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COATINGS DISCRETE SURFACES CONSTRUCTION BY SUPERPOSITIONS OF ADJUSTED MESH FRAMES

The method of curve surface discrete geometric modeling on the basis of two discrete frames superimpositions, formed by a static-geometric method, and on the basis of a single surface curve nodal points superimposition, also formed by static-geometric method, is considered. It has been determined that the suggested method allows to model balanced discrete structures formed on the specified contour nodes, as well as those passing through the specified nodal points without composing and solving equations systems.

Keywords: *discrete geometric modeling, discrete surface frame, coating surfaces, static-geometric method, the external shaping load value, geometric apparatus of superimpositions, coefficients of superimposition.*

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ФОРМУВАННЯ ДИСКРЕТНИХ ПОВЕРХОНЬ ПОКРИТТІВ НА ОСНОВІ СУПЕРПОЗИЦІЙ ЗАДАНИХ СІТЧАСТИХ КАРКАСІВ

Розглянуто спосіб дискретного геометричного моделювання кривих поверхонь на основі суперпозицій двох дискретних каркасів, сформованих статико-геометричним методом, та на основі суперпозицій вузлових точок однієї кривої поверхні, сформованої також статико-геометричним методом. З'ясовано, що запропонований спосіб дозволяє моделювати врівноважені дискретні структури, сформовані на заданих контурних вузлах, а також ті, що проходять через задані вузлові точки без складання і розв'язання систем рівнянь.

Ключові слова: *дискретне геометричне моделювання, дискретний каркас поверхні, поверхні покриттів, статико-геометричний метод, величина зовнішнього формоутворюючого навантаження, геометричний апарат суперпозицій, коефіцієнти суперпозиції.*

Introduction. Geometric designing is important for designing modern building structures, architectural forms of coatings, when the main geometric shapes are determined at the sketch stage with their advantages and disadvantages.

An expedient way to obtain the medial coating surfaces is inefficient. A surface must satisfy the pre-set conditions and requirements that are often competing with each other.

Calculations of such structures considering material physical properties, manufacturing technology and installation features require information about the object presented in the discrete form, therefore it is advisable to form them in the discrete form in the very beginning.

Discrete geometric modeling is the most promising trend in the applied geometry development in the current period, which can be conventionally divided into studies on the discretization of continuous geometric images and the shaping based on discrete source data [1, 2].

The static-geometric method of discrete geometric modeling of curved lines and surfaces [3] allows obtaining discrete frames of curvilinear surfaces under the influence of external shaping loads and, moreover, it is simple and descriptive. Using the static-geometric method allows obtaining discrete frames of curvilinear surfaces on an arbitrary reference contour.

Analysis of recent research sources and publications. The basis of the static-geometric method mathematical apparatus is solution of cumbersome linear equations systems, which complicates the process of computer-implemented calculations. Publication [4] is devoted to the issue of expanding the shape-forming possibilities of the static-geometric method by means of the numerical sequences mathematical apparatus, which permits, in particular, to avoid composing of linear equations systems in the discrete images formation.

In publication [5], the problems of the discrete surfaces frames construction by functional addition based on two pre-calculated frames by means of the static-geometric method were studied. However, grids verification obtained as a result of the initial grids superimposition with different braced stretching coefficients showed that the resulting grid is not balanced with a set of external load on the nodes, thus the results of such superimpositions are not accurate but approximate.

In study [6] the notion of the sets superimposition apparatus in applied geometry is defined. A number of properties have been proved permitted to draw conclusions about the deep comprehensive studies prospect concerning the superimposition apparatus.

The articles reports [7-11] show the approaches to determination of certain functional dependences discrete analogues based on the superimpositions geometric apparatus of one-dimensional point sets permitting the formation of discrete images without composing and solving cumbersome equation systems. The shaping control of discretely submitted curves is performed by varying the superimposition coefficients values.

Identification of previously unsettled parts of the general problem. The classical finite difference method, static-geometric method, mathematical apparatus of numerical sequences have their advantages and disadvantages concerning the solution of specific practical tasks. Therefore, their studies, enrichment with new efficient algorithms, studying the possibility of their compilation, and on this basis, expansion of the output data set, are topical. The above methods further development and improvement in general are topical as well. Using the geometrical superimpositions apparatus in combination with the above-mentioned methods permits to improve significantly the efficiency and to extend the continuous geometric images discrete modeling possibilities.

