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MATHEMATICAL MODEL OF PRESSURE CHANGE IN AUTOMOBILE PNEUMATICAL TIRE DEPENDING ON OPERATING TEMPERATURE

Orysenko Oleksandr[1], Nesterenko Mykola[2], Vasyliev Oleksiy[3*], Rohozin Ivan[4]

[1] Poltava National Technical Yuri Kondratyuk University https://orcid.org/0000-0003-3103-0096
[2] Poltava National Technical Yuri Kondratyuk University https://orcid.org/0000-0002-8961-2147
[3] Poltava National Technical Yuri Kondratyuk University https://orcid.org/0000-0002-9914-5482
[4] Poltava National Technical Yuri Kondratyuk University https://orcid.org/0000-0002-9052-4806
*Corresponding author E-mail: a.s.vasiliev.76@gmail.com

It has been established that in the process of operation pressure ratings in the tires of many cars differs from those recommended by the production plant. Is leads to performance degradation of tires traveling properties and their loss of life. The pressure excursion from the normative value may be caused either by an error during tire inflation, or by the fact that the difference between the operating temperature and the temperature of the inflating air has not considered. Using athematical-statistical methods of data processing, there has been deduced the mathematical relationship between pressure in the pneumatical tire at the operating temperature and the required pressure of inflating air into the tire, if the temperatures of inflation and operation differ.

Keywords: pneumatical tire, inflation pressure, three-level plan, planning matrix, mathematical model.

МАТЕМАТИЧНА МОДЕЛЬ ЗМІНИ ТИСКУ В АВТОМОБІЛЬНІЙ ПНЕВМАТИЧНІЙ ШИНІ ЗАЛЕЖНО ВІД ТЕМПЕРАТУРИ ЕКСПЛУАТАЦІЇ

Орисенко О.В.[1], Нестеренко М.П.[2], Васильєв О.С.[3*], Рогозін І.А.[4]

[1, 2, 3, 4] Полтавський національний технічний університет імені Юрія Кондратюка
*Адреса для листування E-mail: a.s.vasiliev.76@gmail.com

Установлено, що в процесі експлуатації значення тиску в шинах багатьох автомобілів відрізняється від рекомендованого заводом-виробником, що призводить до погіршення експлуатаційних характеристик шин та скорочення їхнього ресурсу. Відомо, що відхилення тиску від нормативного значення може бути спричиненим як похибкою при накачуванні шини, так і неврахуванням різниці між температурою експлуатації та температурою повітря, що закачується. Для проведення дослідження виділено найбільш значущі фактори, які впливають на значення тиску в пневматичній шині, що дозволило скласти рівняння регресії, котре описує явище. Доведено адекватність моделі через визначення похибки та перевірку критеріїв. За допомогою математико-статистичних методів оброки даних виведено математичну залежність між тиском у пневматичній шині при температурі експлуатації та необхідним тиском закачування повітря в шину, якщо температури накачування та експлуатації відрізняються. Отже, отримано математичну залежність, яка дозволяє при відомих температурі в приміщенні, де здійснюється накачування, температурі навколишнього середовища, де буде експлуатуватися шина, та рекомендованому заводом-виробником тиску в шині автомобіля визначити необхідний тиск закачування повітря. Практичним використанням результатів дослідження є значення необхідного тиску накачування залежно від температур.

Ключові слова: пневматична шина, тиск накачування, трирівневий план, матриця планування, математична модель.

Introduction

The durability of pneumatic tires depends on many factors. Gas pressure in the tire is one of the most important ones. Tires of the same model are installed on the cars of different vehicle brands. The car production plant carries out research and establishes the optimum tire pressure when it has the vehicle maximum loss of life. In the case of pressure excursion from the normative value, the service life of the tire is significantly reduced [1, 2]. Also, the safety and comfort of the vehicle movement, fuel consumption, durability of a car suspender depend on the tire pressure [3-6].

However, it is known that the pressure does not meet the norm in 60 – 90% of tires in operation. Due to the fact, that the pressure does not conform to this norm, there are lost from 6 – 15% of the tire life and from 1.5 to 3.0% of fuel [7, 8].

Thus, there is a problem of securing operation of tires with standard pressure. The solution to this problem is complicated by the fact that the pressure change is determined by a large number of different factors. Some factors influence has not been studied sufficiently. Consequently, the crucial task is the study changing gas pressure process in pneumatical tires and the development of measures aimed at the reduction of its deviations from the norm.

