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## Numerical simulation of hard airdrome coatings stress-strain state when interacting with weak ground base

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The technique and its numerical realization of the hard airdrome coatings specified calculations with the consideration of their interaction with heterogeneous multilayer weak soil half-space is proposed. Numerical simulation of the hard airdrome coatings stress-strain state is carried out on the basis of the relations of the elasticity nonlinear theory with the help of the finite element method momentary scheme. The task of calculating the actual concrete coating on a rigid artificial basis of the International Airport "Odesa" runway is solved. A finite-element model for calculating hard surfaces with the use of a momentary finite element scheme and a universal spatial shell finite element has been developed.

**Keywords:** finite element method, weak ground base, rigid airdrome coverage, numerical simulation, stress-strain state

## Чисельне моделювання напружено-деформованого стану жорстких аеродромних покриттів при взаємодії зі слабкою ґрунтовою основою

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Запропоновано методику і її числову реалізацію уточнених розрахунків жорстких аеродромних покриттів з урахуванням їх взаємодії з неоднорідним багат шаровим слабким ґрунтовим півпростором. Чисельне моделювання напружено-деформованого стану жорстких аеродромних покриттів здійснено на основі співвідношень нелінійної теорії пружності за допомогою моментної схеми методу скінченних елементів. Розв'язана задача розрахунку реального бетонного покриття на жорсткій штучній основі злітно-посадкової смуги Міжнародного аеропорту «Одеса». Розроблено скінченно-елементну модель для розрахунку жорстких покриттів з використанням моментної схеми скінченних елементів і універсального просторового оболонкового скінченного елемента. Чисельний розрахунок напружено-деформованого стану жорстких аеродромних покриттів при взаємодії зі слабкою ґрунтовою основою виконаний на дію навантаження від літака В 767 -300. Розрахункова схема покриття побудована так, щоб було включене колісне навантаження від всього шасі повітряного судна з урахуванням того, що основна опора літака розміщувалася б на середній плиті розрахункового фрагмента. Аналіз отриманих чисельних результатів показує, що значення погонного згинального моменту під лівим нижнім колесом основної опори літака В767-300 досягає величини -99,724 кН·м/м, що відповідає напруженню в нижньому волокні умовної плити 950,88 кН·м/м<sup>2</sup>. Розглянута конструкція жорсткого аеродромного покриття задовольняє умовам граничного стану та забезпечена міцність на можливі допустимі напруження. Розрахунок покриттів при колісному впливі крупно фюзеляжних повітряних суден при наявності включень слабких шарів ґрунту необхідно виконувати тільки на основі чисельних досліджень. Максимальне значення прогину при постійному коефіцієнті постелі становить 0,41 мм, а при змінному – 0,75 мм.

**Ключові слова:** метод скінченних елементів, слабка ґрунтова основа, жорсткі аеродромні покриття, чисельне моделювання, напружено-деформований стан



## Introduction

The calculation and design of rigid airfield coverings have been carried out in accordance with the normative document [1]. However, modern literary sources [4, 9] suggest that the existing norms of calculation are clearly obsolete, since they do not consider the parameters of heavy-duty large body aircraft of type B 767-300 and others and the presence of weak layers of soil in the bases.

The basis of the existing regulations for calculating the rigid airfield coverage is the analytical relationship between the calculation of a continuous inseparable plate on elastic basis with the use of the direct proportionality hypothesis, with very close consideration of the transition to finite-size plates in the presence of cross-stitches and jump joint.

## Review of research sources and publications

Since 2015, the volume of overhaul and new building of aerodrome elements is rapidly increasing in Ukraine (in the years 2015-2030, such airports in Boryspil, Odesa, Vinnytsa, Chernivtsi, Kherson, and Nikolaev are planned to be reconstructed).

These problems in airfield construction can be solved only by applying modern numerical methods for calculating the stress-strain state of hard airfield coverings before the construction or reconstruction of real aerodromes.

The forest soils, which occupy almost 75% of the territory of Ukraine, are in stressed state under the influence of wheel loading or their own weight of coverings and soil, they can sink and can reduce their structural strength when soaked [3, 9-11, 13-17]. In order to solve this problem and to improve the calculation of rigid airfield coverings in the presence of weak soil active layer of weak soil and soil heterogeneity inclusions, [4-6] modeling of the soil basis was proposed by a discrete non-homogeneous half-space, considering the discrete-local zones of state equations for anisotropic material, equivalent to real weak soils.

In the framework of existing regulatory documents, considering modern scientific researches, it is possible to consider the interaction of aerodrome coatings with inhomogeneous soil half-space using the functions of the variable coefficient of subgrade resistance [3; 4]. Such approach is consistent with the problem solution of research of rigid airfield coverings which mainly work on the bend and distribute the load on sufficiently large plane of soil half-space. These coatings should be considered as thin plates using discrete finite element models (FEM).

It is proposed to approximate the function of the variable coefficient of subgrade resistance by a trigonometric spline function in the local coordinate system on the median surface of the plate.

## Problem statement

The purpose of the study is to test the strength of the actual airfield coverage using numerical methods.

## Basic material and results

Numerical simulation of the stress-strain state of the rigid aerodrome coating in interaction with weak ground base is performed on the example of the International Airport "Odessa". The engineering-geological section with the largest thickness of weak soils is used as the source data.

In this case, geological data are selected from the most unfavorable conditions, which are shown in Figure 1.

