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Method and example of calculation of combined reinforced bending elements

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A method for calculating rectangular cross-section bending elements, reinforced with ordinary and prestressed reinforcement, as well as steel fiber, based on the deformation method. This takes into account stress losses in the reinforcement from creep deformations and shrinkage of steel-fiber-concrete. The increase in compressive strength of reinforced concrete under biaxial compression conditions is also taken into account. The bearing capacity calculation results of a standard road plate P60.38 and a similar plate with metal fiber are given.

Keywords: bearing capacity, steel-fiber-concrete, bending moment, curvature, prestressed reinforcement, relative deformations, stresses in reinforcement, stresses in steel-fiber-concrete

Методика та приклад розрахунку комбіновано армованих згинальних елементів

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Сучасне будівництво неможливе без використання залізобетонних конструкцій з попередньо-напруженою арматурою. Для покращення міцністних та деформативних характеристик бетону використовують фіброве армування. Серед таких фібр найбільш широко використовується стальна фібра. Вона значно покращує міцність сталефібробетону на розтяг. Це дає можливість враховувати роботу сталефібробетону в розтягнутій зоні перерізу згинальних елементів. Діючі нормативні документи не дають не дають рекомендацій щодо розрахунку сталефібробетонних плоских елементів, які працюють у двох напрямках. Відсутні рекомендації щодо розрахунку сталефібробетонних елементів з попередньо-напруженою арматурою. Дослідження несучої здатності, тріщиностійкості та деформацій двохосно попередньо-напружених сталефібробетонних плит практично відсутній. Запропонована методика розрахунку згинальних елементів прямокутного перерізу, армованих звичайною та попередньо-напруженою арматурою, а також стальною фіброю, на основі деформаційного методу. При цьому враховуються втрати напружень в арматурі від деформацій повзучості та усадки сталефібробетону. Також враховується зростання міцності сталефібробетону на стиск в умовах двохосного обтиску. У результаті порівняльного розрахунку несучої здатності стандартної дорожньої плити П60.38 та аналогічної плити, у якій арматурні сітки були замінені стальною фіброю, встановлено, що несуча здатність плити зі стальною фіброю вища від стандартної на 24,4%. Ефективність плит зі стальною фіброю полягає в тому, що використанні стальної фібри дає можливість зменшення кількості високоміцної попередньо-напруженої арматури до 10…15%. Завдяки хорошим властивостям сталефібробетону протидії стиранню тривалість експлуатації плит аеродромних та дорожніх покриттів набагато більша від залізобетонних.

Ключові слова: плити для покриття доріг, несуча здатність, сталефібробетон, згинальний момент, кривизна, попередньо-напружена арматура, відносні деформації, напруження в арматурі, напруження в сталефібробетоні

Introduction

Modern construction is impossible without using reinforced concrete structures with prestressed reinforcement. Fiber reinforcement is used to improve the concrete strength and deformation characteristics. Among such fibers, steel fiber is the most widely used. It significantly improves the tensile strength of steel-fiber-concrete (SFC). This makes it possible to take into account the SFC operation in the bending elements stretched cross-sectional area. Current regulations do not provide recommendations for the SFC flat elements calculation that work in two directions. There are no recommendations for the SFC elements with prestressed reinforcement calculation. The bearing capacity study and crack biaxially prestressed SFC road slabs resistance is practically absent. The purpose of the work is a comparative calculation of standard road slabs and slabs with metal fiber.

Review of research sources and publications

In recent decades, extensive research on structures with inclusions of various fibers [10-13, 16, 17]. Fiber significantly improves the tensile strength of concrete. Steel fiber has proved itself very well. Recent studies of steel-fiber-concrete (SFC) have shown a positive effect on the work of bending elements. SFB is well resistant to abrasion. Therefore, it can be very effectively used in slabs of road and airfield surfaces.

Definition of unsolved aspects of the problem

The current regulations do not recommend the calculation of rectangular elements reinforced with ordinary and pre-stressed reinforcement, as well as steel fiber. There are also no recommendations for the calculation of the SFB of flat elements that work in two directions.