Review of research sources and publications

It the paper it has been established that environmental temperature shift affects the number of technical and operational parameters of a pneumatical tire [9, 10]. Pressure change is particularly notable.

In the car operation process, the temperature fall can reach several dozens of degrees [7]. For example, during car tire fitting at the premises of a service station or a garage, the air, which temperature is equal to the air temperature in the room, is inflated into the tire. However, outdoors, especially when it is cold, the atmospheric temperature can vary significantly. At the same time, the pressure change takes place in the tire.

Definition of unsolved aspects of the problem

As it has been noted above, the air temperature inside and outside the tire can vary significantly in different periods of time. It affects the pressure amount, when the air temperature inside the tire is equal with ambient temperature. But the tire pressure is regulated by the manufacturer [4, 11]. So, there is a need for further study of this effect for the necessary adjustments and description in the mathematical dependencies field [12 – 14].

Therefore, the issue of forecasting the pressure change in the tire during its operation at different temperatures has not been solved fully.

Problem statement

The goal of the research is determination of mathematical relationship between pressure in the pneumatic tire at the operating temperature and the required pressure of inflating air into the tire if the inflating and operation temperatures differ.

Dasic material and results

To derive a mathematical expression that establishes the functional relationship between the values of standard pressure in the tire, the tire air pressure after finished inflation, the inflating air temperature and the operating temperature for the tire, mathematical-statistical methods have been used [15].

In the investigation process the following assumptions have been introcced:

– the tire volume does not change with the tire pressure change;

– the composition of the inflating air corresponds to the composition of an air at the sea level;

– the inflation is carried out at an atmospheric pressure of 101.3 kPa.

– the main factors of the influence on the standard tire pressure value during operation are: pressure and temperature of the tire inflating air, ambient temperature. Other factors are neglected.

When computations are conducting, the selected factors vary on three levels – middle (main), lower and upper, which are remote from the main level to the same value. This value is called the variation interval (Table 1)

It simplifies the records and subsequent calculations, when the upper level of factors is denoted by the symbol «+», the middle one is «0», and the lower one is «-», which is equivalent to the transfer of factors to the new code scale.

$$x_i = \frac{X_i - X_{i0}}{\Delta X_i}, \quad (1)$$

where x_i – value of *i*th factor on the new code scale;

X_i – value of *i*th factor on a natural scale;

X_{i0} – main *i*th factor level;

ΔX_i – *i*th factor variation interval.

In our case, it assigns the following notations to the factors that consider this calculation:

– the tire air pressure after finished inflation is x_1

$$x_1 = \frac{X_1 - 202.6}{50.65}; \quad (2)$$

– the inflating air temperature is x_2

$$x_2 = \frac{X_2 - 10}{10}; \quad (3)$$

– the operating temperature for the tire is x_3

$$x_3 = \frac{X_3 - 0}{30}. \quad (4)$$

The matrix of the second order for processing of the computational results has been used, since the studied dependence is unknown [15]. Calculations are made according to the matrix plan, and they are reduced to table 2.

Table 1 – The limits of change and factors variation intervals

Factors	The limits of factors change	Variation intervals
Tire air pressure after finished inflation P_i, kPa (atm)	151.95 – 253.25 (1.5 – 2.5)	50.65
Inflating air temperature t_i, °C (K)	0 – 20 (273 – 293)	10
Ambient temperature when the tire is operated t_o, °C (K)	–30 – +30 (243 – 303)	30

Table 2 – Three-level plan of the second order with number of factors $k=3$ $(N=N_1+N_\alpha+n_0)$

Calculation number		Planning matrix (x_i)			Squared variables (x_i^2)			Factor interaction $(x_i x_j)$			Tire pressure, kPa
		x_1	x_2	x_3	x_1^2	x_2^2	x_3^2	x_1x_2	x_1x_3	x_2x_3	y_i
1		2	3	4	5	6	7	8	9	10	11
N_1	1	+	+	+	+	+	+	+	+	+	266
	2	–	+	+	+	+	+	–	–	+	160
	3	+	–	+	+	+	+	–	+	–	281
	4	–	–	+	+	+	+	+	–	–	169
	5	+	+	–	+	+	+	+	–	–	214
	6	–	+	–	+	+	+	–	+	–	128
	7	+	–	–	+	+	+	–	–	+	225
	8	–	–	–	+	+	+	+	+	+	135
N_α	9	+	0	0	+	0	0	0	0	0	246
	10	–	0	0	+	0	0	0	0	0	148
	11	0	+	0	0	+	0	0	0	0	192
	12	0	–	0	0	+	0	0	0	0	203
	13	0	0	+	0	0	+	0	0	0	219
	14	0	0	–	0	0	+	0	0	0	175
n_0	15	0	0	0	0	0	0	0	0	0	196
	16	0	0	0	0	0	0	0	0	0	197
	17	0	0	0	0	0	0	0	0	0	198