The proposed coverage of the International Airport "Odessa" runways consists of the following constructive layers:

- top layer – high-strength concrete grade B40 cement concrete Btb 4,8/60,  
 $R_{bn} = 29 \text{ MPa}$ ,  
 $R_{btn \text{ axial tension}} = 2,1 \text{ МПа}$ ,  
 $R_{btb \text{ bending}} = 4,1 \text{ MPa}$ ,  
 $E_b = 3,53 \cdot 10^4 \text{ МПа}$ ,  
 $\rho = 2500 \text{ kg/m}^3$ ,  
 $\nu_1 = 0,22$ ;
- the bottom layer is a leached concrete grade B7,5 cement concrete Btb 4,8/60,  
 $R_{bn} = 5,5 \text{ MPa}$ ,  
 $R_{btn \text{ axial tension}} = 0,7 \text{ MPa}$ ,  
 $R_{btb \text{ bending}} = 1,5 \text{ MPa}$ ,  
 $E_b = 1,6 \cdot 10^4 \text{ МПа}$ ,  
 $\rho = 2490 \text{ kg/m}^3$ ,  
 $\nu_1 = 0,23$ .

The artificial foundation is represented by such constructive layers:

- ground cement M75  
 $R_{bn} = 3,5 \text{ MPa}$ ,  
 $R_{btn \text{ axial tension}} = 0,55 \text{ MPa}$ ,  
 $R_{btb \text{ bending}} = 1,3 \text{ MPa}$ ,  
 $E_b = 3,53 \cdot 10^4 \text{ МПа}$ ,  
 $\rho = 2500 \text{ kg/m}^3$ ,  
 $\nu_1 = 0,22$ ;
- crushed stone from natural stone, enclosed by a method of decomposition with limiting compressive strength 100 MPa  
 $E_n = 4,5 \cdot 10^2 \text{ MPa}$ ,  
 $K_s = 4,5 \cdot 10^2 \text{ МН/М}^3$ ,  
 $E_b = 3,0 \cdot 10^3 \text{ kgp/sm}^2$ ;
- synthetic gauze  
 $K_{se} = 16 \text{ kgp/sm}^2$ ,  
 $h_{рек.} = 20,0 \text{ см}$ .

Soil foundation has the following characteristics:

- layer from the element EGE-1 ( $E_b = 672,2 \text{ kgp/sm}^2$ ,  $h_1 = 140 \text{ см}$ );
  - layer EGE -2 ( $E_b = 672,2 \text{ kgp/sm}^2$ ,  $h_2 = 170 \text{ см}$ );
  - layer EGE -3 ( $E_b = 525,15 \text{ kgp/sm}^2$ ,  $h_3 = 250 \text{ см}$ );
  - layer EGE -4 ( $E_b = 266,86 \text{ kgp/sm}^2$ ,  $h_4 = 90 \text{ см}$ ).
- Equivalent coefficient of subgrade resistance [7]:  
 $K_{se} = 55,285 \text{ МН/М}^3$

In accordance with [1], this ground base belongs to the weak group.

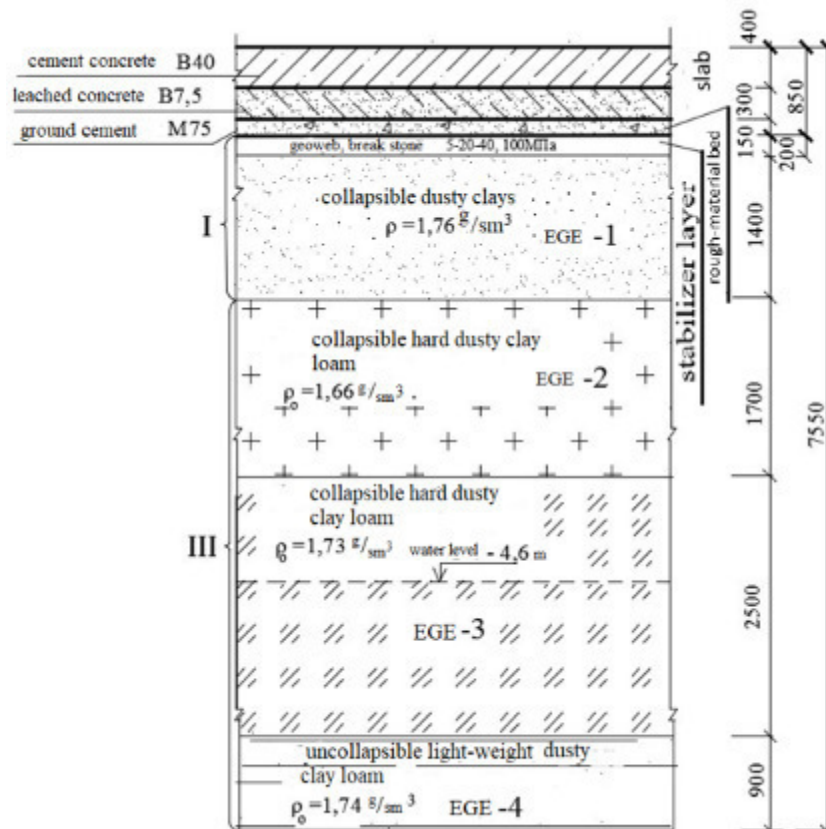


Figure 1 – Aerodrome coating construction considering the active layer of soil foundation

Aerodrome plates, reducing the average pressure under the cover at the expense of a large area, sharply increase the depth of the compressed layer of the soil half-space, that is, attracting to the work of deep, but weak, strongly compressed layers of water saturated dust-loamy soils.

