of the element.

After simple transformations using assumptions, it is got:

$$\int_0^{A_c} \sigma_c\left(x,\varepsilon_c\right) dA_c = E_s A_{si} \sum_{i=1}^{n_s} \left(\frac{k_i d}{x} - 1\right), \qquad (6)$$

$$\int_0^{A_c} \sigma_c\left(x,\varepsilon_c\right) x dA_c + E_s A_{si} \sum_{i=1}^{n_s} \varepsilon_c x \left(\frac{k_i d}{x} - 1\right)^2 = M_{Ed}\ , \qquad (7)$$

where $\sigma_c(x, \varepsilon_c)$ – the function of stress distribution in the concrete compressed zone obtained under the premise 2;

A_{si} – area of the i-th bar in the section of reinforced concrete element.

It is necessary to equate (6) and (7) apply the criteria of destruction to determine the bearing capacity of bending element in the section under consideration. The next dependencies are taken as criteria of destruction:

$$\begin{cases} \dfrac{dM_{Ed}}{d\varepsilon_c} = 0 \quad when \quad \varepsilon_c \le \varepsilon_{cu}, \\ \sigma_{si} = \varepsilon_{si} E_s \le f_{yd}, \quad i = 1 \ldots n, \\ \quad \varepsilon_{si} \le \varepsilon_{su}, \quad i = 1 \ldots n. \end{cases} \qquad (8)$$

In the system of equations (8) ε_{su} – relative ultimate deformations of reinforcing steel meet the yield strength, ε_{cu} – maximum possible relative deformations of compressed concrete.

Expressions (4), (5) and (6) make it possible to calculate the bearing capacity of bending concrete elements, after taking the stress-strain diagram for concrete. This calculation is usually performed by iterations using a computer.

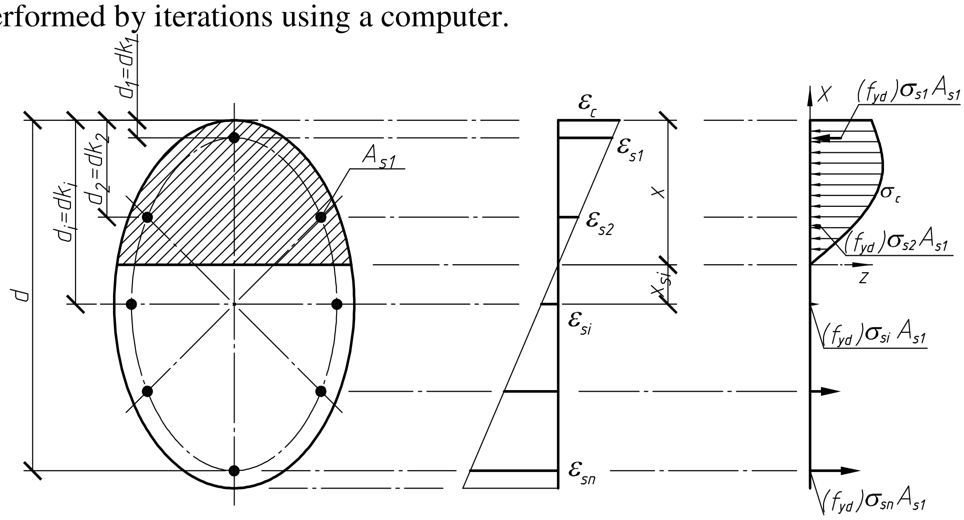

Figure 1 – Stress-strain state of the reinforced concrete bending element

To get the basic equation of reinforced concrete strength considering that in case of rectangular cross sections

$$A_c = b\,d\,, \quad W_c = bd^2/6\,, \tag{9}$$

and in case of circular cross sections

$$A_c = \pi d^2 / 4\,, \quad W_c = \pi d^2 / 32\,, \tag{10}$$

equations (6) and (7) for element with a circular cross section (including addictions (10) and symbols in Figure 2) are brought into view:

$$\frac{\int_0^{A_c} \sigma_c dA_c}{\pi d^2 / 4} = \frac{E_s \rho_f}{n_s} \sum_{i=1}^{n_s} \left(k_i k - 1\right), \tag{11}$$

$$\frac{\int_0^{A_c} \sigma_c x dA_c}{\pi d^3 / 32} + \frac{E_s \rho_f \varepsilon_c}{k n_s / 8} \sum_{i=1}^{n_s} \left(k_i k - 1\right)^2 = \frac{M_{Ed}}{W_c}\,. \tag{12}$$

