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PROBABILITY OF THE UTILITY NETWORKS DOUBLE-RING STRUCTURES' CONNECTIVITY FOR SITES WITH VARIOUS RELIABILITY

The analytical description study results on probability of connectivity for the structures used to model the reliability of various complicated systems are presented. Expressions are formed to calculate the connectivity probability of systems that have structural redundancy. The characteristic components of the formulas are distinguished and they are systematized according to their increasing complexity and the number of elements. The features of the equations' structure permitting to conveniently formulate the probability of the structures connectivity in the process of their construction and transformations are determined. The examples show the formation of formulas and their structural parts at various levels of complexity. The use of the ratio value of the network structure element's unreliability and its reliability is justified, thus reducing the awkwardness of exact expressions for the connectivity probability of network structures and substantially improves the compactness and convenience of using the equations.

Keywords: structural modeling, connectivity probability, reliability of a system with different reliability of elements

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ЙМОВІРНІСТЬ ЗВ'ЯЗНОСТІ ДВОКІЛЬЦЕВИХ СТРУКТУР ІНЖЕНЕРНИХ МЕРЕЖ З РІЗНОЮ НАДІЙНІСТЮ ДІЛЯНОК

Наведено результати дослідження аналітичного опису ймовірності зв'язності структур, що використовуються для моделювання надійності різних складних систем. Побудовано вирази обчислення значення ймовірності зв'язності систем, що мають структурний резерв. Виділено характерні складові формул, систематизовано їх у порядку збільшення складності та кількості елементів. Установлено особливості будови рівнянь, які дозволяють зручно складати вирази ймовірності зв'язності структур у процесі їх конструювання й перетворень. На прикладах показано утворення формул і їх структурних частин різних рівнів складності. Обґрунтовано використання величини співвідношення значення ненадійності елемента структури мереж та його надійності, що скорочує громіздкість точних виразів імовірності зв'язності структур мереж і суттєво підвищує компактність та зручність використання рівнянь.

Ключові слова: структурне моделювання, ймовірність зв'язності, надійність системи з різною надійністю елементів.

Introduction. Development of science and technology widely uses different technical systems (TS), including engineering networks in all branches of industry and management. Qualitative characteristics of compound systems are steadily increasing. However, the problem of security remains the major one. Functions performed by the present day engineering systems are complicated enough, and their tasks are extremely responsible.

The need to integrate different scientific fields in the problem of systems' reliability is explained by the fact that the problem is complicated. Theoretical studies indicate [1 – 7] that it is impossible to achieve a significant increase in the systems' reliability level taking separate isolated measures. Integrated solution of tasks enhances the efficiency of each method and helps obtain qualitatively new results.

Determining the probability of connectivity is an important task in designing and reconstruction of utility networks: water, gas, mobile operators' networks, etc. [1].

Analysis of recent research sources and publications. Recent studies [5] have analyzed the reliability of the network structures with elements that have the same reliability values. In the operating objects, the properties of reliability and efficiency are in the mutual conflict [1, 5]. To raise the reliability of different systems additional expenditures are necessary, thus reducing their efficiency. The study [8] represents the optimum network design with series-parallel structures. In [10], the method of rational valuation of reliability in systems with parallel and serial-parallel structures is suggested.

In [11], the main reasons of inadequate development on redundant utilities' reliability problems are highlighted. The concept of interval and boundaries of the efficient improvement of the redundant utilities' reliability is suggested. The attention is focused on the approaches to the identification of the basic patterns in technical systems' failures, the probability model of the technical systems' non-failure is suggested. Comparison of the probability time of technical systems' non-failure operation with different configurations is performed, the influence of structure forms on the technical systems' uptime is shown.

Determination of still unsolved aspects of the general problem. In mathematical modeling of reliability it is necessary to accurately determine the probability of connectivity for structures with varying reliability of elements.

Problem statement (formulation of the aim and methods to research the problem). Probability expressions should be suggested for connectivity of network systems with areas of varying reliability. At a trial designing or reconstruction of structurally complicated redundant systems, this model will be useful for better decision making.

A method should be suggested for constructing analytical expressions of the exact connectivity probability value for structures with varying reliability elements having two cycles, and a random number of sites in cycles in a compact recording format that is convenient for computer simulation.

It is necessary to display dependence expressions on the connectivity probability for structures that are transformed by means of removing or adding new sites.

