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KINEMATIC METHOD OF CALCULATING THE VALUES OF MOMENTS IN THE MOST DANGEROUS SECTIONS OF BEAMS

The method of applying technique of the kinematic limit equilibrium method was given. Formulas for calculating the values of the bending moments for reinforced concrete structures and their advantages over conventional static method, which is based on the equations of equilibrium was also derived. In this article there is shown how this method is used for the most widely-spread reinforced concrete elements in the building industry which have a great deal with bending. The simply supported beams are mostly used as such elements. They are with position fixed ends in this article. There are three kinds of loads for above mentioned beams: by one concentrated force within limits of the beam, by the uniform load over the full span and by two concentrated forces which are pointed symmetrically about the central axe of the beam and spaced at certain intervals apart.

Keywords: *the kinematic method, the limit equilibrium method, the virtual work, the plastic hinge, the bending moment.*

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КІНЕМАТИЧНИЙ СПОСІБ ВИЗНАЧЕННЯ МОМЕНТІВ У НЕБЕЗПЕЧНИХ ПЕРЕРІЗАХ ЗАЛІЗОБЕТОННИХ БАЛОК

Викладено методику застосування кінематичного способу методу граничної рівноваги при виведенні формул обчислення значень згинальних моментів для залізобетонних конструкцій та показано її переваги над звичайним способом статики, що ґрунтується на використанні рівнянь рівноваги. Продемонстровано, як цей спосіб застосовується для найбільш поширених у практиці будівництва залізобетонних елементів, що працюють в умовах згинання. Як розрахункові елементи в роботі розглянуто балки на двох опорах, шарнірно оперті з обох кінців. Вивчено такі види завантаження для балок: однією зосередженою силою в межах прольоту, рівномірно розподіленим навантаженням уздовж усього прольоту та двома силами, симетрично прикладеними відносно осі симетрії балки на деякій відстані одна від одної.

Ключові слова: *кінематичний спосіб, метод граничної рівноваги, віртуальна робота, пластичний шарнір, згинальний момент.*

Introduction. In the design of concrete structures is often necessary to solve engineering problems with definition values of internal bending moments from outside load in the most dangerous sections. For their calculations can be used depending on which of the known strength of materials or structural mechanics. Such dependence is usually obtained using the equations of statics. However, the use of methods proposed structural mechanics and strength of materials, does not always provide the appropriate level taking into account the physical and mechanical properties of concrete (creep of concrete, the establishment and opening of cracks in the element, the possibility of debonding of reinforcement with concrete, flow steel reinforcement, etc.), which affects the valid values the efforts that are sections in construction, such as a beam, and thus to rationally picked sectional area of the steel reinforcement. This disadvantage can be eliminated by using the kinematic method for determining the efforts sections in construction stage of limit equilibrium. For the possibility of using this method of calculation formulas to display calculation of limit values of bending moments in beams is devoted to this work.

Analysis of recent sources of research and publications. On the possibility of using the kinematic method for calculating the values of effort in sections of reinforced concrete slabs and shells statically indeterminate structures as described in many works, in particular, these include [1 – 10].

Singling unsolved aspects of the problem. In these works [1 – 10] given the withdrawal method of calculation formulas for calculating the values of bending moments in statically indeterminate structures is quite complicated and the possibility of using this method for statically defined concrete structures in the state of limit equilibrium generally not considered.

Setting objectives. Purpose is to demonstrate the possibility of using the kinematic mode using limit equilibrium method for removing dependencies of calculating the values of bending moment in the most dangerous sections of beams to specific cases of downloads. Also need to show the benefits of the kinematic model of the method of limit equilibrium in the derivation of required formulas over conventional static method based on the use of equations balance.

