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MATHEMATICAL MODELING OF THE THERMODYNAMIC PROCESS GAS-STEAM BUBBLES

A mathematical model that considers the inertial oscillations and thermodynamic components bubbles in liquid heat exchange processes, heat transfer on the boundary bubbles. Research of the dynamic characteristics of gas-steam bubbles in various size was conducted. After the calculations its temperature, velocity, pressure steam environment inside the bubble in time, graphs bubbles size change graphs were built . It is established that each bubble size has its oscillation frequency. Calculated speed phase transients and found that it is in its maximum during the bubble oscillation. For thermodynamic properties of the surface of contact liquid and gaseous phases defined amount of solid phase formed. The research results can be applied to optimize various of technological processes related to the boil, swelling materials, and the formation of gas hydrates in a fluid cavitation.

Key words: *mathematical model, bubble, heat exchange, gas hydrate, pressure, temperature.*

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ТЕРМОДИНАМІЧНИХ ПРОЦЕСІВ ГАЗОПАРОВОЇ БУЛЬБАШКИ

Запропоновано математичну модель, яка враховує інерційну та термодинамічну складові осиляції бульбашок, теплообмінні процеси у рідині, теплообмін на границі бульбашки. Проведено дослідження динамічних характеристик газопарових бульбашок різних розмірів. Після виконаних розрахунків побудовано графіки зміни розміру бульбашки, її температури, швидкості руху, тиску парогазового середовища всередині бульбашки в часі. Установлено, що кожний розмір бульбашок має свою частоту осиляцій. Розраховано швидкість фазовоперехідних процесів і встановлено, що вона набуває максимальних значень саме під час осиляцій бульбашки. За термодинамічними характеристиками поверхні контакту рідкої та газоподібної фаз визначено кількість утвореної твердої фази. Результати досліджень можуть застосовуватися для оптимізації різноманітних технологічних процесів, пов'язаних з кипінням, спученням матеріалів, утворенням газових гідратів та кавітацією у рідині.

Ключові слова: *математична модель, бульбашка, теплообмін, газовий гідрат, тиск, температура.*

Introduction. One aspect of the use of advanced technology gas storage is the possibility of building gas storage near large consumers (boiler, CHP). Structurally, a storage battery consists of gas tanks placed in the pit or in the hangar. In spring and summer filled with gas storage, which forms the clathrate structure, and in autumn and winter - give them gas at expansion by using low-grade heat source. Building of such storage facilities near CHP can significantly smooth out seasonal unevenness of gas consumption and is a real alternative to the construction of underground gas storage [1].

Technologically, the accumulation of gas occurs in a dissolution of gas bubbles in water under certain of thermobaric conditions [2]. The process of formation of solid phase takes place on the surface of the oscillating bubbles. During the oscillation is very quick change of thermodynamic parameters of the system «gas-steam bubble – liquid». The study of the dynamics of the process to determine the most influential factors to optimize the production process [3] and reduce the volume construction and installation works. Direct observation of phase transients on the surface of the bubbles is a complex engineering and technical challenge: high pressure (up to 20 MPa), small bubbles ($10^{-5}\div10^{-7}$ m) and the high rate of oscillation (10^{-5} s). Easier way is mathematical modeling of heat exchange and mass transfer processes on its surface.

Review of recent research sources and publications. To analyze the dynamics of growth of vapor bubbles commonly used equation Rayleigh-Plasseta [4-6]. To determine the vapor pressure inside the bubble is often used Clausius-Clapeyron equation [7, 8], or consider adiabatic process [9]. According to other researchers, the processes inside the oscillating bubbles are not limited to the phase transition or the lack of heat exchange at the surface bubbles. In [10] the mathematical formulation of the problem more fully. In addition to the equation of Rayleigh-Plasseta it contains the equation of van der Waals forces to determine the pressure within the gas-steam bubbles and allows you to calculate the temperature of gas inside the bubbles based on the first law of thermodynamics. Also added a mathematical model heat and mass transfer across the border bubbles. However, this mathematical model does not account for the impact of traffic on the wall near her heat exchange processes and phase transitions in liquid.