Problem statement

The purpose of this work is to develop a method for calculating the bending elements of rectangular cross section, reinforced with ordinary and pre-stressed reinforcement, as well as steel fiber, based on the deformation method. When calculating it is necessary to take into account stress losses in the reinforcement from creep deformations and shrinkage of steel-fiber-concrete. It is also necessary to take into account the increase in compressive strength of reinforced concrete under conditions of biaxial compression.

Basic material and results

Method of calculation of combined reinforced bending elements.

Consider a bending element of rectangular cross-section, reinforced with steel fiber and rod ordinary and prestressed reinforcement in compressed and stretched cross-sectional areas. The stress-strain state of a rectangular combined-reinforced section is shown in Fig. 1.

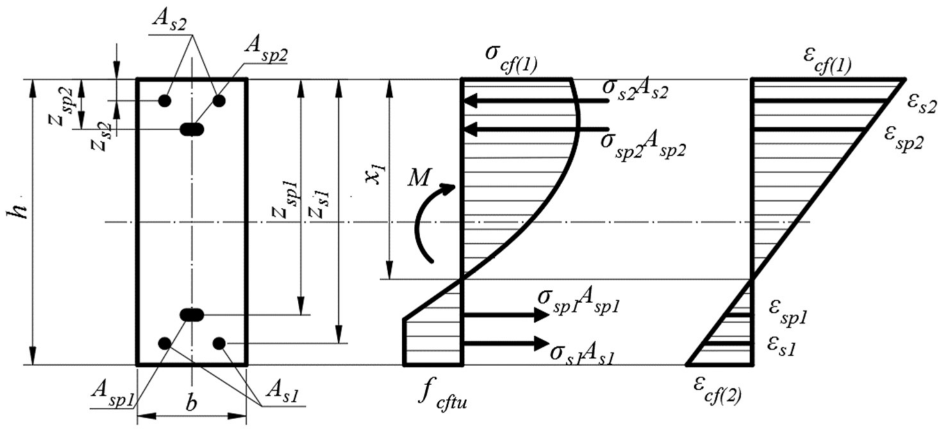

Figure 1 – Stress-strain state of a rectangular combined-reinforced section

Achieving fiber deformations of limit values is accepted as a criterion of bearing capacity exhaustion on a SFB normal section of an element $\varepsilon_{cftu} = -2\, f_{cftu} / E_{cf}$. The value of the ultimate bending moment for the SFB of bending elements of rectangular cross-section with pre-stressed reinforcement is recommended to be determined by the formulas (Fig. 1):

$$\frac{bf_{cf}k_c}{\aleph}\sum_{k=1}^{5}\frac{a_k}{k+1}\gamma^{k+1}-\frac{3}{4}bf_{cft}(h-x_1)+ +\sum_{i=1}^{n}\sigma_{si}A_{si}=0; \quad (1)$$

$$\frac{bf_{cf}k_c}{\aleph^2}\sum_{k=1}^{5}\frac{a_k}{k+1}\gamma^{k+2}-\frac{11}{24}bf_{cft}(h-x_1)^2+ +\sum_{i=1}^{n}\sigma_{si}A_{si}(x_1-z_{si})-M=0. \quad (2)$$

In dependences (1), (2) according to [5] $\aleph$ – the curvature of the curved axis in cross-section (1/m):

$$\aleph=\left(\frac{1}{r}\right)=\frac{\varepsilon_{c(1)}-\varepsilon_{c(2)}}{h}; \quad (3)$$

$\varepsilon_{c(1)}$ – relative deformations of steel-fiber-concrete in the compressed cross-sectional area;

$\varepsilon_{c(2)}$ – relative deformations of steel-fiber-concrete in the stretched cross-sectional area;

γ – the ratio of relative compression strains $\varepsilon_{c(1)}$ to the limit ε_{cf1}:

$$\gamma = \frac{\varepsilon_{c(1)}}{\varepsilon_{cf1}}; \quad (4)$$

x_1 – the height of the compressed zone (m):

$$x_1 = \frac{\varepsilon_{c(1)}}{\aleph}; \quad (5)$$

$\overline{\aleph}$ – relative curvature:

$$\overline{\aleph} = \frac{\aleph}{\varepsilon_{cf1}}; \quad (6)$$

σ_{si} – stresses in reinforceng rod;

z_{si} – distance from the reinforcement gravity center to the extreme verge compressed section;

a_k – the polynomial coefficients, which are determined depending on the value of the SFC compressive strength according to the method [8].

