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## Dynamics research of unbalanced vibrator with variable static moment

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Every year the use of vibrating and vibro-impact equipment becomes more and more common. This machinery is successfully used in the construction of solid foundations for a different building. The main element of these vibrating machines is an unbalanced vibrator. We considered the operation of the unbalanced vibrator in the interaction of mechanical and electro-magnetic processes and the result was obtained as a mathematical model of dynamic processes during working the vibrator. The developed mathematical model makes it possible to carry out the analysis of the transients during the operation of the unbalanced vibrator, taking into account the inseparable interaction of the electric machine and the mechanical part of the drive.

**Keywords:** dynamics, mathematical model, oscillations, static moment, unbalanced vibrator, vibrating deepener, vibrating hammer, vibration

## Дослідження динаміки дебалансного вібратора зі змінним статичним моментом

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З кожним роком все більше набуває використання вібраційного та віброударного обладнання. Ця техніка успішно використовується при спорудженні надійних фундаментів під різні споруди. Основним елементом цих вібраційних машин являється дебалансний вібратор. В даний час, при проектуванні дебалансних вібраторів динамічні фактори при їх експлуатації не враховуються. Тому надійність можна підвищити, якщо на стадії їх проектування враховувати хвильовий характер навантажень. Робота дебалансного вібратора нами розглядалася у взаємодії механічних і електромагнітних процесів і в результаті була отримана математична модель динамічних процесів при роботі вібратора, котра включала нелінійні диференціальні рівняння руху мас вібратора і лінійне диференціальне рівняння електромагнітних явищ в двигуні його приводу. Можна акцентувати, що вібраційному а також віброударному методу мало приділено уваги і широка інформація практично відсутня. Тому являється актуальним створення продуктивних зразків вібраторів, методик їх розрахунків і проведення наукових досліджень динаміки робочих процесів цих машин на що і направлена дана робота. При розрахунку вібраторів на статичну й утомлену міцність коливальні процеси конструкцій та їх динамічні навантаження, в цей час, не враховуються. Однак їх несучу здатність можна значно підвищити, якщо у розрахунках при їх проектуванні враховувати їхні амплітудно-частотні характеристики. Відсутність ж уточненої методики розрахунку дебалансних вібраторів сучасних вібраційних машин ускладнює їхнє проектування і експлуатацію. Метою статті є висвітлення результатів математичного моделювання коливальних процесів при дослідженні дебалансного вібратора зі змінним статичним моментом та визначення динамічних навантажень на його елементи. В роботі теоретично досліджено, з використанням математичного програмного застосунку MathCAD, динаміку механізму приводу дебалансного вібратора і отримано результати які можуть бути використані при проектуванні, розрахунку та визначенні динамічних навантажень подібних вібраторів вібраційних машин.

**Ключові слова:** динаміка, математична модель, колювання, статичний момент, дебалансний вібратор, віброзаглиблювач, вібромолот, вібрація



## Introduction

In our time, unbalanced vibrators have become widely used, which are the basis of vibrating machines used in construction and agriculture. Unbalanced vibrators have become especially widely used in vibrating deepeners and vibrating hammers in the construction of pile foundations for various buildings and structures. But modern requirements for such vibrating machines require changing their vibration parameters during working. Therefore, the creation and research of unbalanced vibrators with variable static moments are relevant.

## Review of the research sources and publications

As we noted earlier, nowadays, much attention is paid to vibration machinery, which is successfully used, especially in the construction of foundations and piles. Much attention is also paid to the improvement of vibration machineries using unbalanced vibrators. To perform the above, you should constantly study the vibration machinery and its main component - the unbalanced vibrator.

Recalling the past, it should be noted that the theoretical foundations of the creation of vibrating machines were laid in the former Soviet Union, as well as the post-Soviet society and modern Ukraine. The following prominent scientists and leading engineers, such as Artobolevsky I., Babichev A., Bliexhman I., Bykhovskiy I., Honcharevych I., Kriukov B., Lavendel E., Lanets O., Nadutyi V., Nazarenko I., Spivakovskiy A., Strelnikov L., Panovko Y., Povidailo V., Poturaiev V., Franhuk V., Sergiev P. and others.