Basic material and results. The form of network structure significantly effects its reliability. Important is the connectivity of the structure that is a topological feature of the network. To determine the structural reliability the concept [5] operating condition is applied, when all components of the structure are connected to each other. The system may have a structural reserve that is formed by closure of connections between the elements in the form of cycles. Accordingly, non-operating condition arises when connection is lost with the only node of the structure. The ultimate operating condition [5] is formed through the connection of all the nodes of the structure by sites of wooden cover. In this case, the system's structure is extensive and has no connection closures between the nodes.

Probability of the network structure connectivity is the probability of the compatible events sum:

$$R = \sum_{i=1}^{T} P(B_i) - \sum_{i \neq j} P(B_i \cap B_j) + \ldots + (-1)^{T-1} P(B_1 \cap B_2 \cap \ldots \cap B_T), \tag{1}$$

where $P(B_i)$ – is the formation probability of the wooden cover B_i that connects all the nodes of the structure;

$P(B_i \cap B_j)$ – is the probability of the simultaneous existence of the wooden covers B_i and B_j in the system's structure.

Let us consider modeling of the structure's reliability in more general terms. Let us assume, that sites are having different reliability: $r_1^d \neq r_2^d \neq \ldots \neq r_m^d$. Limitation: the nodes' reliability is considered absolute: $r_1^v = r_2^v = \ldots = r_m^v = 1$.

Let us take two sites and connect them so as not to close the structure. Then the reliability of the system, with two sites connected in series, will be: $R = r_1 \cdot r_2$, with three sites: $R = r_1 r_2 r_3$, with n sites: $R = r_1 r_2, \ldots, r_n$. The structures of systems with elements connected in series can have a branched (extensive) form. They have no structural reserve, because they have not a high level of reliability. The reliability value (probability of connectivity) varies within the range of rational numbers (0, 1). This indicates that with increasing of the number of sites connected in series, the level of the system's structural reliability is reducing nonlinearly.

Let us close the structure into the cycle for the two sites to have common ends. The simplest structural reserve of the system is formed, when there are not one, but two connections between the nodes. The reliability equation for the redundant structure consisting of two sites with different reliability $r_1 \neq r_2$, in the form of parallel connection between the two nodes v_1 та v_2 looks as follows [1]

$$R = 1 - \prod_{i=1}^{2} (1 - r_i), \tag{2}$$

or:

$$R = -r_1 r_2 + r_1 + r_2. \tag{3}$$

Let's consider the formation of more complicated structures with a single cycle. Let us add to the structure one site with reliability r_3 into the cycle C^2 and the new node $C_1^2 \cup d = C_1^3$. Then another structure will be obtained in the form of a triangle with the following equation of the connectivity probability:

$$R = -2r_1 r_2 r_3 + r_1 r_2 + r_1 r_3 + r_2 r_3. \tag{4}$$

With the increasing number of sites and nodes in the structure S with a single cycle $C_1^i \cup d = C_1^{i+1}$ its connectivity probability expression changes. For the first summand, the coefficient value increases in magnitude per unit, its value being equal to the number of sites smaller by one: $(p - 1)$. The first summand is the product of the coefficient and the reliability values of all sites:

$$-(p-1) \prod_{i=1}^{p} r_i.$$

The number of succeeding additive components is increasing by one, too. The total number of these components is $- p$, i.e. it is equal to the number of sites. Each summand is the

product of the reliability values r_i i.e. it equals to the number of sites, but for the one having the relevant number in ascending order:

$$\sum_{j \neq i}^{p} \frac{\prod_{k=1}^{p} r_k}{r_k}.$$

Table 1 presents graphical models of structures with one cycle C^p with a number of sites $p < 6$ and the respective reliability expressions. The general equation for the reliability of redundant structures, having a single cycle $S \subset C_i^p$, $i = 1$ with p sites is written:

$$R = -(p-1)\prod_{i=1}^{p} r_i + \sum_{j \neq i}^{p} \frac{\prod_{k=1}^{p} r_k}{r_k}, \tag{5}$$

where r_i –is reliability of the i-site.

With the increasing complexity of the structure, the study performed using this approach is getting quite cumbersome. So, let us apply systematization and structuring of the research.