For the implementation of a refined numerical calculation of the coverage under the wheel load of the aircraft chassis one of the most versatile schemes of the FEM is used - the moment scheme of finite elements (MSFE) [5].

In this technique, a simplified mathematical model of elastic basis is used with the assumption of the proportionality between the positive plate deflection and the reaction of the base and depends on the coordinate of the pplate (node) median surface point where the base deflection and reaction must be calculated

$$q(x^2, x^3) = -c(x^2, x^3) \omega, \quad (1)$$

where  $q(x^2, x^3)$  is the function of the proportionality factor (subgrade resistance);

$\omega$  – a positive deflection at this point along the normal to the plate surface in deformed state [8].

The coefficient of subgrade resistance function can be described by two-dimensional approximation [2] using a certain number of the coefficient values of subgrade resistance for the considered section of the geological section, tconsidering the multilayer soils of the underlying coating and its thickness, providing the

value of the equivalent total deformation modulus in this intersection.

Knowing a number of coefficient values of subgrade resistance, depending on the average, the vertical of the base thickness, the general module of soil deformation and using one or another analytical function: spline, trigonometric, index, power, or other, a specific function in this area of the calculation model is obtained.

For example, in the presence of a lens-like soil layer with known the limit values of the subgrade resistance coefficient and, also using an analytic spline according to the law of the sinus, it is got

$$c_N(x^2, x^3) = C_0 - (C_{\max} - C_0) \sin \frac{\pi x_N^2}{l^2} \sin \frac{\pi x_N^3}{l^3}, \quad (2)$$

where  $x_N^2, x_N^3$  – are current local coordinates of nodes in the calculated fragment in the global coordinate system;

$l^2, l^3$  – are the sizes of the calculated fragment in the global axis coordinate system  $z^2$  and  $z^3$ .

In the considered scheme of MSFE, the discrete model has two limiting surfaces - the lower and upper, that is, the plate is considered as single layer - conditional (equivalent to bending and longitudinal stiffness). An equivalent analogue of the conditional plate is constructed from the equivalence condition for the specified stiffnesses for the uniform size of the plate:

$$\begin{cases} E_{(e)} \cdot \frac{t_e^3}{12} \cdot 100 = EI_{(0,x)}^0, \\ E_{(e)} \cdot 100 \cdot t_{(e)} = EF_{(0,x)}^0 \end{cases}, \quad (3)$$

where  $EI_{(0,x)}^0, EF_{(0,x)}^0$  – are the actual rigidities of the construction of a multilayer plate, with the bending (with respect to the coordinate system in the center of the weight of the plate, considering the rigid basis) considers only the upper and lower layers, and the longitudinal rigidity  $EF_{(0,x)}^0$  considers two layers and a rigid foundation. The model of the equivalent analogue of the aerodrome plate construction is shown in Fig. 2

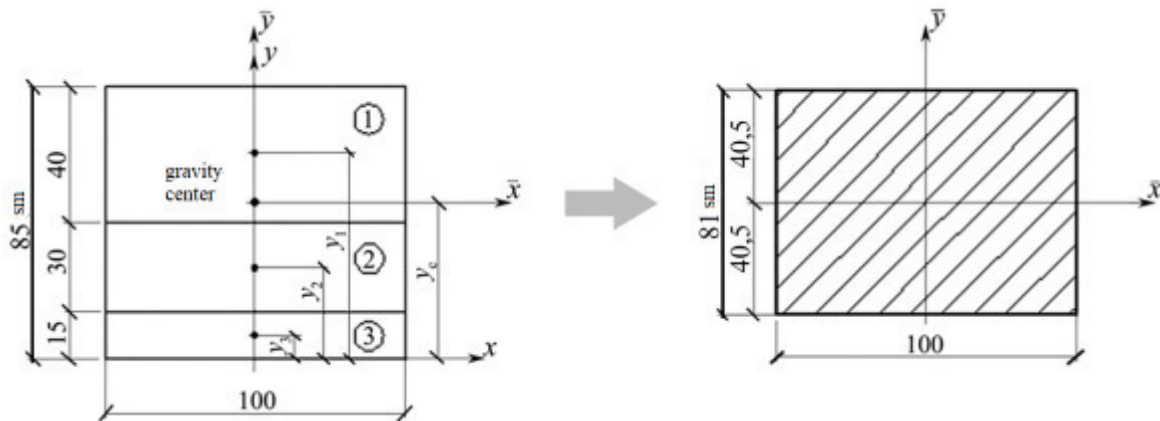


Figure 2 – The model is equivalent to the design of an airfield cover plate

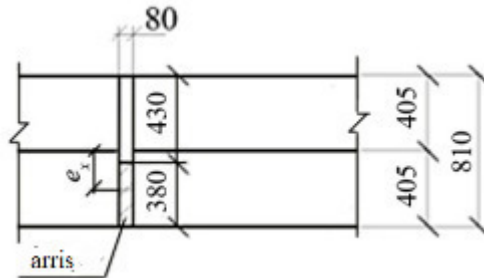


Figure 3 – Model of joints of aerodrome plates

Models parameters of butt inserts have the following meanings:

– the ratio of the rib thickness to the shell thickness:

$$\frac{h_p}{h_o} = \frac{38}{81} = 0.47;$$

– the eccentricity ratio to the shell thickness (in relation to the local coordinate system)

$$\frac{e_x}{h_o} = -\frac{21.5}{81} = -0.265;$$

– elasticity module of the insert material:

$$E_{b(r)} = 2,6 \cdot 10^3 \text{ kg/sm}^2;$$

– poisson coefficient of insertion material  $\nu_{b(r)} = 0.3$ ;

– the ratio of the specific gravity of the insert material to the equivalent specific gravity of the airfield cover:

$$\frac{\gamma_{b(r)}}{\gamma_{cp}^e} = \frac{0.0016}{0.002527} = 0.633;$$

– insertion width:  $b_r^{np} = 8 \text{ cm}$ .

