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***Mitrofanov V.P.**, PhD, Associate Professor*
Center for Advanced Design Methods of Concrete Structures, Poltava
ORCID 0000-0003-1335-2657 vpm.admcs@mail.ru
***Pinchuk N.M.**, PhD, Associate Professor*
Poltava National Technical Yuri Kondratyuk University
ORCID 0000-0002-1720-5497 natali-pinchuk@mail.ru

ESSENTIALS AND PROBLEMS SOLVING ALGORITHMS OF GENERAL STRENGTH THEORY OF RC ELEMENTS UNDER COMPLEX STRESS-STRAIN STATES

The General strength theory of reinforced concrete elements (GSTRCE) passed the long and many-sided examination and showed the considerable advantages: well concordance with experiments, essential economic effect, solving method unity for different strength problems. Simple practical methods for calculating according to GSTRCE and available for use by designers and students are developed. The algorithms of calculating of two practically important problems are stated: strength control and selection of needed longitudinal and lateral reinforcement. Problems solving are represented by using «manual» method and well-known and easy-to-use software complex MS Excel. The results of experimental verification of GSTRCE are stated and the ones show well convergence of theoretical strength to experimental one.

***Keywords:** element of structure, strength of inclined and normal sections, optimization calculation.*

***Митрофанов В.П.**, к.т.н., доцент*
Центр передових методів розрахунку залізобетонних конструкцій, м. Полтава
***Пінчук Н.М.**, к.т.н., доцент*
Полтавський національний технічний університет імені Юрія Кондратюка

ОСНОВИ ТА АЛГОРИТМИ ВИРІШЕННЯ ЗАДАЧ ЗАГАЛЬНОЇ ТЕОРІЇ МІЦНОСТІ ЗАЛІЗОБЕТОННИХ ЕЛЕМЕНТІВ ПРИ СКЛАДНИХ НАПРУЖЕНО-ДЕФОРМОВАНИХ СТАНАХ

Загальна теорія міцності залізобетонних елементів (ЗТМЗБЕ) пройшла довгу та всебічну апробацію і показала значні переваги: добру збіжність з експериментами, істотний економічний ефект, єдиний метод розв'язання для різних проблем міцності. Розроблюються прості методи практичного розрахунку за ЗТМЗБЕ, доступні для використання проектувальниками і студентами. Викладаються алгоритми розрахунку двох практично важливих задач: перевірки міцності та підбору необхідної арматури. Розв'язання задач представлено «ручним» методом та з використанням широко відомого та простого у використанні програмного комплексу MS Excel. Наведені результати експериментальної перевірки ЗТМЗБЕ які показали добру збіжність теоретичної міцності з експериментальною.

***Ключові слова:** елемент конструкції, міцність похилого та нормального перерізів, оптимізаційний розрахунок.*

Introduction. On the 4th International *fib* Congress [1] it was being observed the necessity of a general theory development for the strength calculation of reinforced concrete elements on the normal and inclined sections by unified conception basis and general relations.

Review of the latest research sources and publications. Such general theory is currently worked out [2, 3] and it has the important advantages in comparison with normative designs. The theory is based on the classification of reinforced concrete elements into groups depending on the quantity of longitudinal and transverse reinforcement (Fig. 1). This classification distinguishes the group C of elements which are not overreinforced by longitudinal and transverse reinforcement. The ones are differed by the most economical use of steel and *plastic failure on inclined section*. These elements usually have the broken off or bent up bars of longitudinal reinforcement. So *GSTRCE* is mainly limited by consideration of elements of group C. Herewith, entering some changes and additions in the design scheme of group C elements, the calculation methods of other groups can be obtained.