Let's start with the simplest redundant structure, the structure consisting of the 2 sites, connecting two nodes (Fig. 1). The total of possible unique operating conditions of the redundant structure consisting of 2 sites will be three. Analytically this is expressed as:

$$R = -r_1 \cdot r_2 + r_1 + r_2. \tag{6}$$

Let us assume the concordance that the likely failures of sites are marked with counting numbers in a series of natural numbers. This will help determine patterns of constructing a compound network structure. Then the structure connectivity probability equation is the product of all sites' reliability multiplied by the sum of one and the ratio of unreliable to reliable sites:

$$R = M \left(1 + \sum_{i=1}^{2} e_i \right) \tag{7}$$

where $\quad e_i = \dfrac{1 - r_i}{r_i}, \quad M = \prod_{i=1}^{n} r_i \quad i = 1, 2, \ldots, n$
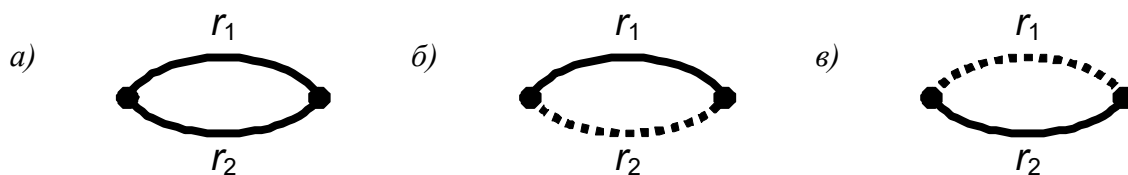

Figure 1 – Operating conditions of the 2-sites redundant structure:
a – all sites are operating; b – failure of site 2; c – failure of site 1
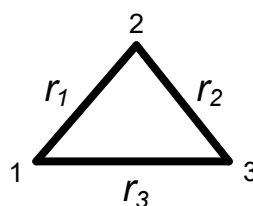

Figure 2 – Operating condition of the elementary structure without failures of sites
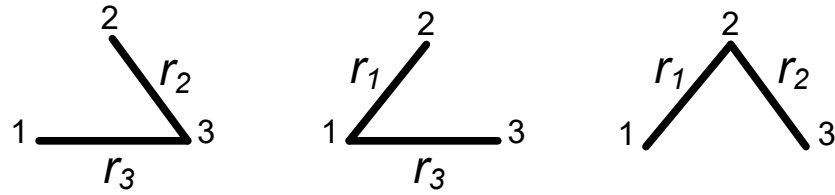
Figure 3 – Operating conditions of the elementary structure when one of the sites fails

Some results of the consistent input of sites into the single-ring structure and defining their connectivity probability expressions are presented in Table 1.

Table 1 – Connectivity probability of single-ring structures comprising 2-5 sites and nodes $S \subset C_i^p$, $i = 1$, $p{<}6$

Numerical character	Graphic symbol of a structure	Connectivity probability expression
$m=2$ $p=2$ $n=1$		$R=M(1+\sum\limits_{i=1}^{2} e_i)$
$m=3$ $p=3$		$R=M(1+\sum\limits_{i=1}^{3} e_i)$
$m=4$ $p=4$		$R=M(1+\sum\limits_{i=1}^{4} e_i)$
$m=5$ $p=5$		$R=M(1+\sum\limits_{i=1}^{5} e_i)$

Thus, reliability of the structure $S^p \subset C_v^l$, $v = 1$ equals:

$$R = M F_d, \qquad (8)$$

where $M = \sum\limits_{i=1}^{p} r_i$ $i = 1,2,...,p$, $F_d = 1 + \sum\limits_{i=1}^{p} e_i$, $e_i = \dfrac{1-r_i}{r_i}$,

r_i – reliability value of the i-site of the structure S.

Property 1. When entering i-site r_i and the top into structure S with a single cycle $v = 1$, as a part of the second summand of the equation (8), there appears the value $e_i = \dfrac{1-r_i}{r_i}$ of this site, which is the ratio of its values of unreliability and reliability. Accordingly, with removal of i-site the value e_i disappears from the expression (8).