In the '50s, the leadership in the field of vibration machinery was for scientists in Western Europe and North America. But, for some time, domestic scientists have already been ahead of foreign scientists in the main areas.

A significant contribution to the improvement of unbalanced drives of vibrators was made by scientists of the National Mining University and the Institute of Geotechnical Mechanics Kriukov B., Franhuk V., Nadutyi V., including their supervisor, Academician Poturaiev V. [1]. Their main assets focused on the imbalance drive, which required the development of simple structures with big perturbing forces for bulky vibrating machines of the mining industry.

The creation of methods and calculation methods, including experimental samples, was carried out by scientists of the Kyiv School of Vibrotechnics Chubuk Y., Nazarenko I., Yakovenko V. [2] and others. Their created vibrotechnics were successfully used for the consolidation of concrete mixes in the construction industry.

The creation of three-mass vibrating machines was carried out by scientists and leading engineers of Lviv Polytechnic Bepalov A., Havrylchenko O., Povidailo V., Silin R., Shchihel V., Sholovii Y., Ufimtsev V. [3]. They have developed and researched a number of small and medium-sized vibrating machines for many areas where dynamic dampers have been used. The designs of these vibrating machines were three-mass, but in the

calculation schemes, they were considered mainly as two-, one- mass.

It should also be noted a significant contribution to the research of oscillatory processes of mechanical systems of the following foreign scientists Kollate L. [4], Tondl A. [5], Jagadish N. [8], Kaplan D. [9].

Until early this century, machines and structures usually had very high mass and damping, because heavy beams, timbers, castings and stonework were used in their construction. Since the vibration excitation sources were often small in magnitude, the dynamic response of these highly damped machines was low. However, with the development of strong lightweight materials, increased knowledge of material properties and structural loading, and improved analysis and design techniques, the mass of machines and structures built to fulfill a particular function has decreased. Furthermore, the efficiency and speed of machinery have increased so that the vibration exciting forces are higher, and dynamic systems often contain high energy sources which can create intense noise and vibration problems [11].

## Definition of unsolved aspects of the problem

Analyzing the received information it is possible to accent that little attention was paid to the vibrating and also vibro-impact methods and the wide information is practically absent. Therefore, it is a revolt to create productive vibrators samples, methods of their calculations and conducting scientific research of the working processes the dynamics of these machines, which is the purpose of this work.

When calculating the vibrators for static and fatigue strength, the oscillatory processes of constructions and their dynamic loads, at this time, are not taken to account. However, their supporting ability can be significantly increased if their amplitude-frequency characteristics are taken to account in the calculations during their design. The absence of a refined method of calculating unbalanced vibrators of modern vibrating machines complicates their design and exploitation.

## Problem statement

The article aims to highlight the results of mathematical modeling of oscillatory processes in the research of unbalanced vibrators with variable static moments and to determine the dynamic loads on its elements.

## Basic material and results

Modern unbalanced vibrators and their drives, which are part of vibrating machines, have a complex structure.

We can also state about the unbalanced vibrator with the electric drive as about the electromechanical machinery. In the process, it performs a vibrating or vibro-impact action on the object. Unbalanced causes vibrating action, resulting arising in vertical centrifugal forces. We accepted for research the designed unbalanced vibrator with a variable static moment and a drive from the electric motor.

Given the fact that the vibrating mechanical system includes elastic elements, these are metal structures and

a vibrator, the acting forces of which are variable in nature, we can state that during the operation of the vibromachine elastic links oscillate and create auxiliary loads.

We use the mathematical program MathCAD to simplify calculations.

Vibration load on the unbalanced vibrator, including deepening of piles is carried out by the special mechanism which is called the vibrating deepener or the vibrating hammer.

This mechanism is an electromechanical machine. It gives the pile a vibrating effect. It is installed on the pile drive and connected to the pile head.