Selection not solved earlier of parts the general problem. For the modeling the transition processes phase on the surface of the bubbles should consider the possibility of dissolution of the gas bubbles in the liquid while liquid phase transition in the solid phase. This process is determined by the rate of heat and mass transfer processes at the surface of the bubbles, which in turn depends on the temperature and pressure inside the gas-steam mixture bubbles. In speed mode cavitation bubbles to change the size of some times can reach several hundred meters per second, which significantly affect the course of the heat - and mass transfer processes at its border. As a result of these processes, thermophysical characteristics of liquid on the boundary of the bubble can also significantly vary. Thus, for a correct formulation of the problem should adequately take into account the complex interrelated mechanical and thermodynamic processes that occur in a limited volume at high speed.

The problem statement. The aim of this work is to create a mathematical model of the dynamics of vapor bubbles that will get reliable information about its thermodynamic characteristics during growth or compression. In general, the mathematical model should include the following components: model the kinetics of gas bubbles in a viscous fluid; model thermodynamic processes inside the gas-steam bubbles; model of heat - and mass transfer processes in border vesicles; Modeling of phase transitions in liquid form with ice or other of solid phase; modeling heat transfer processes in the liquid surrounding the bubble.

Basic material and results. To develop mathematical models of gas-steam bubbles in the liquid is applied following simplifying assumptions: gas-steam bubble has a spherical shape; fluid is viscous and incompressible; gas-steam bubbles inside is a mixture of gas and vapor fluid whose mass may change as a result of mass transfer processes in border bubbles; Gas and vapor bubbles of fluid in the middle considered as real gas (including van der Waals forces).

The velocity of the fluid ($\dot{R}$) on the border of bubbles can be determined by integrating the known equation of Rayleigh-Plasseta [4]. At some times the pressure inside the bubbles can rise sharply and describing its thermodynamic state must take into account the difference of the parameters of state of ideal gas. To determine the partial pressures of the components of gas-vapor mixture, it is advisable to apply the equation of van der Waals forces [10, 11]. In general, the mathematical model of thermodynamic processes of gas-steam bubbles containing the following equation:

$$\frac{d\dot{R}}{d\tau} = \frac{P_{B(\tau)} - P_\infty}{\rho_r R} - \frac{1.5}{R}\dot{R}^2 - \frac{4\mu_r}{\rho_r \cdot R^2}\dot{R} - \frac{2\sigma_r}{\rho_r \cdot R^2}, \quad (1)$$

$$\frac{dR}{d\tau} = \dot{R}, \quad (2)$$

$$P_B = P_g + P_p, \quad (3)$$

$$P_g = \frac{R_\mu T}{\frac{\mu_g}{\rho_g} - b_g} - \rho_g^2 \frac{a_g}{\mu_g^2}, \qquad P_p = \frac{R_\mu T}{\frac{\mu_p}{\rho_p} - b_p} - \rho_p^2 \frac{a_p}{\mu_p^2}, \quad (4)$$

$$\frac{d\rho_g}{d\tau} = \frac{3}{R}\left(-\rho_g \frac{dR}{d\tau}\right), \qquad \frac{d\rho_p}{d\tau} = \frac{3}{R}\left(-\rho_p \frac{dR}{d\tau}\right), \quad (5)$$

$$\frac{d\left(m_g c_g + m_p c_p\right)T}{d\tau} = 4\pi R^2 q - P_B \frac{d\left(4/3\pi R^3\right)}{d\tau}, \quad (6)$$

$$\frac{dT}{d\tau} = \frac{3}{R\left(c_g\rho_g + c_p\rho_p\right)}\left[q - P_B \frac{dR}{d\tau}\right], \quad (7)$$

$$q = q_1 + q_2 - q_3 - q_4, \quad (8)$$

$$q_1 = 0.25\rho_p\left(u_{p(T)} - \dot{R}\right)c_p T_{(R,\tau)}, \quad (9)$$

$$q_2 = 0.25\rho_g\left(u_{g(T)} - \dot{R}\right)c_g T_{(R,\tau)}, \quad (10)$$

$$q_3 = 0.25\rho_p\left(u_{p(T)} - \dot{R}\right)c_p T, \quad (11)$$

$$q_4 = 0.25\rho_g\left(u_{g(T)} - \dot{R}\right)c_g T, \quad (12)$$

$$u_{p(T)} = \sqrt{8R_\mu T/\mu_p\pi} \text{ and } u_{g(T)} = \sqrt{8R_\mu T/\mu_g\pi}, \quad (13)$$