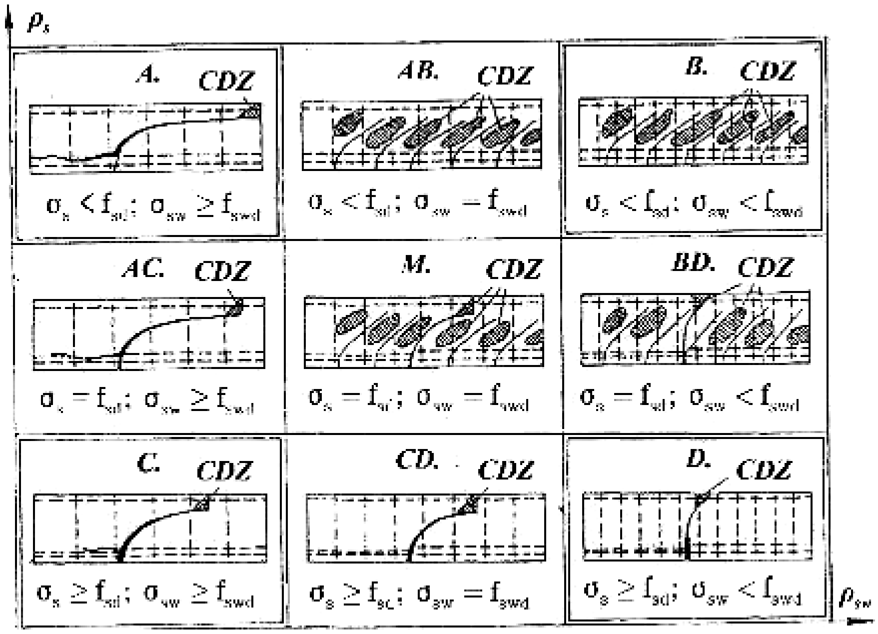

Figure 1 – RC elements classification: ρ_s, ρ_{sw} – respectively longitudinal tensile and lateral reinforcement ratio, σ_s, σ_{sw} – respective reinforcement stress, CDZ – concrete destruction zone.

Depending on the quantity of longitudinal and lateral reinforcement, respective behavior under load and type of failure, RC elements are classified [2] on the 4 main groups A, B, C, D and 5 intermediate groups AB, AC, BD, CD, M:

– group A – elements are overreinforced by longitudinal and underreinforced by lateral reinforcement; stress in longitudinal reinforcement (usually without broken off or bent up bars) does not reach and in lateral reinforcement reaches the limit value; elements are fractured brittly by dangerous inclined crack or by overreinforced normal section;

– group B – elements are overreinforced by longitudinal and lateral reinforcement; stresses in longitudinal and lateral reinforcement do not reach the limit value; brittle failure of elements by concrete compressive inclined strut between regular inclined cracks of element web or by normal overreinforced section is happened;

– group C – elements are underreinforced by longitudinal and lateral reinforcement; stresses in longitudinal reinforcement (usually with broken off or bent up bars) and in lateral reinforcement reach the limit value; plastic failure of elements by dangerous inclined crack or by normal underreinforced section takes place;

– group D – elements are underreinforced by longitudinal and overreinforced by lateral reinforcement; stress in longitudinal reinforcement (usually with broken off or bent up bars) reaches and in lateral reinforcement does not reach the limit value; plastic failure of elements by normal section, situated in action zone of shear forces that significantly influence on strength of normal sections, takes place.

Intermediate elements groups have properties which are transitional between the corresponding main groups.

Definition of unsolved aspects of the problem. The use of GSTRCE in practical designs is constrained by lack of enough simple practical calculations.

Problem statement. Thus, there is necessity of development of simple algorithms to perform calculations by this theory.

Basic material and results. Significant thesis of GSTRCE is made more exact definition of the reinforced concrete element as a part of reinforced concrete bar structure (RCS) on length l of which signs of bending moment M, shear V and longitudinal N forces are constant. This definition of element allows to unify its design scheme, which includes at one dangerous inclined (DIC) and normal (DNC) cracks, reinforcement and forces. The end points A and B of the DIC are considered as theoretical points of curtailment or bent-up of longitudinal reinforcement (Fig. 2).

Concrete strength criterion in zone of concrete failure at the end of DIC (Fig. 2) is determined as the strength condition of concrete wedge, situated over the DIC.

The interlock forces in DIC and the dowel action of reinforcement do not take into account for the group C elements because the element parts I and II (Fig. 2) as fragments of the plastic kinematic mechanism mutually rotate inducing the DIC opening without essential shear of its adjacent surfaces.

It is considered the proportional loading of RCS and its elements and the load parameter F is chosen, through which the forces in inclined section (crack) are expressed:

$$M = Ff_M , \; V = Ff_V , \; N = Ff_N , \tag{1}$$

where f_M, f_V, f_N – load functions, reflecting the element load character and depending on parameters x_A and c (Fig. 2).

Distribution of the normal σ_c and shear τ_c stresses along the concrete failure height x is adopted uniform.

The stresses σ_s, σ'_s σ_{sw} of the longitudinal tensile A_s, compressed A'_s and lateral A_{sw} reinforcement are restricted by the conditions of ultimate state respectively

$$\sigma_s \le f_{yd} , \quad \sigma'_s \le f'_{yd} , \quad \sigma_{sw} \le f_{ywd} , \tag{2}$$

where f_{yd}, f'_{yd}, f_{ywd} – design resistances of corresponding reinforcement.