The calculated values of the pressure in the tire (columns 11 in Table 2) are obtained on the basis of the air composition data at the sea level [16], and under the condition that the test tire has a volume of 25 liters and the gas mass (inside the test tire) corresponds to the air mass at a pressure of 202.6 kPa and a temperature of 20°C. The calculation is carried out according to the formula

$$P_o = \frac{M_a \cdot R_{da} \cdot T_a}{V_a}, \tag{5}$$

where M_a – air mass in the tire, kg;
R_{da} – specific gas constant for dry air [16], J/(kg·K);
T_a – air temperature in the tire, K;
V_a – air volume in the tire, m^3.

The table columns 2 - 4 represent a matrix that defines the initial conditions for conducting calculations. The table columns 5, 6 and 7 show the squared variables obtained as a result of the data columns 2 - 4 squaring. They acquire the values +1 or 0 and are marked as x_i^2. Column 8 is obtained by sequential multiplication of factors (variables interaction). For example, for the 8th calculation according to the plan factors $x_1 = -1$ and $x_2 = -1$ should be set at the lower level, and the estimated value of the interaction is

$$x_1 \cdot x_2 = (-1)\cdot(-1) = +1. \tag{6}$$

The last column 11 shows the calculation results of the tire pressure at various combinations of factors («outputs»).

The calculation results are processed according to the method [15]. In this case, an algebraic equation is obtained. It reflects the relationship between the properties under investigation and the initial factors.

The algebraic equation in general terms is

$$\hat{y}_i = b_0 + \sum_1^k b_i x_i + \sum_1^k b_{ii} x_i^2 + \sum_1^k b_{ij} x_i x_j, \tag{7}$$

or

$$\hat{y}_i = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3, \tag{8}$$

where $i, j = 1, 2, \ldots, k$ – factors order numbers, $i \neq j$;
$\hat{y}_i$ – property under investigation – tire pressure;
$x_1, x_2, x_3, x_4, \ldots x_k$ – initial factors;
$b_0, b_1, b_2, \ldots b_{12}, b_{13}, \ldots b_{ij}, b_{ii}$ – equation coefficients.

The coefficients for the plan of the second order with the number of factors k = 3 are calculated according to the formulas:

$$b_0 = 0.1831[0y] - 0{,}0704\sum_1^k [iiy]; \tag{9}$$

$$b_i = 0.1[iy]; \tag{10}$$

$$b_{ii} = -0.0704[0y] + 0.5[iiy] - 0.1268\sum_{1}^{k}[iiy]; \quad (11)$$

$$b_{ij} = 0.125[ijy], \quad (12)$$

where

$$[0y] = \sum_{1}^{N} y_u ; \quad (13)$$

$$[iiy] = \sum_{1}^{N} x_{iu}^2 y_u ; \quad (14)$$

$$[iy] = \sum_{1}^{N} x_{iu} y_u ; \quad (15)$$

$$[ijy] = \sum_{1}^{N} x_{iu} x_{ju} y_u , \quad (16)$$

according as $i \neq j$;

y_u – value of property under investigation in *u*th calculation;

x_{iu} – value of *i*th factor in *u*th calculation;

x_{ju} – value of *j*th factor in *u*th calculation $(i \neq j)$;

N – total quantity of calculations in plan (null point included).

After processing the calculations for the adopted plan, the following equation is obtained

$$y = 197.19 + 49.2 \cdot x_1 - 5.3 \cdot x_2 + 21.8 \cdot x_3 - 0.26 \cdot x_1^2 + 0.24 \cdot x_2^2 - 0.26 \cdot x_3^2 - 1.25 \cdot x_1 \cdot x_2 + 5.25 \cdot x_1 \cdot x_3 - 0.75 \cdot x_2 \cdot x_3. \quad (17)$$

Statistical analysis of the quadratic dependence is carried out according to the method [15]. In order to check the significance of the coefficients in the factors, the error is artificially set in the calculations of pressure at the null points within ±5%.