We present equations (1), (2) in the form

$$N_{cf} - N_{cft} + N_s = 0; \quad (7)$$

$$M_{cf} + M_{cft} + M_s = M; \quad (8)$$

where: N_{cf}, M_{cf} – efforts in the compressed zone of the SFC;

N_{cft}, M_{cft} – efforts in the stretched zone of the SFC;

N_s, M_s – total effort in reinforcing rods.

Let's describe the value of internal efforts

$$N_{cf} = \frac{bf_{cf}}{\overline{\aleph}} \sum_{k=1}^{5} \frac{a_k}{k+1} \gamma^{k+1}; \quad (9)$$

$$N_{cft} = \frac{3}{4} bf_{cft}(h - x_1); \quad (10)$$

$$N_s = \sigma_{s2} A_{s2} + \sigma_{sp2} A_{sp2} - \sigma_{s1} A_{s1} - \sigma_{sp1} A_{sp1} \quad (11)$$

$$M_{cf} = \frac{bf_{cf}}{\overline{\aleph}^2} \sum_{k=1}^{5} \frac{a_k}{k+1} \gamma^{k+2}; \quad (12)$$

$$M_{cft} = \frac{11}{24} bf_{cft}(h - x_1)^2; \quad (13)$$

$$\begin{aligned} M_s &= A_{s1} E_{s1} \aleph (x_1 - z_{s1})^2 + \\ &+ A_{sp1} E_{sp1} (\aleph (x_1 - z_{sp1}) - \varepsilon_{p01})(x_1 - z_{sp1}) + \\ &+ A_{s2} E_{s2} \aleph (x_1 - z_{s2})^2 + \\ &+ A_{sp2} E_{sp2} (\aleph (x_1 - z_{sp2}) - \varepsilon_{p02})(x_1 - z_{sp2}) \end{aligned} \quad (14)$$

where: ε_{p0i} – strain caused by prestressing reinforcement with all the losses.

Tension in normal and prestressing reinforcement:

$$\sigma_{si} = E_{si} \aleph (x_1 - z_{si}); \quad (15)$$

$$\sigma_{spi} = E_{spi} (\aleph (x_1 - z_{spi}) - \varepsilon_{p0i}). \quad (16)$$

Substituting expressions (5), (6), (15), (16) in equation (9) – (11), we obtain:

$$N_{cf} = \frac{bf_{cf} \varepsilon_{cf1}}{\aleph} \sum_{k=1}^{5} \frac{a_k}{k+1} \gamma^{k+1}; \quad (17)$$

$$N_{cft} = \frac{3}{4} bf_{cft} \left(h - \frac{\varepsilon_{cf1}}{\aleph}\right); \quad (18)$$

$$\begin{aligned} N_s &= A_{s2} E_{s2} \aleph (x_1 - z_{s2}) + \\ &+ A_{sp2} E_{sp2} (\aleph (x_1 - z_{sp2}) - \varepsilon_{p02}) - \\ &- A_{s1} E_{s1} \aleph (x_1 - z_{s1}) - \\ &- A_{sp1} E_{sp1} (\aleph (x_1 - z_{sp1}) - \varepsilon_{p01}). \end{aligned} \quad (19)$$