The basic material and results. To achieve this goal we consider statically determined reinforced concrete beam on two supports with three variants external load: one concentrated force F, two equal forces F and a concentrated uniformly distributed load of intensity q. For such settlement schemes beams external forces (concentrated or evenly distributed) will perform work on the possible linear movements that occur during its deformation. At the same time, counteracting the influence of external forces, internal power factors (points), seeking to return to its original undeformed beam position, and will be working on relevant possible angular movements. Since the balance between internal and external efforts in the state of limit equilibrium should not be disturbed, the work done by external forces, equal work, which is done internally. Expression of virtual equality work undertaken by external and internal efforts in general can be represented as follows:

$$W_{F,q} = W_{M,Q,N}\,, \tag{1}$$

where $W_{F,q}$, $W_{M,Q,N}$ – relevant work undertaken by external forces and internal efforts on possible movements that occur during the beam deformation.

The use of the expression (1) first look for a design scheme of reinforced concrete beams, when the applied force F and the distance a from the left support (fig. 1, *a*). With such load defined beam current limiting its equilibrium state characterized by the fact that a dangerous power under section you can enter the elastic-plastic hinge point of the largest, which is external torque (fig. 1, *b*). For reinforced concrete beams this condition occurs in only one case, namely, when the strain in the tension reinforcement in concrete compressed zone reach critical ε_{s0} and ε_{c1}. This estimated value under stress will f_{yd} and f_{cd}. It is in this state for beams with reinforcement in the area of the stretched zone A_s and the inner shoulder of force couple z_s moment's limited value $M_u = A_s \cdot f_{yd} \cdot z_s$.

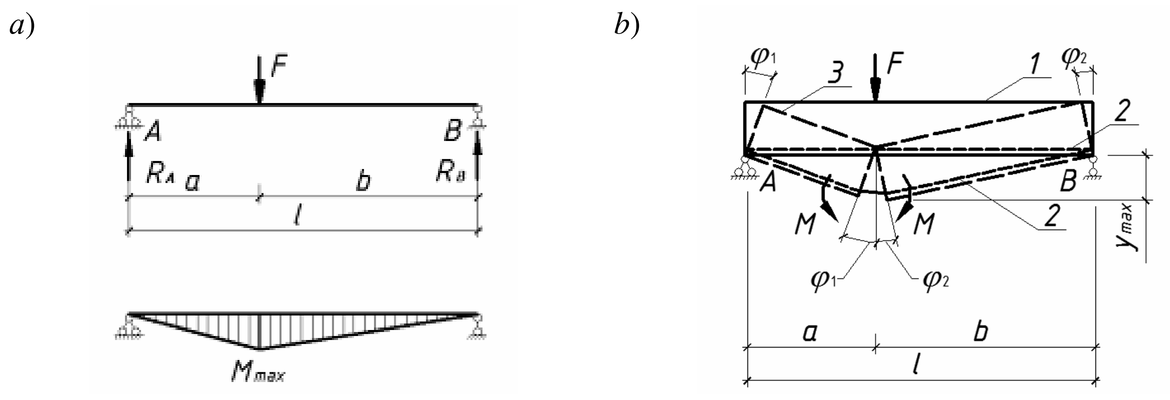

Figure 1 – Schemes to determine M_{max} of the force *F*:
***a* – by static method; *b* – by kinematic method;**
1 – reinforced concrete beam;
2 – steel reinforcement;
3 – reinforced concrete beam in a state of limit equilibrium

Using the adopted design scheme (fig. 1, *b*), the formula for calculating the value of virtual work W_F in equation (1), carried out an external concentrated force *F*, can be written as:

$$W_F = F \cdot y_{max}, \tag{2}$$

where y_{max} - moving the beam in the direction of the force *F*.

Similarly, virtual work W_M, which is carried out by the internal aspect *M* at angles of rotation φ_1 and φ_2, determined by the formula

$$W_M = \sum_{i=1}^{2} M\varphi_i = M \cdot \varphi_1 + M \cdot \varphi_2 = (\varphi_1 + \varphi_2) \cdot M \; . \tag{3}$$

Substituting expressions (2) and (3) in equation (1) will have

$$F \cdot y_{max} = (\varphi_1 + \varphi_2) \cdot M \; . \tag{4}$$

Taking into account that the angles φ_1 and φ_2 are small size, and taking into account that the proposed design scheme for this case $\varphi_1 \approx tg\varphi_1 = y_{max} / a$, and $\varphi_2 \approx tg\varphi_2 = y_{max} / b$, and also $l = a + b$, from equation (4), after substituting it above expressions for φ_1 and φ_2 and relevant simplifications will have

$$M = M_{max} = M_u = \frac{F \cdot y_{max}}{y_{max} / a + y_{max} / b} = \frac{F \cdot a \cdot b}{l} \; . \tag{5}$$

That is, using the kinematic method limit equilibrium method, we were known in course materials resistance and structural mechanics a formula to determine the moment in the section of the force *F*, but much easier than allow it to carry out these courses with known methods.