$$\frac{\partial\left(\rho_r c_r T_{(x,\tau)}\right)}{\partial\tau} + \dot{R}\frac{\partial\left(\rho_r c_r T_{(x,\tau)}\right)}{\partial x} = \frac{1}{x^2}\frac{\partial}{\partial x}\left(\lambda_r x^2 \frac{\partial T_{(x,\tau)}}{\partial x}\right) + q_{v(x,T)}, \quad (14)$$

$$-\frac{\partial\left(\lambda_r T\right)}{\partial x}(x = R, \tau) = -q, \quad (15)$$

$$T_{(x,\tau=0)} = T_0. \quad (16)$$

where $\dot{R}$ – speed liquid bubbles on the boundary, m/s;
τ – time, s;
$P_{B(\tau)}$ – pressure gas-vapor mixture inside the bubbles, Pa;
P_∞ – fluid pressure, Pa;

ρ_r – fluid density, kg/m^3;
μ_r – dynamic viscosity fluids, Pa·s;
σ_r – the surface tension on the boundary liquid gas, N/m;
P_g – the partial pressure of gas, Pa;
ρ_g, ρ_p – under the density of gas and steam, kg/m^3;
R_μ = 8314 – universal gas constant, J/(кmol·К);
μ_g – molecular weight gas, kg/кmol;
μ_p – molecular weight liquid vapor, kg/кmol;
T – the temperature of the gas mixture in the bubble, К;
a_g, a_p – constant Van der Waals forces respectively for gas and steam, (N·m^4)/mol^2;
b_g, b_p – constant Van der Waals forces respectively for gas and steam, m^3/mol;
m_g, m_p – weight respectively gas and pairs, kg;
c_g, c_p – mass heat capacity of the gas and water vapor, J/(kg°C);
q – specific heat flux at the surface bubbles, W/m^2;
$T_{(R,\tau)}$ – surface temperature bubbles, °C;
c_r – heat capacity of liquid, J/(kg°C);
λ_r – fluid conductivity, W/(m°C);
q_v – volumetric power sources or waste water heat, W/m^3.

As a result, phase of the transition processes on the boundary bubbles liquid can change its thermophysical characteristics (λ_r, ρ_r, c_r).Volumetric heat source considers phase transitions in liquid medium.

For the formation of the solid phase certain conditions must be fulfilled, the partial pressure of gas exceed the minimum pressure phase transition at a given temperature. The weight solids are determined by removal of the transition region phase transition heat. The intensity of the heat sources for volume phase transition are adopted linearly proportionally to the temperature difference between the surface and the phase transition temperature equilibrium solid phase. 1-16 system of equations can be solved by using digital techniques such as Runge-Kutta 4th order [12, 13].

In the proposed mathematical model computer program «RELEY41» was compiled and behavior of methane bubbles of different sizes under these initial conditions was investigate Methane bubbles from water vapor impurity in the water at the thermobaric conditions sufficient to hydrate formation were investigated. Size of bubbles effect on the rate phase-transition process was defined. To accomplish this fact calculations of thermodynamic properties of bubbles with radii 2 0,5 0,1 0,05 0,01 mm were done. The calculation results are shown in figure.1÷5.

For the initial bubbles radius of 0.1 mm. the results of calculation of the radius and mass of solid phase formed over time is shown in Figure 1. During initial isothermal conditions. Obviously, the most intensive formation of solid phase occurs during the oscillation process.

Analyzing Figure 2. there are three characteristic temperatures bubbles: warm region, the region damped oscillations and fixed area. Bubbles of small size are heated very quickly and this area of the graph we cannot see. Region warming is noticeable only for relatively large bubbles. Due to the intense of heat removal in this area there is the highest rate of phase the transition processes, but it has low duration and so the amount of solid phase formed is small.

In contrast to the cavitation process where the velocity of the bubbles wall reaches hundreds of meters per second, bubbles with gas have much lower velocity wall, Figure 3. Initial velocity is determined by the initial conditions, and eventually dies speed (due to the loss of energy to friction and heat exchange with the environment) more quickly than smaller bubbles.