Substituting equation (17) – (19) into (7) and after transformations, we obtain the dependence for curvature

$$\aleph = \frac{-b_\Sigma + \sqrt{b_\Sigma^2 - 4 a_\Sigma c_\Sigma}}{2 a_\Sigma}, \quad (20)$$

where:

$$\begin{aligned} a_\Sigma &= A_{s1} E_{s1} z_{s1} + A_{s2} E_{s2} z_{s2} + \\ &+ A_{sp1} E_{sp1} z_{sp1} + A_{sp2} E_{sp2} z_{sp2}; \end{aligned} \quad (21)$$

$$\begin{aligned} b_\Sigma &= \frac{3}{4} bhf_{cft} - \varepsilon_{cf(1)} (A_{s1} E_{s1} + A_{s2} E_{s2} + \\ &+ A_{sp1} E_{sp1} + A_{sp2} E_{sp2}) + \\ &+ A_{sp1} E_{sp1} \varepsilon_{p01} + A_{sp2} E_{sp2} \varepsilon_{p02}; \end{aligned} \quad (22)$$

$$c_\Sigma = -\frac{3}{4} bf_{cft} \varepsilon_{cf(1)} - bf_{cf} \varepsilon_{cf1} \sum_{k=1}^{5} \frac{a_k}{k+1} \gamma^{k+1}. \quad (23)$$

After determining the curvature $\aleph$, its values are substituted into formulas (12) – (14) to determine the moments M_{cf}, M_{cft}, M_s. After that, by formula (8) determine the bending moment M, which corresponds to the curvature $\aleph$. The calculation is performed step by step for each value of relative deformations in the compressed cross-sectional area $\varepsilon_{c(1)}$, which consistently increases in magnitude $\Delta\varepsilon_{c(1)}$.

At each step of the calculation necessary to control the tension in the prestressed reinforcement, which is located in a stretched zone section. To do this, use the diagram "σ - ε" for stressed steel (Fig. 2) [1]. Upon reaching the stress values $\sigma_{sp} \geq f_{pd}$ in the following steps, the stress in the prestressed reinforcement must be determined by the formula [8]

$$\sigma_{sp} = f_{pd} + \left(\frac{f_{pk}}{\gamma_s} - f_{pd}\right) \cdot \frac{\varepsilon_{sp} - \varepsilon_{p0}}{\varepsilon_{ud} - \varepsilon_{p0}} \quad (24)$$

where:

$$f_{pd} = \frac{f_{p0,1k}}{\gamma_s}; \quad \varepsilon_{p0} = \frac{f_{pd}}{E_p};$$

$$\varepsilon_{sp} = \aleph (x_1 - z_{sp}) - 0.0021.$$

To determine the bearing capacity of SFC with prestressed reinforcement developed an algorithm, which is implemented in the program MathCAD.

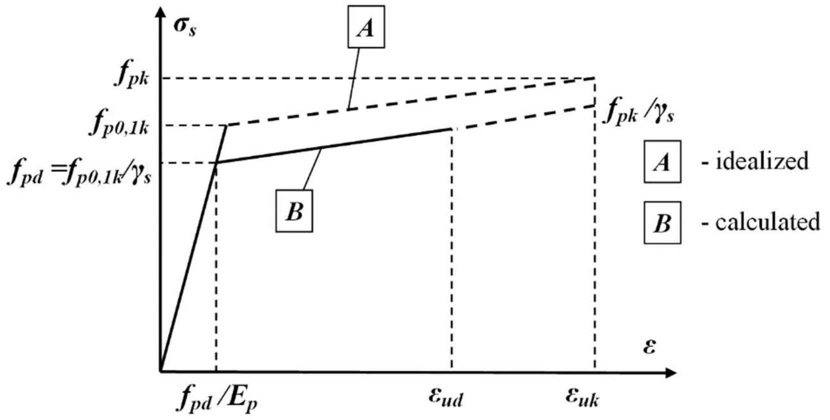

Figure 2 – Idealized and calculated diagram "σ-ε" for stressed steel

Comparative calculation of biaxially prestressed road slab.

Prefabricated reinforced concrete prestressed brands slabs are used for paving P60.38, P60.35, P60.30, which correspond to the current DSTU [4]. Plates of the brand P60.38 have the sizes in the plan 6,0×3,75 m, brand P60.35 – 6,0×3,5 m, brand P60.30 – 6,0×3,0 m and a thickness of 140 mm (Fig. 3).