As a rated aircraft, Boeing 767-300 has been adopted.

The parameters of the round wheel print are calculated by the formula:

$$F_d = \frac{F_n}{n_k} \cdot k_d \cdot \gamma_f, \quad (4)$$

where  $F_d$  – is estimated load on the wheel;  
 $n_k$  – number of wheels of the main support;  
 $F_n$  – load on the main support ( $F_n = 724.9 \text{ kH}$ );  
 $k_d$  – dynamic factor ( $k_d = 1.25$ );  
 $\gamma_f$  – unloading factor ( $\gamma_f = 1.0$ ).

Then  $F_d = 226.53 \text{ kN}$ .

The square footprint for the Aircraft B767-300 is 43 cm.

The calculation scheme of the coating is constructed so that the wheel load from the entire chassis of the aircraft was included, considering that the main bearing of the plane is placed on the middle plate of the calculated fragment.

Considering the symmetry of the Aircraft B767-300, a discrete model containing six plates of a runway covering 10×7.5 m has been constructed. The calculation scheme and the finite-element model of the calculated coating fragments for the Aircraft B767-300 are shown in Fig. 4

Equivalent discrete prints of wheel pneumatics with a pressure  $p_a = 1,21$  MPa according to the finite element model presented, are located on one medium plate, that is, on four discrete areas of uniform surface loading, brought to the nodal.

The discrete model is constructed so that the main four-wheel support is located in the middle of the calculated fragment middle slab, with the aircraft chassis parameters. Fragments of the topological model of wheel prints in the coating calculation scheme are described by the grid coordinates  $S_2, S_3$  – the beginning and the end of the wheel load.

The dimensions of the grid area are:  $M1 \times M2 \times M3$ , that is  $2 \times 30 \times 46$ , and the size of the calculated fragment –  $15000 \times 30000$  mm. Total nodes in the FE-model, presented at fig. 4 -  $N_u = 2 \times 30 \times 46$  which corresponds to the equilibrium equations system  $k_p = N_u \cdot 3 = 2760 \cdot 3 = 8280$ , (without consideration the imposed connections) and the number of finite elements –

$$M_e^p = (M2 - 1) \cdot (M3 - 1) = (30 - 1)(46 - 1) = 1305.$$

According to the butt joint adopted model of the coated plates in the structure of the presented discrete model (see Figure 3), the plates borders introduced inserts (edges) which fragments are also described by network coordinates - only five fragments.

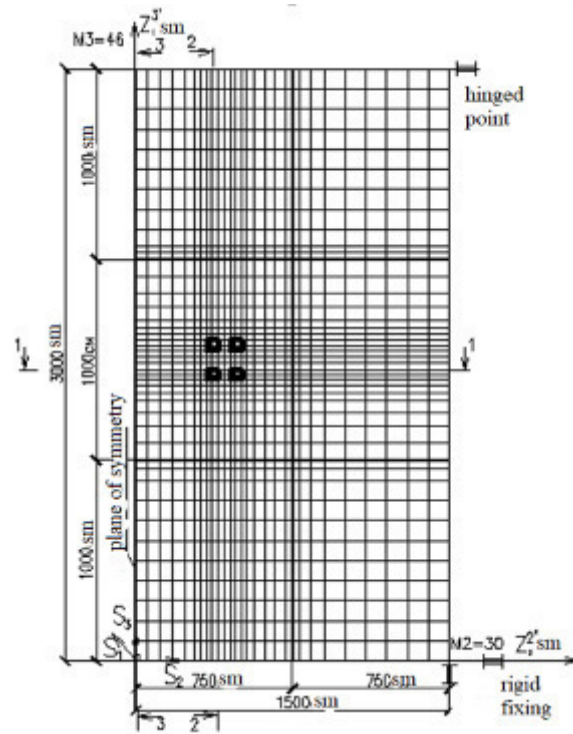
In accordance with the calculated scheme of plates discrete models, boundary kinematic communication conditions in the global coordinate system  $OZ^1 Z^2 Z^3$  are:

– on the coordinate line  $OZ^3$  – the plane of symmetry is imposed by the joints on the movement  $u_N^2$  and angles of rotation  $v_N^2$ ;

– on the coordinate line  $OZ^2$  – The model of hinge tangential fastenings - is imposed on a link to move  $u_N^3, u_N^2$ ;

– on the edges of the estimated fragments - if  $Z_N^2 = 1500$  sm and  $Z_N^3 = 3000$  sm the model of hinged tangential fixings is implemented – the connections are on  $u_N^3, u_N^2$ .

The numerical calculation results of the wheel load from an aircraft in B767-300 are shown on the isopolos and diagrams, which are presented in Fig. 5-13.