Let us raise the complexity of structure S and, maintaining the structural reserve, let us enter the second cycle into the structure. The reliability equation for the three parallel sites r_1, r_2, r_3 is:

$$R = 1 - \prod_{i=1}^{3} (1 - r_i) \tag{9}$$

or, having performed the transformation,

$$R = M\left(1 + e_5 + \sum_{i=1}^{4} e_i(1 + e_5) + \sum_{i=1}^{2} e_i \sum_{i=3}^{4} e_i\right). \tag{10}$$

Let us enter one site and one top into one of the cycles. Accordingly, we'll obtain a change in the connectivity probability of the structure, which is expressed in Table 2.
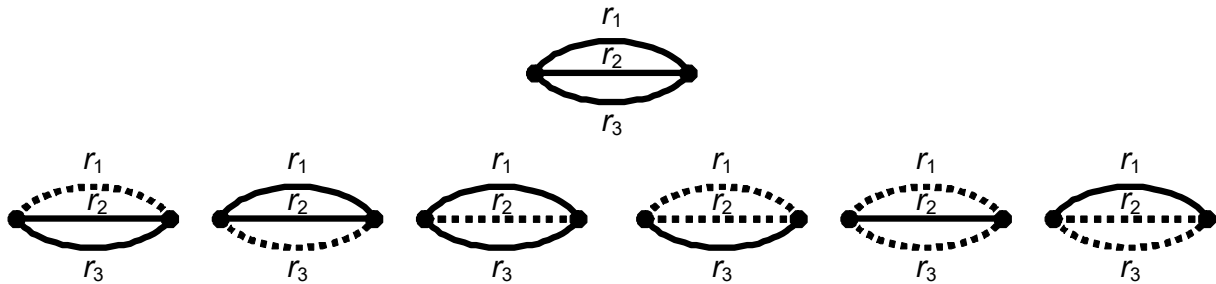

Figure 4 – Seven operating conditions of the redundant structure consisting of 3 sites

Analytically, the connectivity probability of the structure equals:

$$R = r_1 \cdot r_2 \cdot r_3 - r_1 \cdot r_2 - r_1 \cdot r_3 - r_2 \cdot r_3 + r_1 + r_2 + r_3, \tag{11}$$

or:

$$R = \prod_{i=1}^{3} r_i - \sum_{j=1}^{3} \frac{\prod_{j=1}^{3} r_j}{r_j} + \sum_{k=1}^{3} r_k. \tag{12}$$

Assuming that variable $r_i = \dfrac{1}{1 + e_i}$, $M = \prod_{i=1}^{3} r_i$, $i = 1,2,3$, after transformations, we'll obtain the equations included into Table 2.

Table 2 – Connectivity probability for double-ring structures
with three sites $S^p \subset C_v^l$, $v = 2$, $p < 4$

№	Network structure	Connectivity probability formula
1		$R = M\left(1 + e_3 + \sum_{i=1}^{2} e_i(0 + e_3) + \sum_{i=1}^{1} e_i \sum_{i=2}^{2} e_i\right)$
2		$R = M\left(1 + e_4 + \sum_{i=1}^{3} e_i(1 + e_4) + \sum_{i=1}^{1} e_i \sum_{i=2}^{3} e_i\right)$

Property 2. At entering another cycle with *h*-sites into the single-cycle structure, the structural connectivity is formed, which is described by the additive component:

$$F_C = e_h + \sum_{i=1}^{h-1} e_i (1 + e_h) + \sum_{i=1}^{j} e_i \sum_{i=j+1}^{h} e_i \ .$$
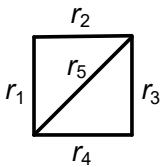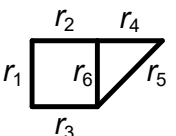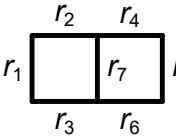
In the connectivity probability equation:

$$R = M \left(1 + \sum_{i=1}^{p} e_i + F_C\right), \tag{13}$$

where $e_i = \dfrac{1 - r_i}{r_i}$, $M = \prod_{i=1}^{p} r_i$, $i = 1, 2, ..., p$, r_i – reliability values of *i*-site of structure S, j – is the number of sites in the first cycle C_1^l.

With the increasing number of sites in the cycles the additive component of the connectivity probability equation varies. The connectivity probability of structures with two cycles and the number of sites from five to seven are presented in Table 3.