Slabs are made of concrete class C25/30 and reinforced with pre-stressed reinforcement in two directions (Fig. 3). The plate P60.38 is reinforced with 24Ø10800 fittings located in the longitudinal direction of the plate in two levels and 18Ø12A800 located in the transverse direction of the plate in the center. The plate P60.35 is reinforced with 22Ø10A800, fittings located in the longitudinal direction of the plate in two levels and 18Ø12A800 located in the transverse direction of the plate in the center. The plate P60.30 is reinforced with 20Ø10A800, fittings located in the longitudinal direction of the plate in two levels and 18Ø12A800 located in the transverse direction of the plate in the center.

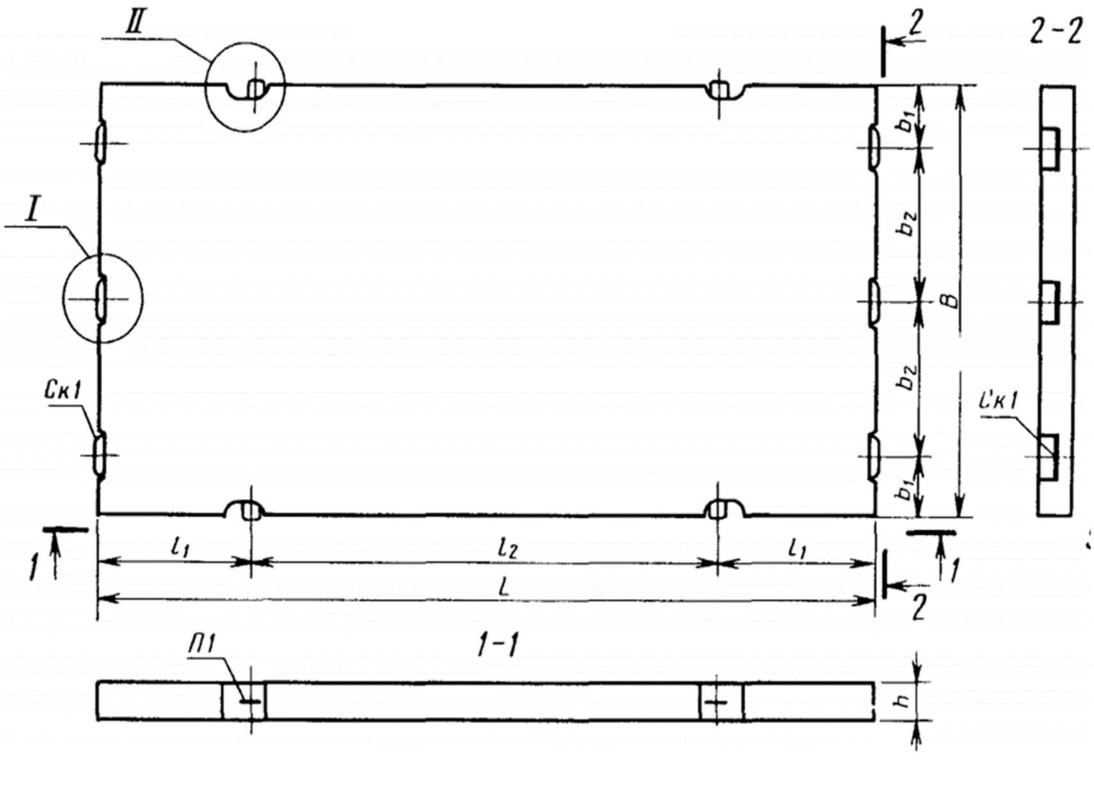

Figure 3 – Prefabricated reinforced concrete slabs of brands P60.38, P60.35, P60.30

Calculation of the bearing capacity of the plate P60.38 without metal fiber

Section dimensions b=3750 mm, h=140 mm (Fig. 4).

Prestressed reinforcement in two levels on 12Ø10A800: A_{sp1}=A_{sp2}=942 mm^2; E_{sp}=190000 MPa; f_{pk}=840 MPa; $f_{p0,1k}$=765 MPa; ε_{uk}=0,018. Elongation of reinforcement taking into account all losses makes ε_{sp0}= -0,002. distance from the center of gravity of reinforcement to the extreme verge compressed section z_{sp1}=105 mm, z_{sp2}=35 mm. Reinforcement pitch along the axis perpendicular to the calculated one S=350 mm.