**Figure 4 – Coverage scheme of the SZPS with the wheelbase chassis of the Aircraft B 767-300 chassis**

Based on the results of calculations, data have been obtained to determine the maximum values of the main stresses in the conventional plate and stresses in the upper and lower layers of the real coating design.

$$\sigma_{1,\text{sup}} = k_{1,\text{sup}} \sigma_{\text{max}}^{(e)}; \quad \sigma_{2,\text{inf}} = k_{2,\text{inf}} \sigma_{\text{max}}^{(e)}.$$

The analysis of the obtained numerical results shows that the value of the lateral bending moment under the left lower wheel of the main support of the Aircraft B767-300 reaches the value  $M_{552}^{33} = -99.724$  kWhm/m, which corresponds to the stress in the lower fiber of the conditional plate  $\sigma_{552}^{\text{max}} = 950.884$  kWhm/m<sup>2</sup>.

To switch to a real 2-layer coating it should be used the formulas [4]:

$$\sigma_{1,\text{sup}} = 1.4389 \sigma_{\text{max}}^{(e)};$$

$$\sigma_{2,\text{inf}} = 0.4528 \sigma_{\text{max}}^{(e)}.$$

Considering the design resistance of the concrete strength for the upper and lower layers, respectively  $R_{btm}^{\text{sup}}$  and  $R_{btm}^{\text{inf}}$ , it can be written:

$$\sigma_{1,\text{sup}} = 1.4389 \times 950.88 \leq R_{btm}^{\text{sup}} \cdot \gamma_c \cdot k_u;$$

$$\sigma_{2,\text{inf}} = 0.4528 \times 950.88 \leq R_{btm}^{\text{inf}} \cdot \gamma_c \cdot k_u,$$

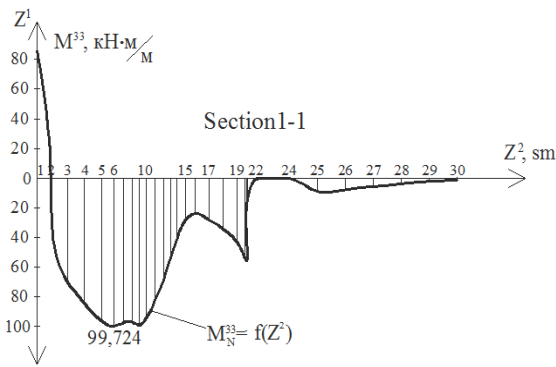
where  $\gamma_c$  – is coefficient of work conditions, with [1]  $\gamma_c = 0.75$ ;

$$k_u = 2 - 0.167 \cdot l_g \cdot U_d,$$

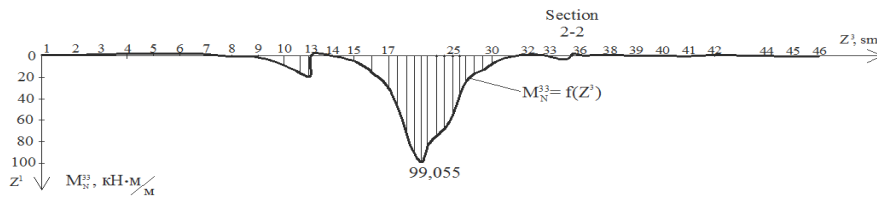
$U_d$  – average annual application of wheel loads Aircrafts.

$$\sigma_{1,\text{sup}} = 1.395 \text{ MPa} < 2.131 \text{ MPa};$$

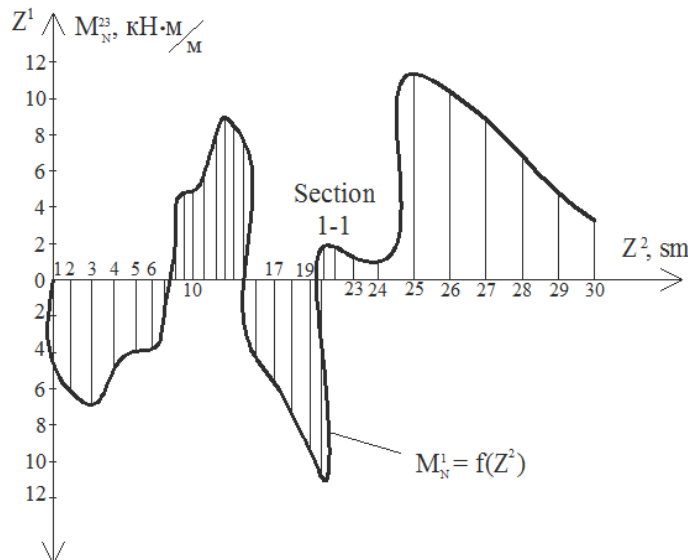
$$\sigma_{2,\text{inf}} = 0.439 \text{ MPa} < 0.711 \text{ MPa}.$$



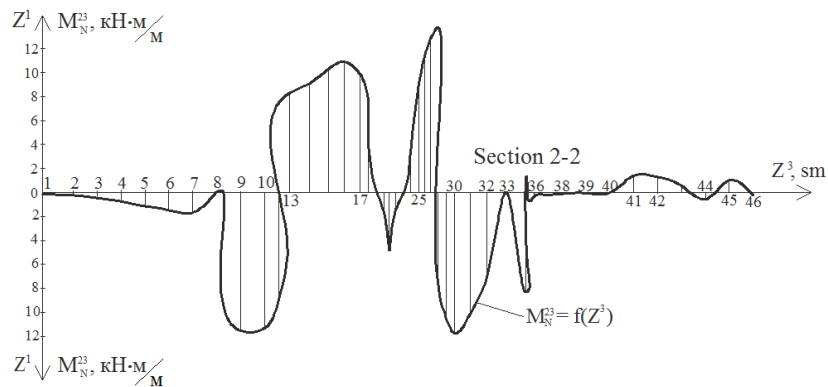
**Figure 5 – Circle of bending moments  $M^{33}$ ,  $\text{kH}\cdot\text{m}/\text{m}$  from the action of the towers Aircraft B 767-300 in the section 1-1**