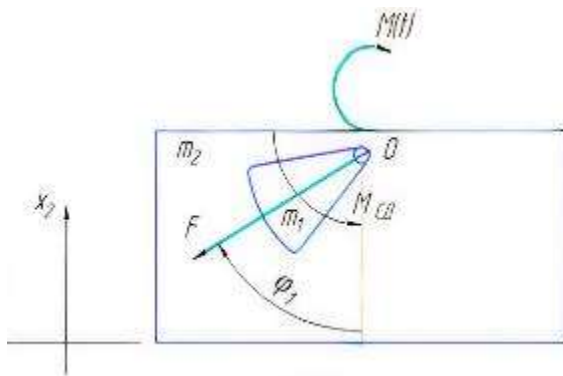
Unbalances create a vibrating action of the deepener, resulting in vertical centrifugal forces, which are transmitted through the head to the pile. When performing the process of deepening the pile there is a destruction of the soil structure in which there are irreversible deformations.

During operation, transients take place in the drive mechanism of the unbalanced vibrator, which causes the rise of additional forces. In turn, it is necessary to note that inertial and hard parameters of links of the mechanism significantly influence passable dynamic transients.

The beginning of the movement, and also a stop of the electric drive of the mechanism of the unbalanced vibrator, is carried out at loading, that is, during its operation.

Considering the dynamic processes that take place when starting directly the drive of the unbalanced vibrator, we take the movement under load (work process).

The composite system of differential equations of motion and electromagnetic state of the electric motor has the following form (Fig. 1):



**Figure 1 – Calculation scheme of the electric drive unbalanced vibrator with a variable static moment during transients**

$$J_1 \ddot{\varphi}_1 - m_1 e x_1 \sin \varphi_1 = M(t); \quad (1)$$

$$(m_1 + m_2) \ddot{x}_2 - m_1 e (\varphi_1 \sin \varphi_1 + \varphi_1^2 \cos \varphi_1) = -M_{CD}. \quad (2)$$

The calculation scheme is presented in Figure 1: where  $J_1$  – the moment of inertia of the drive rotor of the electric motor;

$m_1$  – the given mass of unbalances;

$m_2$  – the given mass of the bearing hull of the unbalanced vibrator;

$\Pi_1, x_1, x_2$  – angular and linear displacements of all two masses, respectively;

$x = 0.0174e\Pi$  – numerical dependence between angular and linear mass displacements;

$e$  – eccentricity of the established unbalances.  $e_1 = 100$  mm,  $e_2 = 300$  mm;

$M_{CD}$  – the given moment of the certain forces of resistance of imbalances;

$M_{CP}$  – the given moment of the certain forces of resistance to the immersion of a pile;

$M(t)$  – the moment developing the electric motor.

The moment of the electric motor we are written by the following differential expression [6, 7]

$$M(t) = A_0 + A_1 M'(t) + A_2 x_1'(t), \quad (3)$$

where  $A_0, A_1, A_2$  – engine constants.

The constants have the next form:

$$A_0 = \frac{2M_k}{S_k}; \quad A_1 = \frac{1}{\omega_0 S_k}; \quad A_2 = \frac{2M_k}{\omega_0 S_k}, \quad (4)$$

where  $M_k$  – the critical moment of the engine;

$S_k$  – critical sliding of the electric motor rotor;

$\omega_0$  – the angular speed of the electric motor;

$t$  – time.

The limits of application of equation (3) have the following limitation [6, 7]

$$-0,8 M_k \leq M \leq 0,8 M_k. \quad (5)$$

After performing the transformation, the nonlinear equations (1–2) after the replacement and lowering the order, to be able to solve them, using the program MathCAD, will have the next form:

$$\begin{aligned} q'(t) &= w(t); \\ w'(t) &= \frac{-0.0174e \cdot q(t) + M(t)}{J_1 - 0.0174e \cdot \sin q(t)}; \\ u'(t) &= p(t); \\ p'(t) &= \frac{m_1 \cdot e \cdot w^2(t) \cdot \cos q(t) - M_{CD}}{(m_1 + m_2) - 2831 \cdot \sin q(t)}; \\ M'(t) &= \frac{M(t)}{A_1} - \frac{A_2}{A_1 \cdot 0.0174e} w(t) - \frac{A_0}{A_1}; \\ o(t) &= \frac{-0.0174 \cdot e \cdot q(t) + M(t)}{J_1 - 0.0174 \cdot e \cdot \sin q(t)}; \\ d(t) &= \frac{m_1 \cdot e \cdot w^2(t) \cdot \cos q(t) - M_{CD}}{(m_1 + m_2) - 2831 \cdot \sin q(t)}; \end{aligned} \quad (6)$$

$$F(t) = m_1 \cdot e \cdot w^2(t),$$

where  $o(t), d(t)$  – angular and linear accelerations, respectively, of the electric motor rotor and mass  $m_2$ ;

$F(t)$  – centrifugal force of inertia of unbalances.