Setting objectives. The purpose of the present article is to study the method of two-dimensional geometric image modeling in the form of discretely presented curved surface by means of the superimpositions geometric apparatus of one-dimensional point sets based on a single curved surface formed using the static-geometric method.

Main material and results. It is supposed the surface differential equation specified as $z = x^2 + y^2$ (fig. 1) : $\frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2} = 4$ for a limited area $-2 \le x \le 2$; $-2 \le y \le 2$, and the marginal conditions are set as four lines:

$$\begin{aligned} &1.\ y = 2; \quad z = x^2 + 4; \\ &2.\ y = -2; \quad z = -x^2 + 4; \\ &3.\ x = 2; \quad z = y^2 + 4; \\ &4.\ x = -2; \quad z = -y^2 + 4. \end{aligned} \tag{1}$$

It is determined discrete point surface frame in the specified area with step $h = 1$.

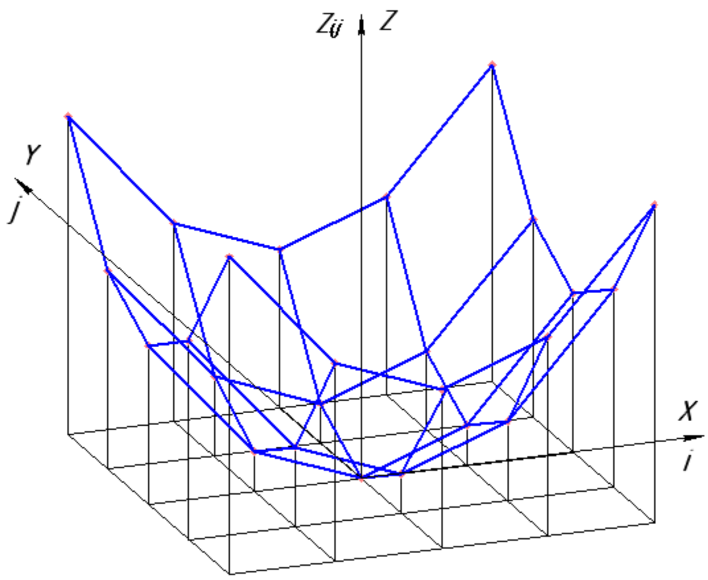

Figure 1 – Discretely presented surface $z = x^2 + y^2$ **.**

According to dependences (1) the applicative points values are determined on the surface boundary lines with step h = 1. In the Table 1 they are located in the boxes surrounded with a heavy line.

Table 1 – Values of surface applicate points

	$i = -2$	$i = -1$	$i = 0$	$i = 1$	$i = 2$
$j = 2$	8	5	4	5	8
$j = 1$	5	2	1	2	5
$j = 0$	4	1	0	1	4
$j = -1$	5	2	1	2	5
$j = -2$	8	5	4	5	8

Let us replace the differential equation with the finite differences expression

$$z_{i-1,j} + z_{i+1,j} - 4z_{i,j} + z_{i-1,j} + z_{i+1,j} - 4 = 0 \tag{2}$$

and compose the system (3) of finite differences equations for all unidentified nodes of the area.

$$\begin{cases} z_{10} + z_{-10} - 4z_{00} + z_{01} + z_{0-1} - 4 = 0 \\ z_{20} + z_{00} - 4z_{10} + z_{11} + z_{1-1} - 4 = 0 \\ z_{21} + z_{01} - 4z_{11} + z_{12} + z_{10} - 4 = 0 \\ z_{11} + z_{-11} - 4z_{01} + z_{02} + z_{00} - 4 = 0 \\ z_{01} + z_{-21} - 4z_{-11} + z_{-12} + z_{-10} - 4 = 0 \\ z_{00} + z_{-20} - 4z_{-10} + z_{-11} + z_{-1-1} - 4 = 0 \\ z_{0-1} + z_{-2-1} - 4z_{-1-1} + z_{-10} + z_{-1-2} - 4 = 0 \\ z_{1-1} + z_{-1-1} - 4z_{0-1} + z_{00} + z_{0-2} - 4 = 0 \\ z_{2-1} + z_{0-1} - 4z_{1-1} + z_{10} + z_{1-2} - 4 = 0 \end{cases} . \quad (3)$$