Calculation of the inclined section strength in reinforced concrete elements by the GSTRCE is based on the design scheme of Figure 2 and on relationships:

– criterion of concrete strength above the DIC under shear force V_c and compressive force N_c action

$$V_c = Af_{cd}bx + a_N N_c , \tag{3}$$

where A, a_N – coefficients that are taken depending on the case of failure [2, 3];

f_{cd} – concrete strength under axial compression;

b – width of cross section of element;

x – height of concrete failure zone;

– equations of equilibrium of element part I (Fig. 2):

$$\sum X = 0;\; Ff_N + \sigma_s A_s - \sigma'_s A'_s - N_c = 0; \tag{4}$$

$$\sum Y = 0;\; Ff_V - \left(\sigma_{sw} A_{sw} c\right)/s - V_c = 0; \tag{5}$$

$$\sum M_0 = 0;\; Ff_M - \left(\sigma_{sw} A_{sw} c^2\right)/2s - \sigma'_s A'_s z_s - N_c\left(d - x/2\right) = 0\,, \tag{6}$$

where A_s, A'_s A_{sw} – area of cross-section of longitudinal tensile, longitudinal compressed and lateral reinforcement respectively;

σ_s, σ'_s σ_{sw} – stresses in longitudinal tensile, longitudinal compressed and lateral reinforcement;

s – step of lateral reinforcement bars;

z_s, d – are shown on Fig. 2.

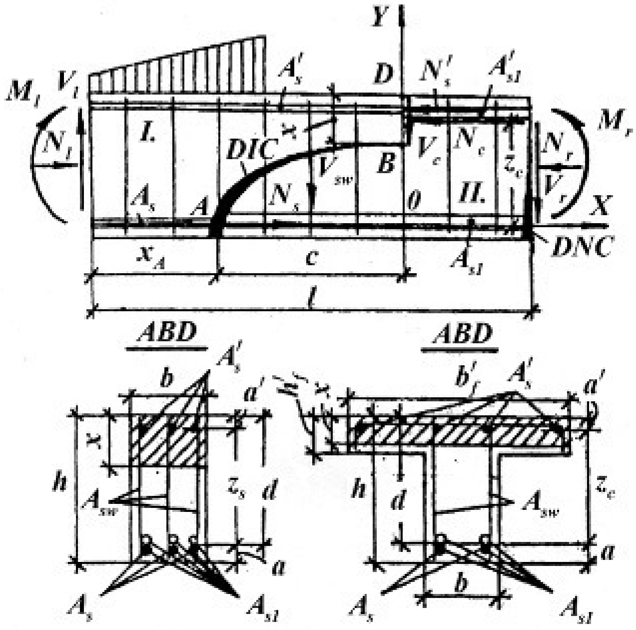

Figure 2 – Design model of reinforced concrete element with dangerous normal DNC and inclined DIC cracks, reinforcement and forces

From equations (3 – 6) the resulting relationship is obtained:

$$\begin{gathered} P^2 f_N\left(f_V - f_N\alpha_N\right)/2A + P\left[f_M/d + \omega f_V - f_N\left(1 + m_{sw}\xi_{sw}c/2Ad - \eta\right)\right] - \\ -\left(m_{sw}\xi_{sw}/2\right)\left(c^2/d^2 + 2\omega c/d\right) - B = 0, \end{gathered} \tag{7}$$

that is quadratic equation for the relative load parameter

$$P = F/f_{cd}bd\,. \tag{8}$$

In (7) f_M, f_V, f_N – load functions, expressing the forces M, V, N in element inclined section through the load parameter F,

$$\eta = m_s\xi_s - m'_s\xi'_s, \qquad \omega = \eta/2A, \qquad B = \eta\left(1 - \eta/2\right) + m'_s\xi'_s z_s/d\,, \tag{9}$$

$$\xi_s = f_{yd}A_s/f_{cd}bd\,, \qquad \xi'_s = f'_{yd}A'_s/f_{cd}bd\,, \qquad \xi_{sw} = f_{ywd}A_{sw}/f_{cd}bs\,, \tag{10}$$

$$m_s = \sigma_s/f_{yd}\,, \qquad m'_s = \sigma'_s/f'_{yd}\,, \qquad m_{sw} = \sigma_{sw}/f_{ywd}\,, \tag{11}$$

m_s, m'_s, m_{sw} – factors of resistance use completeness of respective reinforcement.