The next replacement is made:

$$\begin{aligned} \varphi_1 &= q(t); & \varphi_1' &= w_1(t) = q'(t); \\ x_2 &= u(t); & x_2' &= p(t) = u'(t); \\ o(t) &= w'(t) = q''(t); & d(t) &= p'(t) = u''(t). \end{aligned} \quad (7)$$

The initial conditions are as follows:

$$\begin{aligned} (t) &= 0; & q(0) &= 0; & w(0) &= 0; & u(0) &= 0; \\ p(0) &= 0; & o(0) &= 0; & d(0) &= 0; \\ M(0) &= 0; & F(0) &= 0. \end{aligned} \quad (8)$$

The solution of the system of nonlinear equations (6) is carried out to drive an unbalanced vibrator with an electric drive, which has the parameters shown in tables 1 and 2.

When implementing the MathCAD application, we get the magnitudes of the moment of the electric motor of the unbalanced vibrator drive mechanism, angular, linear displacements of unbalances, vibrator hull, electric motor and their speed, as well as their angular accelerations (Fig. 2–7).

Constructed, as a result of the implementation of this program, graphs of changes in the moment of the electric motor of the unbalanced vibrator drive mechanism  $M(t)$  and the centrifugal force of unbalances  $F(t)$  as a function of time (Fig. 6, 7) show that the growth of inertia of unbalances  $F(t)$  continues about 3 s from the moment of its inclusion reaching, thus, the maximum value.

Considering the constructed graph (Fig. 6), we can also state that the centrifugal force of inertia of unbalanced after 4 s becomes constant.

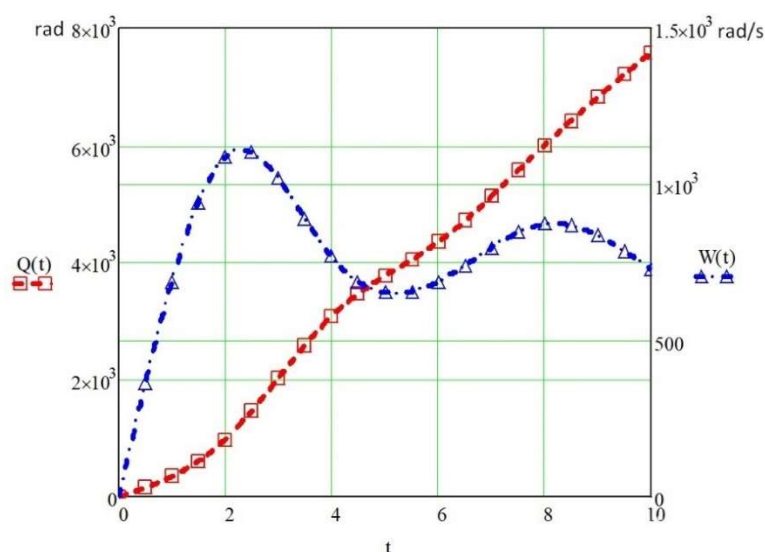
Figure 4 shows the change in the mass movement, which can modulate the process of deep piles during the operation of the unbalanced vibrator in the mode of the vibrating deepener.

**Table 1 – Parameters of the drive unbalanced vibrator**

| Parameters | Units of measurement | Numerical values |
|------------|----------------------|------------------|
| $J_1$      | кг·м <sup>2</sup>    | 1,275            |
| $m_2$      | кг                   | 11600            |
| $M_{CD}$   | Nm                   | 50               |
| $e$        | М                    | 0,1 – 0,3        |
| $v$        | N <sub>s</sub> / м   | 25               |
| $M_{CP}$   | Nm                   | 1550             |

**Table 2 – Values of the constant electric motor drive unbalanced vibrator**

| Type of electric motor                   | Mode of operation of the electric motor | The angular speed of the electric motor rotor, rad/s |
|--|---|--|
| MTB 512-8<br>N = 50 kWt<br>n = 720 t/min | Performance                             | 75,4   |
| Constant of electromotor                 |   |  |
| $A_0$                                    | $A_1$                                   | $A_2$  |
| 67105                                    | -0,9471                                 | -1495  |