Considering the initial data symmetry, the system of equations can be significantly reduced by writing the symmetry conditions

$$z_{01} = z_{10} = z_{-10} = z_{0-1}\ ; \quad z_{11} = z_{-11} = z_{-1-1} = z_{1-1}\ .$$

It is substituted the marginal contour applicative-nodes values and the symmetry conditions into the system (3):

$$\begin{cases} 4z_{10} - 4z_{00} - 4 = 0 \\ 4 + z_{00} - 4z_{10} + 2z_{11} - 4 = 0 \\ 8 + 2z_{10} - 4z_{11} - 4 = 0 \end{cases} . \quad (4)$$

The results of solving the system (4) are presented in the Table 1.

To form the surface discrete frame by the static-geometric method, the model of the given surface in the form of a discrete grid with a uniform step $h = 1$ along the axis Ox: $x_{i+1} = x_i + h$; and the axis Oy: $y_{i+1} = y_i + h$; can be imagined as being balanced by certain external efforts in nodes and forces in the braces so that these efforts are directly proportional to these braces lengths (Fig. 2).

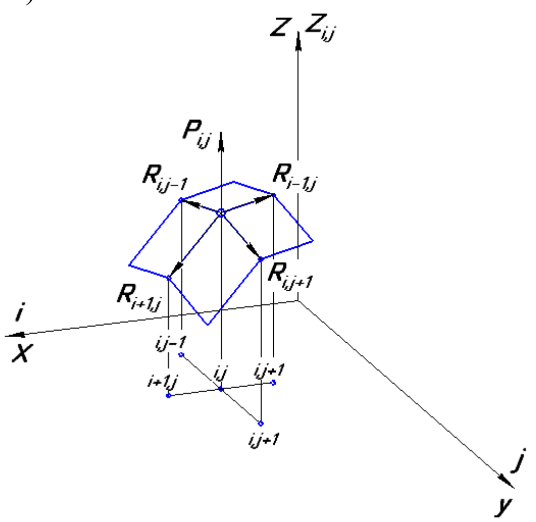

Figure 2 – Diagram of the balanced discrete grid nodes

Then it can be determined the external effort KP_{ij} that balances the effort $R_{i-1,j}$, $R_{i+1,j}$, $R_{i,j-1}$, $R_{i,j+1}$ of the respective braces

$$\overline{P}_{i,j} = \overline{R}_{i-1,j} + \overline{R}_{i+1,j} + \overline{R}_{i,j-1} + \overline{R}_{i,j+1}\ . \quad (5)$$

For the type II grid [12] with the reference system of nodes, as it is shown in Fig. 2, the equation (5) in the coordinate form is:

$$u_{i-1,j} + u_{i+1,j} - 4u_{i,j} + u_{i,j-1} + u_{i,j+1} + KP_{i,j} = 0\ , \tag{5}$$

where u is a generalized designation of the respective coordinate.

Thus, to form the discrete frame of the surface shown in Fig. 1 by the static-geometric method, it is necessary to compose and solve a system of equations on the internal nodes equilibrium:

$$\begin{cases} 4z_{10} - 4z_{00} - KP_{ij} = 0 \\ 4 + z_{00} - 4z_{10} + 2z_{11} - KP_{ij} = 0 \\ 8 + 2z_{10} - 4\,z_{11} - KP_{ij} = 0 \end{cases}, \tag{7}$$

where KP_{ij} is the value of the external shaping load.

It is considered an example of discrete surface model formation based on two discrete surface frames superimpositions formed by the static-geometric method in a single reference contour (Fig. 3).