For determination of reinforcement cross-section area the formula of volume ratio of element reinforcement is used:

$$V_s = f_{cd}\xi_s X_A / f_{yd} l + f_{cd}\xi'_s \left(X_A + c\right) / f'_{yd} l + f_{cd}\xi_{sw} / f_{ywd} + \\ + f_{cd}\xi_{s1}\left(1 - X_A/l\right) / f_{yd} + f_{cd}\xi'_{s1}\left[1 - \left(X_A + c\right)/l\right] / f'_{yd}, \tag{12}$$

where

$$\xi_{s1} = f_{yd} A_{s1} / f_{cd} bd, \; \xi'_{s1} = f'_{yd} A'_{s1} / f_{cd} bd. \tag{13}$$

The problem of strength control is solved as optimization one with objective function of the relative load parameter P, depending on parameter c. In this problem set quantities are the next: element with all sizes, reinforcement, characters of materials (b, h, d, z_s, l, X_A, A_s, A_{s1}, A'_s, A'_{s1}, A_{sw}, s, f_{cd}, f_{ctd}, f_{yd}, f'_{yd}, f_{ywd}) , character and situation of loads. The ultimate load parameter F_u and respective value c_u are unknown.

Design algorithm of ultimate load parameter

1. The static calculation of structure is carried out.
2. The structure is divided on elements and each element design is fulfilled separately.
3. The length l of each element is determined; in element the location of DIC and DNC with input X_A, c, internal forces in the inclined section N_s, N'_s, V_{sw}, V_c, N_c are shown.
4. The load parameter F is selected and functions f_M, f_V, f_N are determined, taking into account the location and distribution of element loads.
5. The obtained functions f_M, f_V, f_N are substituted in resulting relationship (7) and the relation $P(c)$ is got.
6. The obtained function $P(c)$ is investigated on minimum by the conditions

$$m_s = m'_s = m_{sw} = 1, \tag{14}$$

corresponding to the group C elements, and the value c_u is determined.

The ultimate load parameter is calculated using the found value c_u.

If as result of calculation is $X_A + c > l$, the *widened element* is considered and points 2-6 are repeated.

7. The conditions of balance reinforcing are checked

$$A_s \le A_{s,opt}, \; A'_s \le A'_{s,opt}, \; A_{sw} \le A_{sw,opt}, \tag{15}$$

where the optimal values respective reinforcement $A_{s,opt}$, $A'_{s,opt}$, $A_{sw,opt}$ are adopted from the problem solution of the needed reinforcement determination by the load parameter that was found in the solution of previous strength control problem.

In practice of designing the problem of calculation the required longitudinal and lateral reinforcement in inclined section has the main importance.

The problem of needed reinforcement design is solved as optimization with objective function as volume ration of reinforcement in element V_s (12) together with additional conditions (7), (2) and $X_A + c \le l$. In this problem set quantities are the next: element with all sizes, reinforcement and characters of materials, the character of loads and numerical value of load parameter. From the previous calculation of normal (cross) section the A_{s1} and A'_{s1} values are known.

The analysis of this problem solving on the basis of Kuhn-Tucker conditions [4] shows that the optimal reinforcement is achieved at the highest possible stresses in reinforcement when the (14) is observed. It is appropriate to do the *incomplete optimization* of reinforcement. In this case the previously performed design of the normal (cross) section strength and the calculated A_{s1} and A'_{s1} values allow to adopt the areas $A_s < A_{s1}$ and $A'_s < A'_{s1}$ with the result that A_{sw}, X_A, c are only stayed as the unknown values.

Design algorithm of needed reinforcement

1. The static calculation of structure is carried out.

2. The structure is divided on elements and design of each element is fulfilled separately.

3. The length l of each element is determined; in element the location of DIC and DNC with input X_A, c, internal forces in the inclined section N_s, N'_s, V_{sw}, V_c, N_c are shown.

4. The load parameter is selected and functions f_M, f_V, f_N are determined, taking into account the location and distribution of element loads. The relative value of the load parameter (8) is calculated. Further the design for elements under cross bending without axial force N are given as example.

5. From function

$$\gamma_{18} = \frac{f_M}{d} + f_V \omega \tag{16}$$

the partial derivatives

$$\gamma_{14} = \frac{\partial \gamma_{18}}{\partial (c/d)}; \quad \gamma_{15} = \frac{\partial \gamma_{18}}{\partial (X_A/l)} \cdot \frac{d}{l}, \tag{17}$$

are determined.

6. The parameters (9), ξ_s, ξ'_s according to (10) and

$$D = \left[(\xi_{s1} - \xi_s) \frac{f_{ywd}}{f_{yd}} + (\xi'_{s1} - \xi'_s) \frac{f_{ywd}}{f'_{yd}} \right] \frac{d/l}{P}, \tag{18}$$

$$D_1 = \frac{(\xi'_{s1} - \xi'_s)}{P} \cdot \frac{f_{ywd}}{f'_{yd}} \cdot \frac{d}{l} \tag{19}$$

are calculated.