**Figure 2 – Movement of mass  $Q(t) = \phi_1$  and its speed  $W(t) = \phi_1' = Q'(t)$**

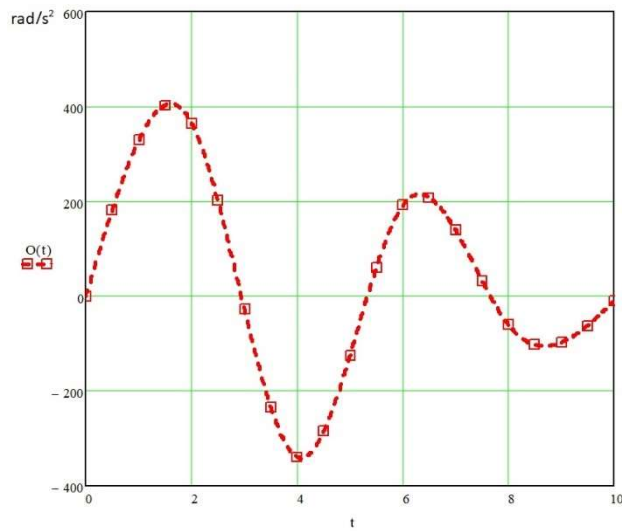


Figure 3 – Change in acceleration  $O(t)$  of mass  $Q(t) = \phi_1$

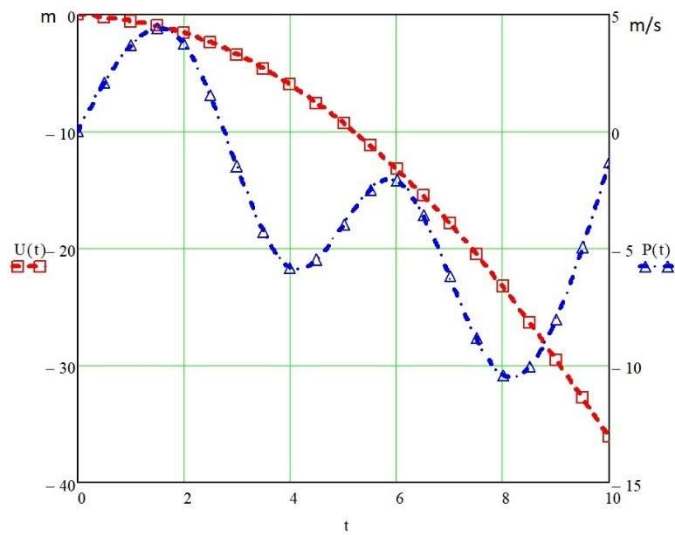


Figure 4 – Movement of mass  $U(t) = m_2$  and its speed  $P(t) = U(t)$

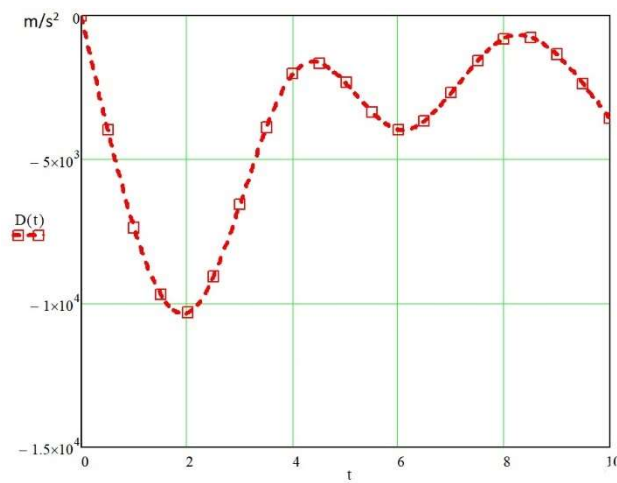


Figure 5 – Changing in acceleration  $D(t)$  of mass  $U(t) = m_2$

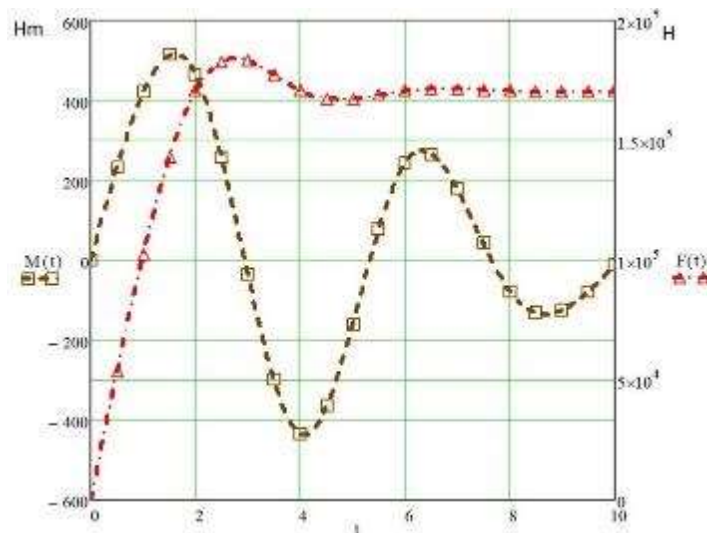


Figure 6 – Graphs of change of the moment  $M(t)$  of the electric motor of the vibrator and centrifugal force of inertia of unbalances  $F(t)$  at work (with attenuation)

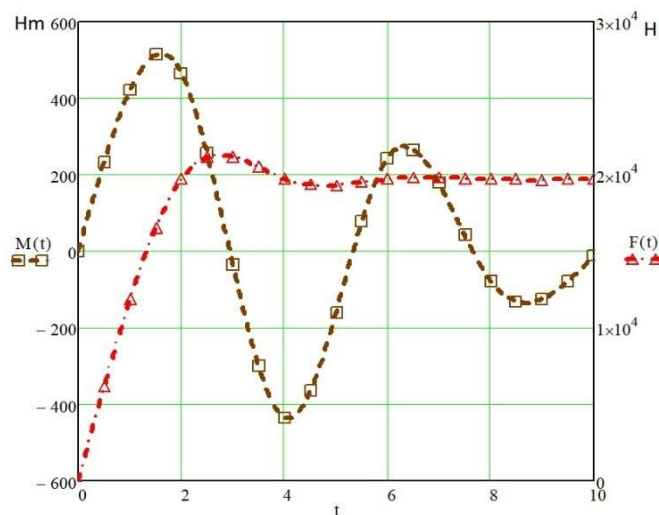


Figure 7 – Graphs of change of the moment  $M(t)$  of the electric motor of the vibrator and centrifugal force of inertia of unbalance  $m$  at work (without attenuation)

The graphs constructed by us (Fig. 2 – 7) showed that nonlinear oscillating processes are observed during the operation of the unbalanced vibrator. The following information can be obtained from nonlinear equations (1 – 2):

- uneven rotation of imbalances is carried out;
- oscillations of the base of the unbalanced vibrator and the object are non-sinusoidal;
- oscillations of the angular velocity of rotation of the unbalances and oscillations of the base of the unbalanced vibrator and the object mutually affect each other;
- statement of the problem of oscillations of the basis of the unbalanced vibrator and the object is nonlinear.

At the dynamic phenomena shown in figures 2 – 7, fluctuations of angular and linear displacements, speeds, and also their accelerations are characteristic. Based on this, mathematical modeling of the unbalanced vibrator should be carried out using the equations of electromechanical interaction of the system.

The use of numerical methods for integrating nonlinear differential equations of motion and electromagnetic state expands the possibility of using the developed method to determine dynamic loads in electromechanical and mechanical systems vibrating deepener (vibrating hammer) where unbalanced vibrators are successfully used, including vibrators with the variable static moment.

## Conclusions

The developed mathematical model makes it possible to carry out the analysis of the transients during the operation of the unbalanced vibrator, taking into account the inseparable interaction of the electric machine and the mechanical part of the drive.

In the work, the dynamics of the unbalanced vibrator drive mechanism are theoretically researched using the mathematical program MathCAD.

Results obtained from research of the drive mechanism of the unbalanced vibrator with using the mathematical program MathCAD can be used in the designing, calculation and determination of dynamic loads of similar vibrators of vibrating machines are obtained.

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