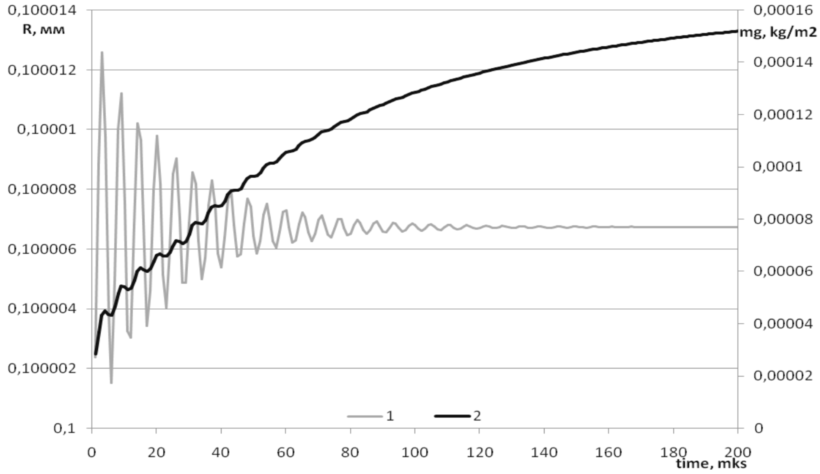

Figure 1 – Change of the radius of bubbles and the specific weight of solid phase:
1 – the radius of the bubbles (R), mm; 2 – the amount of solid phase (mg), kg/m^2

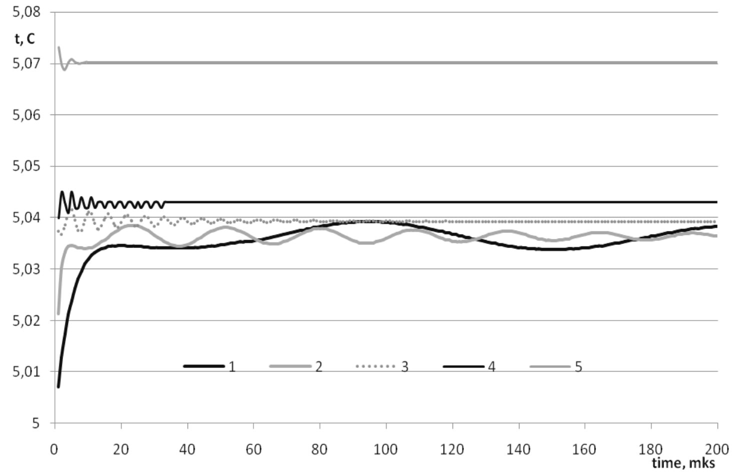

Figure 2 – Temperature of gas-steam bubbles environment:
1 – 2 mm, 2 – 0,5 mm, 3 – 0,1 mm, 4 – 0,05 mm, 5 – 0,01 mm

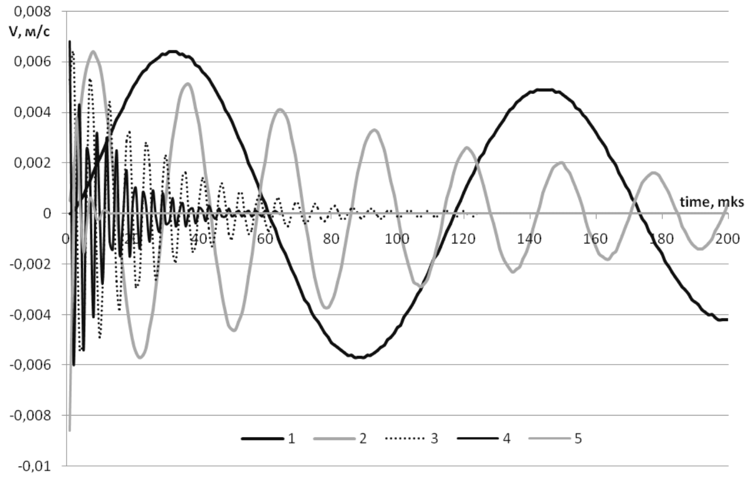

Figure 3 – The rate of change of radius of bubbles for bubble radii:
1 – 2 mm, 2 – 0,5 mm, 3 – 0,1 mm, 4 – 0,05 mm, 5 – 0,01mm