Concrete class C25/30: f_{cd}=17,0 MPa, E_{cd}=25000 MPa, ε_{c1}=0,00169, ε_{cu}=0,00328.

Polynomial coefficients:

a_1	a_2	a_3	a_4	a_5
2,7404	-2,7649	1,3416	-0,35004	0,03295

The calculation results are summarized in the graph "moment-curvature", which is shown in Fig. 5. The plate bearing capacity is M_u=144,9 kNm.

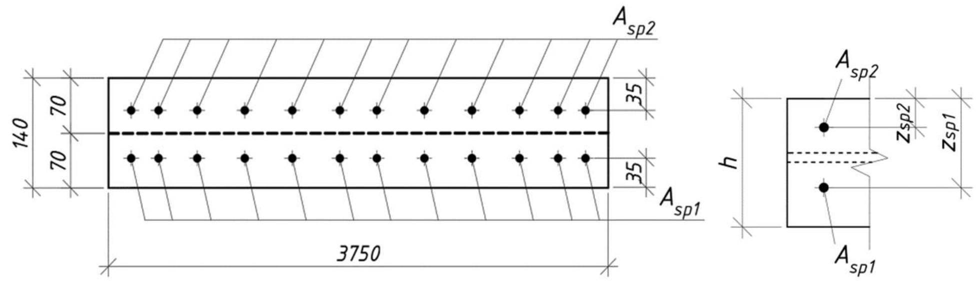

Figure 4 – The calculated cross-section of the plate P60.38 (a) and calculated parameters (b)

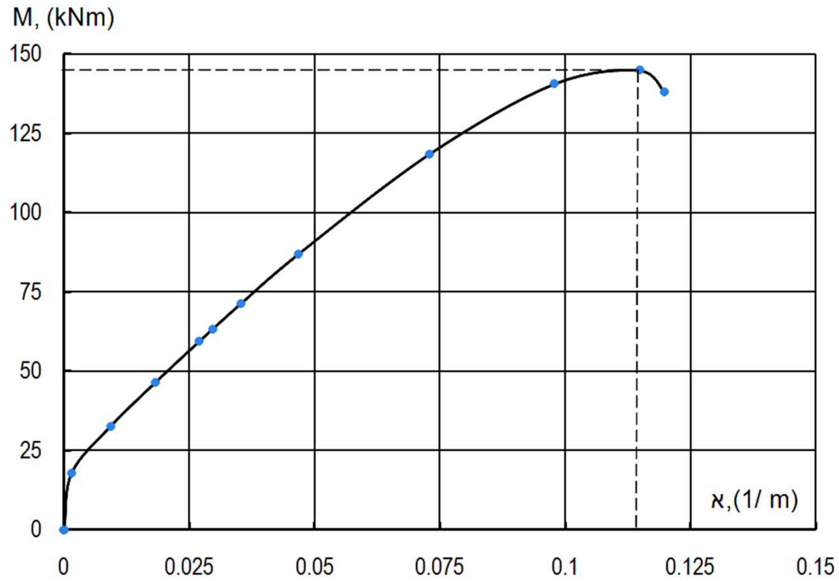

Figure 5 – Graph "moment-curvature" when calculating the plate P60.38 without metal fiber

Calculation of the bearing capacity of the plate P60.38 with metal fiber

Section dimensions b=3750 mm, h=140 mm (Fig. 4). Prestressed reinforcement in two levels on 8Ø10A800: A_{sp1}=A_{sp2}=628 mm^2; E_{sp}=190000 MPa; f_{pk}=840 MPa; $f_{p0,1k}$=765 MPa; ε_{uk}=0,018. Elongation of reinforcement taking into account all losses makes ε_{sp0}= -0,002. distance from the center of gravity of reinforcement to the extreme verge compressed section z_{sp1}=105 mm, z_{sp2}=35 mm. Reinforcement pitch along the axis perpendicular to the calculated one S=350 mm.