7. From the system of equations

$$\begin{cases} 0{,}5D\left(c^2/d^2 + 2\omega c/d\right) - \gamma_{15} = 0; & (20) \\ \gamma_{18} - B/P - 0{,}5D\left(c^2/d^2 + 2\omega c/d\right)\left[0{,}5D_1\left(c^2/d^2 + 2\omega c/d\right) + (\gamma_{14} - \gamma_{15})\right] / (c/d + \omega) = 0 & (21) \end{cases}$$

the values X_A/l, c/d are found.

8. The necessary intensity of lateral reinforcement is determined by the equation:

$$\xi_{sw} = P\left[0{,}5D_1\left(c^2/d^2 + 2\omega c/d\right) + \gamma_{14} - \gamma_{15}\right] / (c/d + \omega) \tag{22}$$

9. The A_{sw} and s values are calculated by the found ξ_{sw} (10).

10. The relative height of the concrete failure zone is determined by the formula:

$$\xi = \frac{x}{d} = \frac{Pf_V}{A} - \frac{\xi_{sw}}{A} \cdot \frac{c}{d} + \eta. \tag{23}$$

The condition $x > a'$ is checked, where a' is thickness of the protective concrete layer of compressed reinforcement A'_s.

The stated methods of problem solving can be called «manual» when the whole process of solution is considered and the unknown parameters are found in succession. This method is especially important in mastering of the GSTRCE. After GSTRCE mastering it is advisable to use «computer» method, when the optimization problems are solved by using software packages, such as processor MS Excel.

Experimental verification of the GSTRCE. The many-sided verification of the GSTRCE was conducted under V.P. Mitrofanov guidance. The strength criterion (3) was thoroughly experimentally verified by the specimens-wedges of usual heavy [5] and light-weight

[6] concretes. The significant control of (3) was fulfilled on the beams with artificial DIC, which allowed to exclude the interlock between crack sides and enabled to find reliably the experimental V_c and N_c quantities [5]. The noted all-round tests confirmed the realityof theoretical failure cases [5] of concrete wedges over the DIC and reliability of the relationship (3).

It is necessary to note that in all countries of the world the strength investigations of the reinforced concrete elements under the shear forces action were being conducted mostly on the elements of groups A and B (see Fig. 1) without curtailment of longitudinal reinforcement in zone action of shear forces. These tests led to the inexact notion that RC elements failure by the DIC has only brittle character. In our researches of the group C elements the longitudinal reinforcement was used in the form of two-bars bundles in which one bar had the break off in zone action of shear force and its end was welded to the being continued bar. The tested elements revealed the positive for practice features of the group C elements: clear-cut plastical behavior in ultimate state and the most economical expense of steel. Our investigation included the tests:

– 26 columns under joint action of M, V, N forces [7];

– 21 usual [5], 22 prestressed and 12 usual [8] simple beams;

– 22 simple and 13 cantilever beams with failure by the cross sections where the concrete failure zone was undergone the essential influence of shear force action [9];

– 4 simple beams with inclined compressive side [10];

– 6 two-span continuous beams [10];

– more 120 specimens-wedges [5, 6];

– 5 simple beams with shear span changing from 4,0 to 1,6.

All tested columns and beams were reinforced in accordance with the GSTRCE for the given load.

The test data of 131 beams and columns showed the mean ratio F^{test} / F^{calc} of experimental ultimate load parameter F^{test} to the theoretical one F^{calc} equal to 1,073 on the coefficient of variation $V = 9{,}525$ %. The comparison of the test data with design results by the Code SNiP 2.03.01-84* (Gosstroy, Moscow, 2000) led to the mean ratio $F^{test} / F^{calc} = 0{,}838$, $V = 23{,}06$ %. The like comparison with Eurocode 2-1992 produced the worst results. There are design examples according to the GSTRCE in [3].

Conclusions. The GSTRCE implementation into practice supposes perceptible economical effect at the expense of the complete use of all reinforcement strength and more exact optimization designs. The GSTRCE secures the simultaneous plastic element failure both on the cross and on the inclined sections (cracks), i.e. leads to the elements of equal resistance. The general conception basis for designs on the inclined and on the cross sections witnesses about more high development level of the GSTRCE in comparison with known designs systems. For the GSTRCE practical use it is offered two design methods: «manual» and «computering».

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