Table 3 – Connectivity probability of structures with two cycles

and the number of sites from five to seven $S^p \subset C_v^l$, $v = 2$, $p<6$

	Networks structure	Connectivity probability formula
$m=4$ $p=5$	r_2, r_5, r_1, r_3, r_4	$R = M \left(1 + e_5 + \sum_{i=1}^{4} e_i (1 + e_5) + \sum_{i=1}^{2} e_i \sum_{i=3}^{4} e_i\right)$
$m=5$ $p=6$	r_2, r_4, r_1, r_6, r_5, r_3	$R = M \left(1 + e_6 + \sum_{i=1}^{5} e_i (1 + e_6) + \sum_{i=1}^{3} e_i \sum_{i=4}^{5} e_i\right)$
$m=6$ $p=7$	r_2, r_4, r_1, r_7, r_5, r_3, r_6	$R = M \left(1 + e_7 + \sum_{i=1}^{6} e_i (1 + e_7) + \sum_{i=1}^{3} e_i \sum_{i=4}^{6} e_i\right)$

Thus, reliability of the structure $S^p \subset C_v^l$, $v = 2$ equals:

$$F_C = e_h + \sum_{i=1}^{h-1} e_i (1 + e_h) + \sum_{i=1}^{j} e_i \sum_{i=j+1}^{h} e_i$$

$$R = M \left(1 + e_p + \sum_{i=1}^{p-1} e_i (1 + e_p) + \sum_{i=1}^{j} e_i \sum_{i=j+1}^{p-1} e_i\right) \tag{14}$$

where $e_i = \dfrac{1 - r_i}{r_i}$, $M = \prod_{i=1}^{p} r_i$, $i = 1, 2, ..., p$,

r_i – reliability values of *i*-site of structure S^p,

p –is the number of sites in the structure,

j – is the number of sites in the first cycle C_1^l,

v – is the number of cycles,

l – is the number of sites in the *i*-cycle.

Conclusions. The equation for the exact value of the connectivity probability of structures with varying reliability of elements, having two cycles and a random number of sites in the cycles, is obtained.

A compact form to record reliability equations is obtained, their structure being convenient for computer simulation.

The structure of the suggested dependences permits obtaining new expressions of connectivity probability for structures, transformed by removing or adding new sites. This approach helps apply them in the computer modeling of connectivity probability of network structures in various fields of engineering.

References

1. *Reinschke K. Application of Graph Theory for Reliability Analysis / K. Reinschke, I. Ushakov. – Berlin: Verlag Technik, 1987. – 209 p.*
2. *Barlow R. E. Mathematical Theory of Reliability / R. E. Barlow, F. Proschan. – New York: John Wiley & Sons, 1965. – 592 p.*
3. *Надежность технических систем: Справочник / Под ред. И.А. Ушакова – М. : Радио и связь, 1985. – 606 с.*
4. *Advanced Computer Graphics / Edited by Tosiyasu L. Kunii – Tokyo, Springer-Verlag. 1986. – 508 p.*
5. *Probabilistic constrained optimization. Methodology and applications / Ed. S. P. Uryasev. – Dordrecht/Boston/London: Kluwer Academic Publishers. 2000. – 307 p.*
6. *Onisava T. Reliability and Safety Analyses under Fuzziness / T. Onisava, J. Kacprzyk. – Phisica-Verlag. A Springer-Verlag company, 1995. – 376 p.*
7. *Kaufman Z. W. Mathematical models for the study of the reliability of system / Z. W. Kaufman, D. Groucho, R. Cruon. – NY: Academic Press, 1977. – 221 p.*
8. *Gutjahr W. Configurations of Series-Parallel Networks with Maximum Reliability / W. Gutjahr, G. C. Pflug, A. Ruszczynski. – Laxenburg, Austria: IIASA Working Paper, 1993. – 15 p.*
9. *Новохатній В. Г. Порівняльний аналіз надійності і топології структур водопровідних мереж / В. Г. Новохатній, І. С. Усенко // Збірник наукових праць. Серія: Галузеве машинобудування, будівництво. – Полтава: ПолтНТУ, 2002. – Вип. 10. – С. 50 – 55.*
10. *Усенко В. Г. Раціональне нормування надійності елементів в системах з паралельно-послідовними структурами // Містобудування та територіальне планування: наук.-техн. зб. – К.: КНУБА, 2012. – Вип. 46. – С. 578 – 582.*
11. *Усенко В. Г. Границы эффективности повышения структурной надежности в резервируемых инженерных сетях / В. Г. Усенко, И. С. Усенко // Сборник научных трудов SWorld. – Иваново: МАРКОВА АД, 2013. – Вып. 4., Т. 13. – С. 78 – 83.*