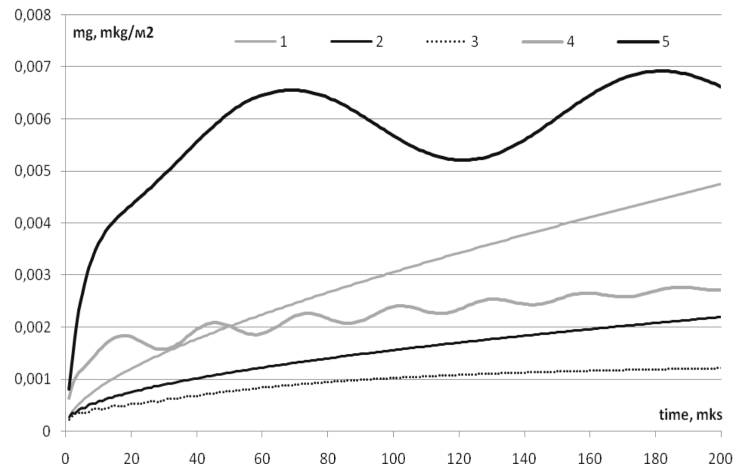

Figure 4 – The specific amount of solid phase that formed on the surface of bubbles of different sizes

1- 0,01 mm, 2- 0,05 mm, 3- 0,1 mm, 4- 0,5 mm, 5- 2 mm

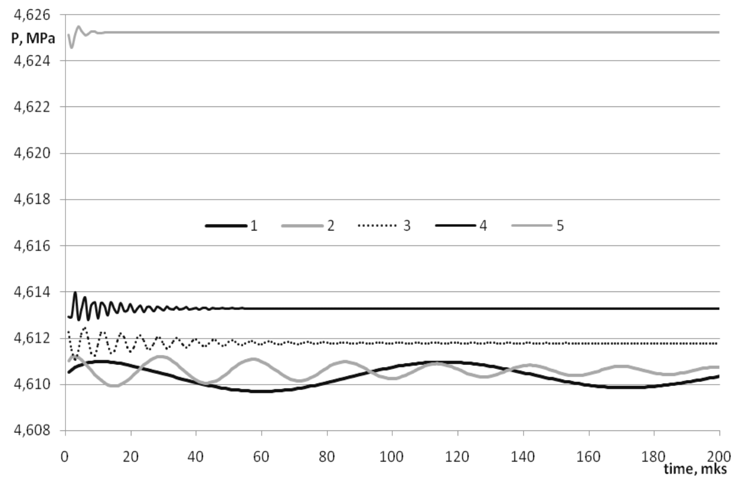

Figure 5 – Graphs of pressure changes bubbles of gas-steam environment

1 – 2 mm, 2 – 0,5 mm, 3 – 0,1 mm, 4 – 0,05 mm, 5 – 0,01mm

Analysis of the dependency indicates the significant influence of two factors on the rate of solid phase, the size of bubbles and its oscillations. Reducing the size of the bubbles leads to increase in their total surface area. However oscillations of small bubbles rapidly damped (range 0.01-0.1 mm) and do not create significant impact on the phase transition processes.

In large bubbles (radius 2 mm) dominant influence on the phase transition processes are oscillatory parameters. Their damped of oscillations continued relatively long time and during that time formed the main part of the solid phase. However, large bubbles have low total surface area of contact interface.

The pressure in the middle of the bubbles is another important factor for mass transfer, Fig. 5. At the same initial temperature conditions pressure low inside the bubbles is greater. This phenomenon is caused by surface tension, also it intensifies the process of phase transition.

Conclusions. It the investigation mathematical model for comprehensive consideration of the impact of various factors on the thermodynamic state oscillating gas-steam bubbles is achieved. At the beginning of solid phase oscillations formation (damped oscillations) bubbles are observed. The starting mechanism for these fluctuations is the temperature difference between the gas-steam bubbles and medium temperature phase transition, which is determined by the pressure medium. During solidification of the liquid phase, and then, through heat exchange and steam bubbles environment locally increasing liquid temperature occurs . Increased gas temperature leads to increased pressure bubbles and begins the process of increasing its diameter. Each size has its bubbles frequency oscillations. Gradually, viscous fluid damped oscillations and phase transition process is supported by heat removal in the outer layers of liquid. At the time of the oscillations top speed of solid phase formation was observed. Over time, the speed phases in the transition processes decreases gradually. The research results can be applied to optimize technological processes related to the boil, swelling materials, and the formation of gas hydrates in fluid cavitations.

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