According to the results of calculations at the null points, it is determined:

a) arithmetical average

$$\bar{y}_0 = \frac{\sum_{1}^{n_0} y_{0u}}{n_0}, \quad (18)$$

where y_{0u} – value of property under investigation at the null point in *u*th calculation;

n_0 – quantity of calculations at the null point;

b) dispersion at the null point

$$S_{\bar{y}}^2 - S_0^2 = \frac{\sum_{1}^{n_0} \left(\bar{y}_0 - y_{0u}\right)^2}{n_0 - 1}; \quad (19)$$

c) mean-square deviation

$$S_{\bar{y}} = S_0 = \sqrt{S_0^2} = \sqrt{\frac{\sum_{1}^{n_0} \left(\bar{y}_0 - y_{0u}\right)^2}{n_0 - 1}}; \quad (20)$$

d) root-mean-square error in determining coefficients

$$S\{b_i\} = \frac{S_{\bar{y}}}{\sqrt{N_1}}. \quad (21)$$

The estimated value of Student-test is determined in the following form:

$$t_c = \frac{|b_i|}{S\{b_i\}}, \quad (22)$$

and compare the resulting value t_c with the tabular one t_t with number of freedom degrees $f_{\bar{y}}$, by which it was determined $S_{\bar{y}}$ [15];

$$f_{\bar{y}} = n_0 - 1. \quad (23)$$

In the case $t_c < t_t$ with a level of significance $\alpha = 0.05$, the coefficient is taken to be equal to zero, and the corresponding equation member is rejected.

After mathematical processing of data, the refined equation is obtained in the form:

$$y = 197.19 + 49.2 \cdot x_1 - 5.3 \cdot x_2 + 21.8 \cdot x_3 + 5.25 \cdot x_1 \cdot x_3. \quad (24)$$

To verify the suitability of the obtained refined equation, the adequacy dispersion is calculated

$$S_{ad}^2 = \frac{\sum_{1}^{N_1} \left(y_u - \hat{y}_u\right)^2}{N_1 - m}, \quad (25)$$

where y_u – value of property under investigation in *u*th calculation;

$\hat{y}_u$ – value of property under investigation in *u*th calculation, which is calculated according to the refined equation;

m – quantity of significant coefficients together with b_0.

The estimated value of F-test F_p is obtained

$$F_p = \frac{S_{ad}^2}{S_{\bar{y}}^2} \quad (26)$$

and compare it with the table value *F* [15] for degrees of freedom with which were defined S_{ad}^2 and $S_{\bar{y}}^2$, that is

$$f_{ad} = N_1 - m, \quad (27)$$

$$f_{\bar{y}} = n_0 - 1. \quad (28)$$

In our case $F_p < F$, therefore, the equation is considered suitable for calculations.

The resulting equation (17) connects tire pressures at different ambient temperatures and the required inflating pressure at a given temperature of the inflating air.

The designations $P_o = y$, $P_i = x_1$, $t_i = x_2$, $t_o = x_3$, are introduced, and the values of x_1, x_2, x_3 from the formulas (2 – 4) are substituted into (17). Then the equation of tire pressure dependence P_o from the named factors in the natural form is obtained after simple mathematical transformations:

$$P_o = 5.69 + 0.971 \cdot P_i - 0.53 \cdot t_i + 0{,}027 \cdot t_o + 0.0035 \cdot P_i \cdot t_o. \quad (29)$$

Since, in the conditions of service stations, the practical interest is not so much the pressure in the tire depending on the ambient temperature, but the required pumping pressure depending on the temperature when the pump is carried out. So the value of the pump pressure P_i, kPa, is set from the equation (29):

$$P_i = \frac{P_o - 5.69 + 0.53 \cdot t_i - 0.027 \cdot t_o}{0.971 + 0.0035 \cdot t_o}. \quad (30)$$

Conclusion

As a result of the undertaken research, mathematical relationship between pressure in the pneumatic tire at the operating temperature and the required pressure of inflating air into the tire, if the inflating and operation temperatures differ, has been obtained.

Using the given equation at the known temperature in the room, where the inflation is made, and at the known temperature of the environment, where the tire is operated and at the pressure in the car tire, recommended by the production plant, it is possible to determine the required pressure of air inflation.

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