Now apply this method of determining internal efforts to reinforced concrete beams loaded at its two forces (fig. 2).

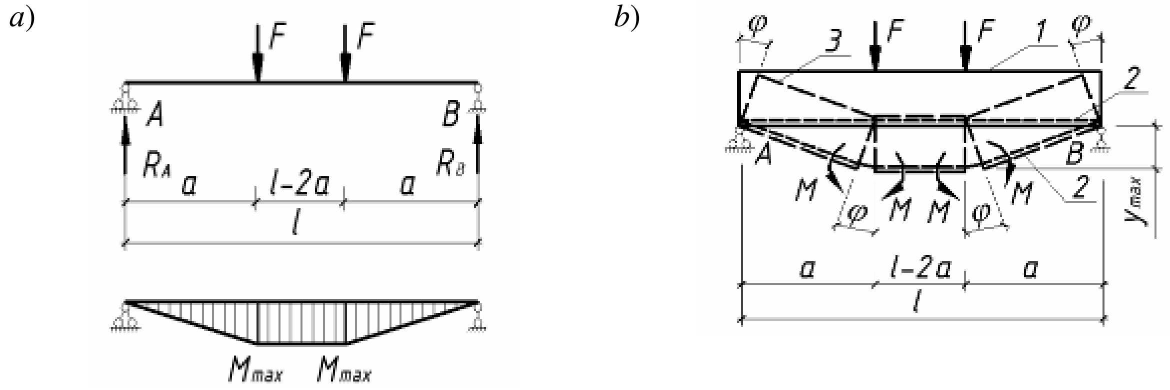

Figure 2 – Schemes to determine M_{max} from the action of two forces F:
a – by static method; b – by kinematic method;
1 – reinforced concrete beam;
2 – steel reinforcement;
3 – reinforced concrete beam in a state of limit equilibrium

For loading beams with two forces take the following assumptions: current limit state beam equilibrium characterized by the fact that dangerous under sections can be entered by elastic-plastic hinges equal in magnitude in their internal moments $M = A_s \cdot f_{yd} \cdot z_s$ beams and so, that equal the external moment in these sections (fig. 2, *b*).

In accordance with the prerequisites for on figure 2, *b* design scheme formula for determining the virtual work W_F in expression (1), which provides two external concentrated forces F, would look like this:

$$W_F = \sum_{i=1}^{2} F_i y_i = F \cdot y_{max} + F \cdot y_{max} = 2F \cdot y_{max} . \tag{6}$$

Similarly, virtual work W_M, which is carried out by internal elements M at angles of rotation φ

$$W_M = \sum_{i=1}^{2} M_i \varphi_i = M \cdot \varphi + M \cdot \varphi = 2M \cdot \varphi . \tag{7}$$

Substituting expressions (6) and (7) in equation (1) will have

$$2F \cdot y_{max} = 2M \cdot \varphi . \tag{8}$$

Taking into account that the angles φ are small size, and taking into account that the proposed design scheme for this case $\varphi \approx tg\varphi = y_{max} / a$, from equation (8), after substituting in the above expression for φ and relevant simplifications will have

$$M = M_{max} = M_u = \frac{F \cdot y_{max}}{y_{max} / a} = F \cdot a . \tag{9}$$

Apply the above method of determining the internal forces for statically determinate beams, which operates uniformly distributed load along its entire length (fig. 3). For this circuit the load is evenly distributed load beam is clear that the current limit state of beam is characterized that the dangerous section is located in the middle of the beam. Here you can enter the elastic-plastic hinge with point $M = A_s \cdot f_{yd} \cdot z_s$ in a limit state of beam, which is equal to moment in this sections (fig. 3, *b*) from outside load.