**Figure 6 – Circle of bending moments  $M^{33}$ ,  $\text{kH}\cdot\text{m}/\text{m}$  from the action of the towers Aircraft B 767-300 in the section 2-2**



**Figure 7 – Circuit running torque  $M^{23}$ ,  $\text{kH}\cdot\text{m}/\text{m}$  from the action of the towers Aircraft B 767-300 in the section 1-1**



**Figure 8 – Circuit running torque  $M^{23}$ ,  $\text{kH}\cdot\text{m}/\text{m}$  from the action of the towers Aircraft B 767-300 in the section 2-2**

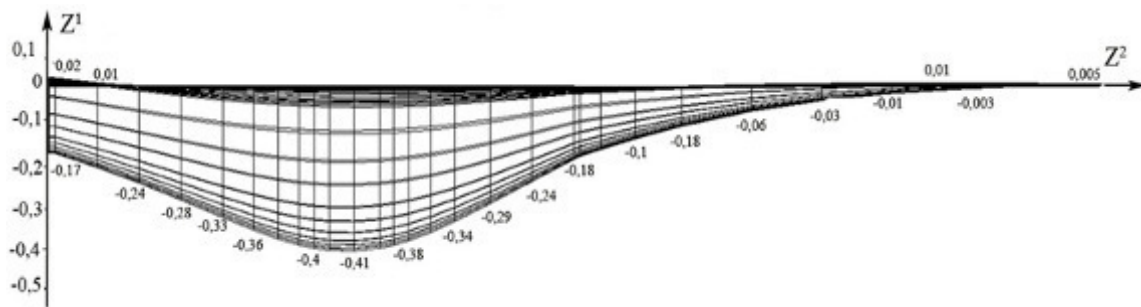


Figure 9 – The course of nodal displacements is a constant coefficient of subgrade resistance from the wheel loading of the bearings Aircraft B767-300 in the section 1-1

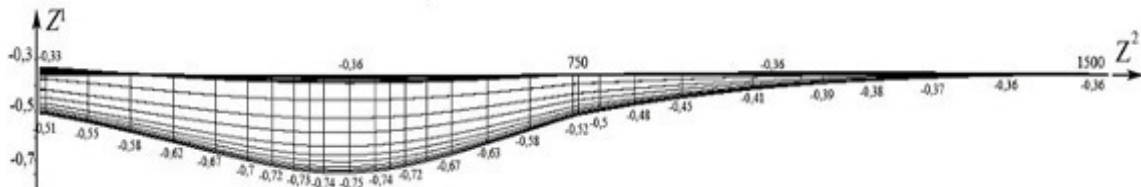


Figure 10 – Episode nodal displacement with variable coefficient of subgrade resistance from the wheel loading of the bearings Aircraft B767-300 in the section 1-1

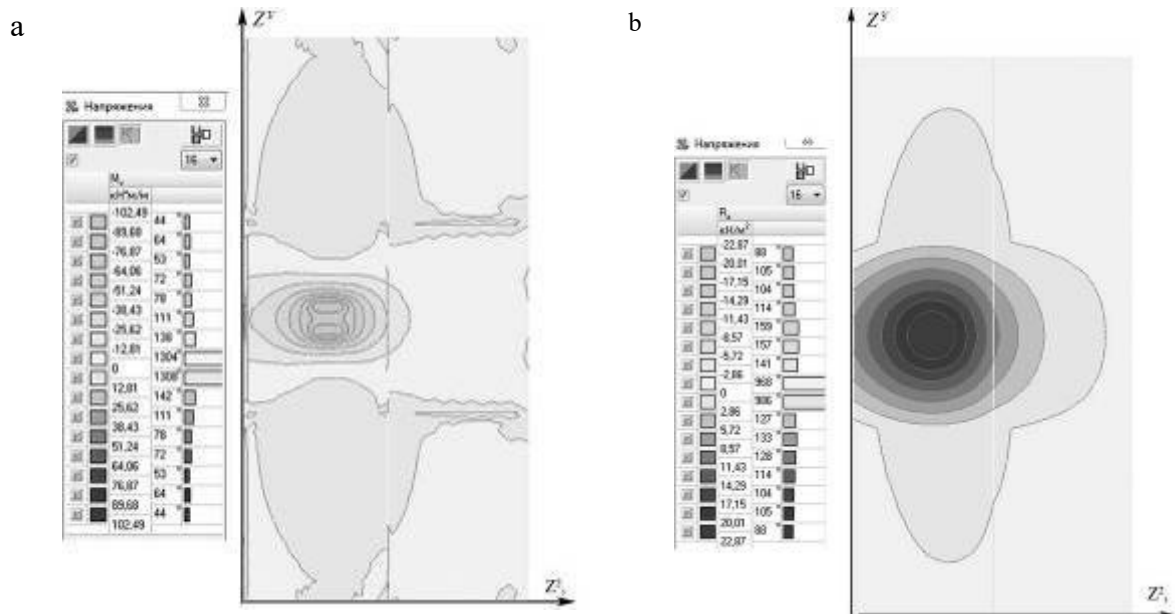


Figure 11 – Isolation of the calculation scheme of the coating from the action of the Aircraft B767-300 supports:  $a$  - bending moments  $M^{33}$ ,  $kH \cdot m/m$ ;  $b$  - basic reactions  $R_{22}$ ,  $kN/m$