The following designations are introduced in deriving dependencies (9) and (10):

$$\rho_f = \frac{4 A_{si} n_s}{\pi d^2}, \quad k = \frac{d}{x}, \quad n_a = \frac{a_s}{d} \tag{13}$$

All designations are conformed to ones given by [4, 5].

The type of function-approximation «$\sigma_c - \varepsilon_c$» of stress-strain state of concrete is substantially irrelevant for further change dependencies (11) and (12). The main thing is that it contains conventional parametric points.

Integrals in expressions (11) and (12) can be established by the direct integration of functional relationships or by replacing the curved chart into barchart. The integral sign can be replaced by corresponding summation symbol in this case. Inacurracy of this method is equally predetermined when a sufficient number of areas. The transcendental equations will be received with direct integration of analytical expression «$\sigma_c - \varepsilon_c$», solution of which can be performed by iterative methods.

The second method is considered in more details. The compressed zone of circular cross-section is presented with a certain number of parts (Fig. 2). The strain values is assumed constant within each of parts and those that are corresponded to the value of strain at the gravity centre of each rectangle (Fig. 2).

In this case, the internal forces in the concrete compressed zone are determined by the formulas:

$$N_c = \int_0^{A_c} \sigma_c\left(x, \varepsilon_c\right) dA_c = \sum_{i=1}^{n_c} \sigma_{ci}\left(\varepsilon_{ci}\right) b_i h_n\,, \tag{14}$$

$$M_c = \int_0^{A_c} \sigma_c\left(x, \varepsilon_c\right) dA_c = \sum_{i=1}^{n_c} \sigma_{ci}\left(\varepsilon_{ci}\right) b_i h_n x_{li}\,. \tag{15}$$

The strains of the concrete compressed zone of the *i*-th part are defined by the expression

$$\varepsilon_{c,i} = \varepsilon_c \left(1 + \frac{1 - 2i}{2 n_c}\right), \tag{16}$$

where n_c – number of areas on which the concrete compressed zone is broken; $i = 1 \ldots n_c$.

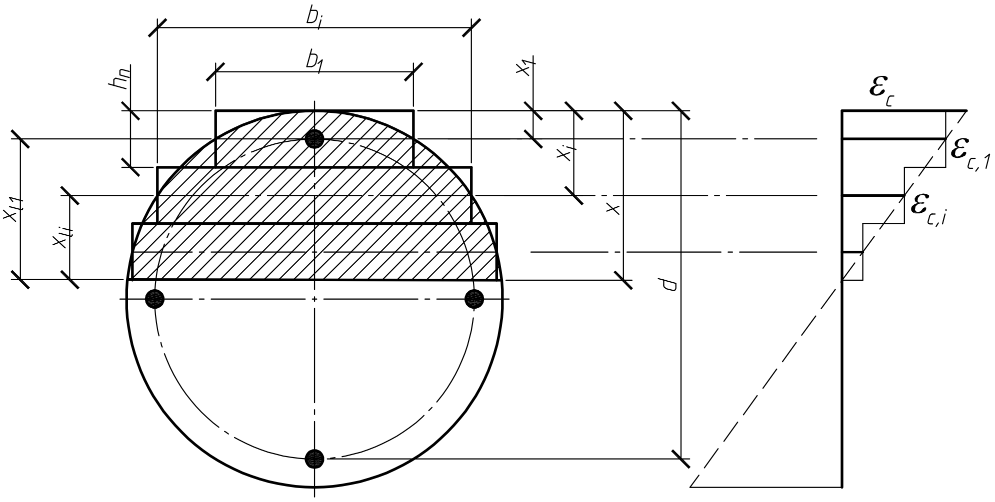

Figure 2 – To the definition of internal forces in the concrete compressed zone of a circular profile element