Concrete class C20/25: f_{cd}=14,5 MPa, E_{cd}=23000 MPa; ε_{c1}=0,00165, ε_{cu}=0,00344.

Metal fiber STAFIB 50/1.0: f_f=1000 MPa; l_f=50 mm; d_f=1 mm; μ_{fv}=0,01.

The calculated compressive strength of reinforced concrete is determined according to DSTU [6]: f_{cf}=22.36 MPa; f_{cft}=1.49 MPa.

The modulus of elasticity of SFC E_{cf}=24940 MPa.

Polynomial coefficients:

a_1	a_2	a_3	a_4	a_5
2,51816	-2,14804	0,71003	-0,04839	-0,03169

Relative SFC deformations at compression ε_{cf1}=0,00176; ε_{cftu} =0.00293.

Relative SFC strains at tension ε_{cf1}=0,00018; ε_{cftu} =0.00035.

The calculation results are summarized in the graph "moment-curvature", which is shown in Fig. 6. The plate bearing capacity is M_u=180,3 kNm.

As a comparative bearing capacity calculation result of the standard road slab П60.38 and a similar plate with metal fiber, it was found that the bearing capacity of the plate with steel fiber is higher than the standard by 24.4%.

The steel fiber plate efficiency is that the steel fiber makes it possible to reduce the number of high-strength reinforcement from 24Ø10A800 to 16Ø10A800. At the same time, the plate bearing capacity with steel fiber is much higher. The number of high-strength reinforcement in the transverse direction is also reduced by 15…20%.

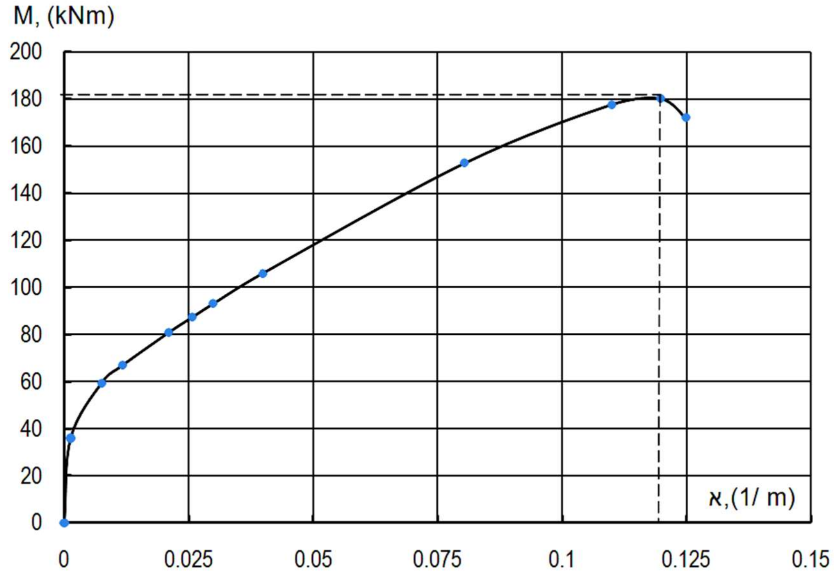

Figure 6 – Graph "moment-curvature" when calculating the plate P60.38 with metal fiber

Conclusions

The general algorithm of bending elements calculation of rectangular section reinforced by usual and prestressed reinforcing rod, and also steel fiber is offered.

The calculation method is based on the deformation theory of reinforced concrete structures calculation taking into account the complete diagram "σ-ε" for concrete and reinforced concrete under compression.

The method makes it possible to calculate biaxially prestressed plates. This takes into account the increase in strength of concrete and steel-fiber-concrete in the biaxial compression conditions.

As a result of comparative bearing capacity calculation of the standard road slab P60.38 and a similar plate with metal fiber, it was found that the plate bearing capacity with steel fiber is higher than the standard by 24.4%.

The plates' efficiency with steel fiber is that steel fiber makes it possible to reduce the number of high-strength prestressed reinforcement to 10…15%.

Due to the good anti-abrasion properties of reinforced concrete, the service life of aerodrome and road surface slabs is much longer than reinforced concrete.

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