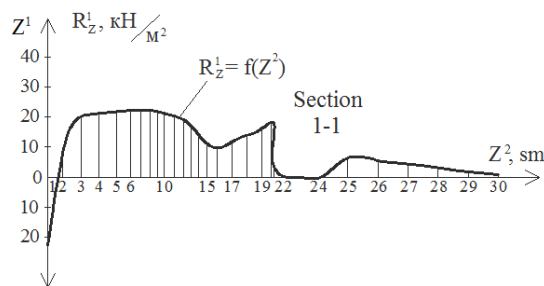


Figure 12 – Episode of nodal ground-level reactions in section 1-1

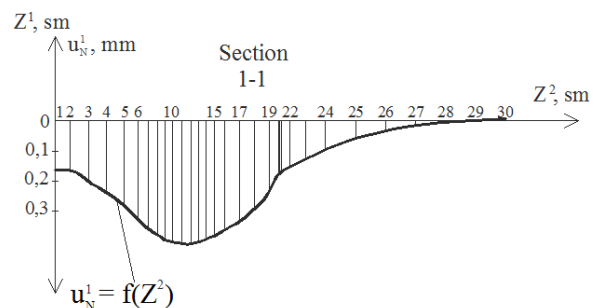


Figure 13 – The section of the nodal displacements from the wheel load of the Aircraft B767-300 in the section 1-1

Thus, the strength of the possible permissible stresses is ensured. The considered construction of the coating satisfies the conditions of the boundary state.

Calculation of coatings under the wheel influence of large-scale aircraft in the presence of weak soil layers inclusions should be performed only on the basis of numerical studies [4].

The results of numerical calculations of hard airdrome coverage are obtained in two variants:

1) with constant coefficient of subgrade resistance  $C \equiv k_{se} = 55.8 \text{ MN/m}^3$ ;

2) with variable coefficient of subgrade resistance within the calculated fragment by the formula (2) (in the presence of soil weak layer) with  $C \equiv k_{se} = 55.8 \text{ MN/m}^3$  and  $C_{min} = 33 \text{ MN/m}^3$ .

Grades of the deflections have a smooth character, the maximum value of the deflection at constant coefficient of subgrade resistance is  $-0.41 \text{ mm}$  fig. 9, and with a variable coefficient of subgrade resistance ( $c_0 = 33 \text{ MN/m}^3$ ,  $c_{max} = 56 \text{ MN/m}^3$ ) is  $-0.75 \text{ mm}$  in the knot 201 FE-model fig. 10.

$$u_{201}^I = -0.75 \text{ mm.}$$

## Conclusions

A numerical calculation of rigid two-layer coating on rigid artificial basis, in interaction with inhomogeneous multilayer weak ground sub-space on the wheel influence of the chassis of the V767-300 subsystem, has been performed. According to the calculations of the real construction of the runway cover SIA "Odessa" the presence of soil weak layers on the basis of the coating, it can be concluded that the proposed methodology for numerical studies of hard coatings from the the wheel load action of super-heavy aircraft is sufficiently effective and reliable, meets the requirements for ensuring the reliable operation of modern airports airfields.