Finally, equations (11) and (12) are taken the following form after simple transformations:

$$\sum_{i=1}^{n_c}\left(\sigma_c(x...)\frac{8\varepsilon_c(1+m)}{\pi n_c k}\sqrt{\frac{(1+n_a)^2}{4}-\left(\frac{1+n_a}{2}+\frac{m}{k}\right)^2}\right)=\frac{E_s\rho_f}{n_s}\sum_{i=1}^{n_s}(k_i k-1), \tag{17}$$

$$\sum_{i=1}^{n_c}\left(\sigma_c(x...)\frac{\varepsilon_c(1+m)^2}{\pi n_c k^3/64}\sqrt{-m(k+n_a k+m)}\right)+\frac{E_s\rho_f\varepsilon_c}{kn_s/8}\sum_{i=1}^{n_s}(k_i k-1)^2=\frac{M_{Ed}}{W_c}, \tag{18}$$

where

$$m=(1-2i)/n_c. \tag{19}$$

The left side of expression (18) in conjunction with criteria (8) is the formula for determining the design strength of reinforced concrete of the circular section of bending elements, ie,

$$f_{zM}=\sum_{i=1}^{n_c}\left(\sigma_c(x...)\frac{\varepsilon_c(1+m)^2}{\pi n_c k^3/64}\sqrt{-m(k+n_a k+m)}\right)+\frac{E_s\rho_f\varepsilon_c}{kn_s/8}\sum_{i=1}^{n_s}(k_i k-1)^2. \tag{20}$$

Similar expressions are obtained for reinforced concrete bending elements with rectangular cross-section with a single, double and distributed reinforcement [8]. The concept of the strength of reinforced concrete can be used in calculations of bearing capacity of reinforced concrete elements. The tables of the strength of reinforced concrete values are stacked depending on the reinforcement ratio and the strength class of concrete and reinforcement.

Tables 1 – 3 shows the values of the design strength of reinforced concrete for various classes of materials and reinforcement ratios for cross-section of the bending elements. Function (3.4) in the norms [1] is accepted as the stress-strain curve of concrete "$\sigma_c - \varepsilon_c$" in calculating these values.

The general condition of normal sections strength of the reinforced concrete elements in bending is given by

$$f_{zM} \geq \frac{M_{Ed}}{W_c}, \tag{21}$$

where $f_{zM} = f(C, \rho_f, f_{yd})$– design strength of reinforced concrete in the bending elements taken from Tables 1 – 3;

W_c – elastic moment of resistance of the effective section of concrete: $W_c = bd^2/6$ – for the rectangular cross-sections;

$W_c = \pi d^3/32$ – for the circular sections.

Condition (21) allows to solve all problems with the design of reinforced concrete elements: to check for their carrying capacity in the considered section, to define reinforcement. This method allows to reduce calculation method of reinforced concrete elements to classical strength of materials method, but considering its nonlinear deformation. It should be noted that the proposed method of calculation is not simplified. After all, the strength of the cross-section calculated using the formula (21) for the corresponding values of reinforcement ratios coincides with the strength determined by computer according to the norms [1] in the relevant software systems.

The method is universal. It can be used for various loads: permanent, cyclic, impact, dynamic and others. It is necessary to obtain stress-strain diagram for concrete under corresponding loads at first. The proposed method of calculation significantly reduces the process of designing the reinforced concrete elements and in many cases allows to avoid errors when using the computer.

The application of the proposed method is illustrated by the example of definition of the required amount of reinforcement in reinforced concrete elements.