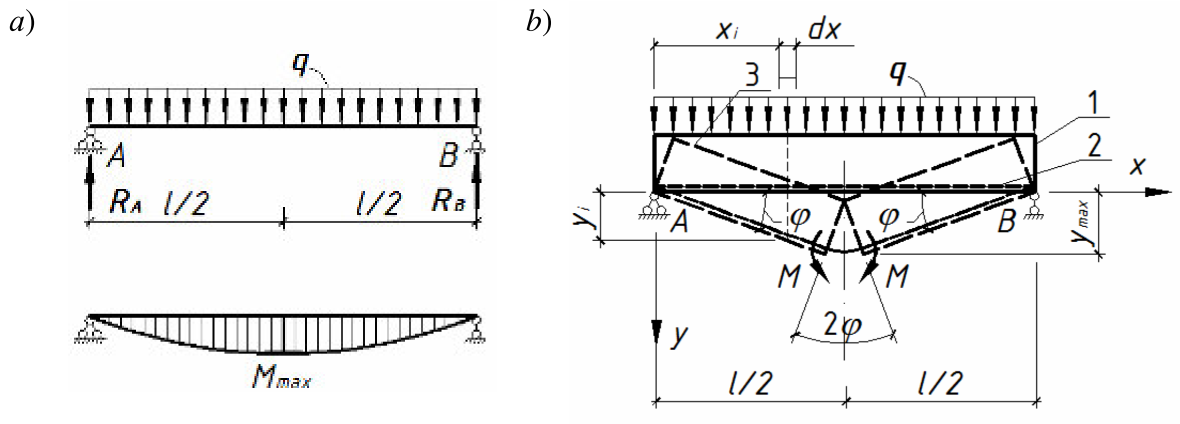

Figure 3 – Schemes to determine M_{max} from action of uniformly distributed load intensity *q*:
***a* – by static method; *b* – by kinematic method;**
1 – reinforced concrete beam;
2 – steel reinforcement;
3 reinforced concrete beam in a state of limit equilibrium

Using the adopted design scheme (fig. 3, *b*) and according to the above assumptions applied linear relationship $y_i = \varphi \cdot x$ formula for calculating the value of virtual work W_F in expression (1), which is implemented uniformly distributed load of intensity *q*, will look like

$$W_q = 2\int_0^{l/2} y_i q dx = 2q \cdot \int_0^{l/2} \varphi \cdot x dx = 2q\varphi \frac{l^2/4}{2} = \frac{q\varphi \cdot l^2}{4}. \quad (10)$$

Similarly, virtual work W_M, carried out by the internal moment *M* on the corner of turn 2*φ*,

$$W_M = M \cdot 2\varphi. \quad (11)$$

Substituting expressions (10) and (11) in equation (1) will have

$$\frac{q\varphi \cdot l^2}{4} = 2\varphi \cdot M . \quad (12)$$

From equation (12) after appropriate simplifications we obtain

$$M = M_{max} = M_u = \frac{q \cdot \varphi \cdot l^2}{4 \cdot 2\varphi} = q \cdot l^2 / 8. \quad (13)$$

Now, for example, if the formula (13) written as:

$$A_s \cdot f_{yd} \cdot z_s = q \cdot l^2 / 8, \quad (14)$$

we get the dependency, which can be easily determined required reinforcement area A_s for reinforced concrete beams.

Conclusions. Using the kinematic method limit equilibrium method, in the dependences for calculation of values of bending moments in the most dangerous sections of beams loaded one and two concentrated forces and evenly distributed load intensity *q*. After comparing the methods of static and kinematic methods output desired formula becomes apparent that the simplest is the second. This conclusion is based on the reduced number of calculations in these examples, as compared to methods that use static equation. So kinematic method in engineering calculations is more convenient, simple and one that more accurately reflects the essence of reinforced concrete structures under load. Also, if we take into account that $M_u = M$, then, substituting instead M_u and M appropriate expressions easily calculate the area of steel reinforcement, which guarantees the limiting equilibrium reinforced concrete beams.

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