## References

1. СНиП 2.05.08.85. (1985). *Аэродромы*. Москва: ЦНТП Госстроя СССР.
2. ДБН В.2.6-98:2009. (2011). *Бетонні та залізобетонні конструкції. Основні положення*. Київ: Мінрегіонбуд України, ДП „Укрархбудінформ“.
3. Гольдштейн, М.Н. (1973). *Механические свойства грунтов*. Москва: Стройиздат.
4. Цыхановский, В.К., Козловець-Талах, С.М., Коряк, А.С. (2008). *Расчет тонких плит на упругом основании методом конечных элементов*. Киев: Изд-во „Сталь“.
5. Баженов, В.А., Сахаров, А.С., Цыхановский, В.К. (2002). Моментная схема метода конечных элементов в задачах нелинейной механики сплошной среды. *Прикладная механика*, 6, 24-63.
6. Шимановский, А.В., Цыхановский, В.К. (2005). *Теория и расчет сильнонелинейных конструкций*. Киев: Изд-во „Сталь“.
7. Білеуш, А.І., Березівський, М.В., Серд, Я. (2002). Патент України 52548А. *Спосіб визначення коефіцієнта постелі ґрунтової основи*. Київ: ДП «Український інститут промислової власності».
8. Карпиловский, В.С., Криксунов, Э.З., Моляренко, А.А. и др. (2015). *SCAD Office. Версия 21. Вычислительный комплекс SCAD++*. Москва: СКАД СОФТ.
9. Кульчицкий, В.А., Макагонов, В.А., Васильев, Н.Б. и др. (2002). *Аэродромные покрытия. Современный взгляд*. Москва: Физматлит.
10. Harr, M.E. (1966). *Foundations of theoretical soil mechanics*. New York: McGraw-Hill.
11. Vynnykov, Y. (2000). *Numerical Solutions of Non-linear Three-dimensional problems of Interaction of Compaction Foundations with Soil*. Proc. of the First Central Asian Geotechnical Symposium «Geotechnical Problems of Construction, Architecture and Geoenvironment on 339 Boundary of XXI Century».
12. Zienkiewicz, O. (1971). *Finite Element Method in Engineering Science*. New York: Wiley.
1. СНиП 2.05.08.85.(1985). *Airfields*. Moscow. TsNTP Gosstroy USSR.
2. SBN B.2.6-98:2009 (2011). *Concrete and reinforced concrete constructions. The main positions*. Kiev: Ministry of Urban Development of Ukraine, State Enterprise «Ukrarchbudinform».
3. Goldstein, M.N. (1973). *Mechanical properties of soils*. Moscow: Stroyizdat.
4. Tsykhanovsky, V.K., Kozlovets-Talah, S.M. & Koryak, A.S. (2008). *Calculation of thin plates on elastic foundation by the finite element method*. Kiev: Publishing House «Steel».
5. Bazhenov, V.A., Sakharov, A.S. & Tsykhanovsky, V.K. (2002). Moment scheme of the finite element method in problems of nonlinear mechanics of continuous medium. *Applied Mechanics*, 6, 24-63.
6. Shimanovsky, A.V. & Tsykhanovsky, V.K. (2005). *Theory and calculation of strongly nonlinear structures*. Kiev: Publishing House «Steel».
7. Bileush, A.I., Berezivskiy, M.V. & Serd, Y. (2002). Patent of Ukraine 52548A. *Method for determining bed coefficient of soil base*. Київ: SE «Ukrainian Intellectual Property Institute».
8. Karpilovsky, V.S., Kriksunov, E.Z., Molyarenko, A.A. et al. (2015). *SCAD Office. Version 21. Computer complex SCAD++*. Moscow: SCAD SOFT.
9. Kulchitsky, V.A., Makagonov, V.A., Vasiliev, N.B. et al. (2002). *Airfield coverings. Modern look*. Moscow: Fizmatlit.
10. Harr, M.E. (1966). *Foundations of theoretical soil mechanics*. New York: McGraw-Hill.
11. Vynnykov, Y. (2000). *Numerical Solutions of Non-linear Three-dimensional problems of Interaction of Compaction Foundations with Soil*. Proc. of the First Central Asian Geotechnical Symposium «Geotechnical Problems of Construction, Architecture and Geoenvironment on 339 Boundary of XXI Century».
12. Zienkiewicz, O. (1971). *Finite Element Method in Engineering Science*. New York: Wiley.



13. Zotsenko, N. & Vynnykov, Yu. (2015). Cast-in-situ piles in punched holes design features. *The special aspects energy and resource saving*. Oradea: Oradea University Press, 4-34.

14. Zotsenko, N. & Vynnykov, Yu. (2016). Long-Term Settlement of Buildings Erected on Driven Cast-In-Situ Piles in Loess Soil. *Soil Mechanics and Foundation Engineering*, 53(3), 189-195.

<https://doi.org/10.1007/s11204-016-9384-6>

15. Zotsenko, N., Klovanich, S., Sckola, A. & Vynnykov, Y. (2003). *Full-scale Tests and Numerical Simulation of Interaction between Foundations with Tamped Rigid Process Wastes Widenings and Soil Environment.* Proc. of the XIIIth European Conf. on Soil Mechanics and Geotechnical Eng, 1, 963-966.

16. Zotsenko, M., Vynnykov, Y. & Yakovlev, A. (2010). *Modern practice of determination of strength characteristics of cohesive soils by penetration methods*. Proc. of XIVth Danube – European Conf. on Geotechnical Eng., 245-253.

17. Zotsenko, N. & Vynnykov, Yu. (1999). *Rapid Investigation Methods of Soil Properties and Interpretation of their Results for Bridge Foundations Design*. IABSE New Delhi Colloquium reports on “Foundations for Major Bridges: Design and Construction”, 19-24.

13. Zotsenko, N. & Vynnykov, Yu. (2015). Cast-in-situ piles in punched holes design features. *The special aspects energy and resource saving*. Oradea: Oradea University Press, 4-34.

14. Zotsenko, N. & Vynnykov, Yu. (2016). Long-Term Settlement of Buildings Erected on Driven Cast-In-Situ Piles in Loess Soil. *Soil Mechanics and Foundation Engineering*, 53(3), 189-195.

<https://doi.org/10.1007/s11204-016-9384-6>

15. Zotsenko, N., Klovanich, S., Sckola, A. & Vynnykov, Y. (2003). *Full-scale Tests and Numerical Simulation of Interaction between Foundations with Tamped Rigid Process Wastes Widenings and Soil Environment.* Proc. of the XIIIth European Conf. on Soil Mechanics and Geotechnical Eng, 1, 963-966.

16. Zotsenko, M., Vynnykov, Y. & Yakovlev, A. (2010). *Modern practice of determination of strength characteristics of cohesive soils by penetration methods*. Proc. of XIVth Danube – European Conf. on Geotechnical Eng., 245-253.

17. Zotsenko, N. & Vynnykov, Yu. (1999). *Rapid Investigation Methods of Soil Properties and Interpretation of their Results for Bridge Foundations Design*. IABSE New Delhi Colloquium reports on “Foundations for Major Bridges: Design and Construction”, 19-24.