Task 1 (see Fig. 3). Calculate the area of reinforcement required for reinforced concrete bending elements with concrete class *C20/25* and steel class *A400C* if the external bending moment $M_{Ed} = 150\ kHm$. The sections of the elements are:

1) rectangular section with single reinforcement, $b \times d = 40 \times 55\ cm$;

2) rectangular section with double reinforcement, $b \times d = 40 \times 55\ cm$;

3) round section diameter $D = 60\ cm$ with the effective depth of the section $d = 55\ cm$.

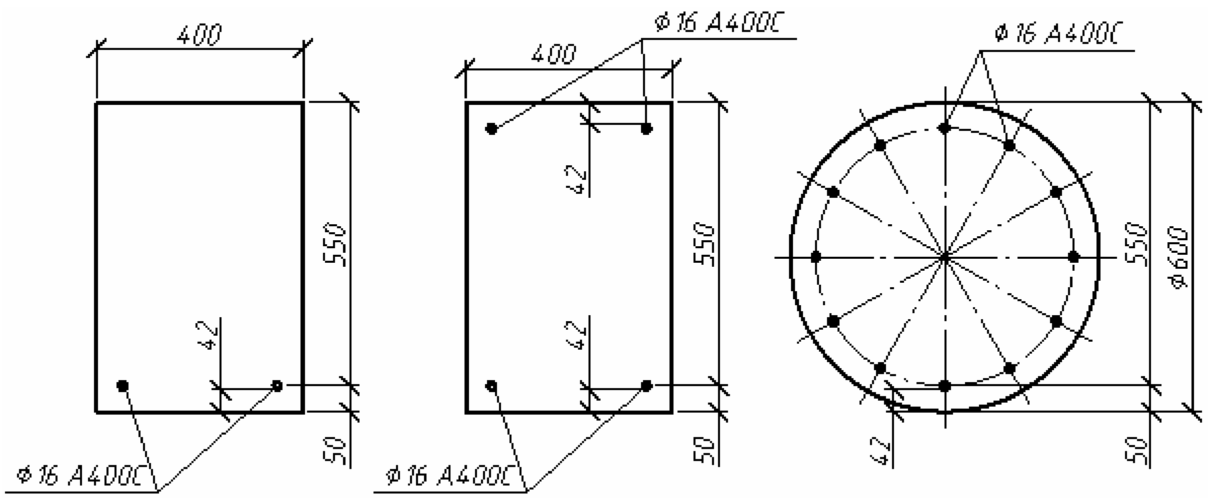

Figure 3 – To determination the cross-sectional area of reinforcement for bending elements

Solution. The required design strength of reinforced concrete:
– for rectangular sections

$$f_{zM} = \frac{M_{Ed}}{W_c} = \frac{6M_{Ed}}{bd^2} = \frac{6\times150\times10^3}{40\times55^2} = 7,44\ MPa;$$

– for circular sections

$$f_{zM,r} = \frac{M_{Ed}}{W_c} = \frac{M_{Ed}}{0,1d^3} = \frac{150\times10^3}{0,1\times55^3} = 9,02\ MPa.$$

The necessary reinforcement ratios are finded in corresponding tables:
– rectangular section with a single reinforcement $\rho_{f1}=0,363\%$;
– rectangular section with double reinforcement $\rho_{f2}=0,355\%$;
– circular section $\rho_{f3}=0,722\%$.
Determine the cross-sectional area of reinforcement:
– rectangular section with a single reinforcement
$A_s=\rho_{f1}\times bd=0,363/100\times40\times55=7,99$ *см*^{2},
take *4∅16 A400C*, $A_s=8,04$ *см*^{2} (see Fig. 3);
– rectangular section with double reinforcement
$A_s=A_{sc}=\rho_{f2}\times bd=0,355/100\times40\times55=7,81$ *см*^{2},
take *4∅16 A400C*, $A_s= A_s=8,04$ *см*^{2},
$A_s+A_s=16,08$ *см*^{2} (see Fig. 3);
– circular section
$A_s=\rho_{f3}\times\pi d^2/4=0,722/100\times3,14\times55^2/4=17,15$ *см*^{2},
take *12∅16 A400C*, $A_s=24,13$ *см*^{2} (see Fig. 3).

Table 1 – The design values of reinforced concrete strength f_{M1} for bending elements with a single reinforcement (f_{yd} = 375 MPa; A400C), MPa

Concrete class	Reinforcement ratio ρ_f							
	0.05	0.5	1	1.25	1.75	2	2.5	3
C8/10	1.078	9.205	13.545	13.992	14.590	14.802	15.123	15.354
C12/15	1.083	9.714	16.956	18.643	19.626	19.982	20.531	20.936
C16/20	1.086	10.033	18.233	21.646	25.196	25.736	26.582	27.217
C20/25	1.088	10.220	18.981	22.814	29.359	30.978	32.138	33.021
C25/30	1.089	10.326	19.403	23.474	30.679	33.807	36.553	37.645
C30/35	1.089	10.405	19.718	23.966	31.643	35.072	40.687	42.062
C32/40	1.090	10.465	19.961	24.345	32.386	36.043	42.630	46.307
C35/45	1.091	10.522	20.189	24.702	33.086	36.957	44.058	50.238
C40/50	1.091	10.561	20.342	24.941	33.554	37.568	45.013	51.679
C45/55	1.091	10.592	20.468	25.137	33.939	38.071	45.799	52.810
C50/60	1.092	10.623	20.590	25.328	34.313	38.560	46.563	53.911

Table 2 – The design values of reinforced concrete strength f_{M1} for bending elements with double symmetrical reinforcement (f_{yd} = 375 MPa, A400C, n = 0,06-0,1), MPa

Concrete class	Reinforcement ratio ρ_f						
	0.05	0.5	1	1.25	1.5	2	2.5
C8/10	1.089	10.341	20.621	25.764	30.907	41.196	51.486
C12/15	1.108	10.370	20.647	25.788	30.930	41.216	51.504
C16/20	1.128	10.405	20.679	25.818	30.959	41.243	51.529
C20/25	1.146	10.438	20.712	25.851	30.990	41.272	51.556
C25/30	1.159	10.466	20.740	25.877	31.016	41.297	51.580
C30/35	1.171	10.494	20.766	25.904	31.042	41.322	51.604
C32/40	1.182	10.520	20.794	25.930	31.068	41.347	51.628
C35/45	1.194	10.550	20.827	25.962	31.098	41.376	51.656
C40/50	1.203	10.574	20.855	25.989	31.124	41.399	51.678
C45/55	1.212	10.597	20.882	26.017	31.151	41.423	51.699
C50/60	1.221	10.623	20.915	26.050	31.184	41.453	51.726

Table 3 – The design values of reinforced concrete strength f_{M1} for bending elements with a circular cross-section (f_{yd} = 375 MPa, A400C), MPa

Concrete class	Reinforcement ratio ρ_f						
	0.05	0.5	1	2	3	4	5
C8/10	0.868	6.852	12.716	23.338	33.651	43.931	54.200
C12/15	0.887	7.161	13.138	24.278	34.619	44.888	55.128
C16/20	0.901	7.473	13.583	25.232	35.634	45.873	56.055
C20/25	0.909	7.643	13.963	25.738	36.480	46.678	56.787
C25/30	0.915	7.769	14.251	26.141	37.089	47.248	57.285
C30/35	0.919	7.877	14.503	26.508	37.592	47.709	57.672
C32/40	0.922	7.969	14.720	26.842	38.033	48.126	58.034
C35/45	0.925	8.061	14.932	27.184	38.420	48.486	58.332
C40/50	0.927	8.122	15.073	27.426	38.685	48.745	58.559
C45/55	0.928	8.171	15.181	27.621	38.856	48.909	58.696
C50/60	0.930	8.218	15.281	27.814	39.015	49.071	58.839

Conclusions. The application of idea of the design strength of reinforced concrete allows to obtain simple calculations of bearing capacity of reinforced concrete elements. The proposed method of calculating the strength of reinforced concrete bending elements with rectangular and circular profiles by applying the design characteristics of strength of reinforced concrete can be used in engineering practice.

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Received 22.11.2016