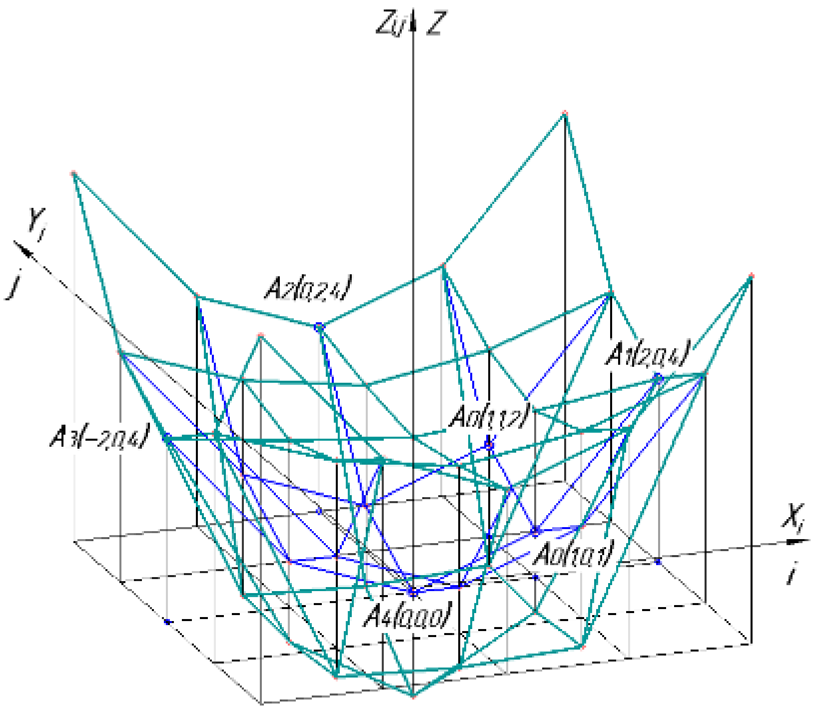

Figure 3 – Formation of discrete surface models based on superimpositions of two discretely determined surfaces.

Discrete values of the first curved surface are formed basing on the initial data $z_{A1} = 4$, $z_{A2} = 4$, $z_{A3} = 4$, $z_{A4} = 4$, $P_{ij} = -1$.

The equation system (7) is:

$$\begin{cases} 4z_{10} - 4z_{00} - 1 = 0 \\ 4 + z_{00} - 4z_{10} + 2z_{11} - 1 = 0 \\ 8 + 2z_{10} - 4\,z_{11} - 1 = 0 \end{cases}.$$

This system solution results in: z_{00}=27/8 , z_{10}=29/8 , z_{11}=65/16;

The discrete values of the first surface application formed by the static-geometric method in the specified reference contour and by the value of the external shape-forming load $P_{ij} = -1$, specified and uniformly distributed between the discrete frame nodes, are presented in Table 2.

Table 2 – Application points values of the first surface with the load value of P_{ij} = –1

	i = –2	i = –1	i = 0	i = 1	i = 2
j = 2	8	5	4	5	8
j = 1	5	65/16	29/8	65/16	5
j = 0	4	29/8	27/8	29/8	4
j = –1	5	65/16	29/8	65/16	5
j = –2	8	5	4	5	8

Statement. Any point coordinates of two-dimensional set of points are the coordinate superimposition (8) of this set four arbitrary points

$$\begin{cases} x_0 - x_4 = k_1\,(x_1 - x_4) + k_2\,(x_2 - x_4) + k_3\,(x_3 - x_4) \\ y_0 - y_4 = k_1\,(y_1 - y_4) + k_2\,(y_2 - y_4) + k_3\,(y_3 - y_4) \\ z_0 - z_4 = k_1\,(z_1 - z_4) + k_2\,(z_2 - z_4) + k_3\,(z_3 - z_4) \end{cases} . \tag{8}$$

It can be computed the superimposition coefficients values for three specified points of the reference contour $A_1(2;\,0;\,4)$, $A_2(0;\,2;\,4)$, $A_3(-2;\,0;\,4)$ and the central node $A_4(0;\,0;\,3{,}375)$ to determine the coordinates of point $A_0(1;\,0;\,3{,}625)$.

Then the system of equations (8) is

$$\begin{cases} 1 - 0 = k_1\,(2 - 0) + k_2\,(0 - 0) + k_3\,(-2 - 0) \\ 0 - 0 = k_1\,(0 - 0) + k_2\,(2 - 0) + k_3\,(0 - 0) \\ 3{,}625 - 3{,}375 = k_1\,(4 - 3{,}375) + k_2\,(4 - 3{,}375) + k_3\,(4 - 3{,}375) \end{cases} .$$

The solution of this system gives the value of the superimposition coefficients: k_1 = 9/20, k_2 = 0, k_3 = –1/20.

To determine the coordinates of point $A_0(1;\,1;\,4{,}0625)$, the superimposition coefficients has the values: k_1 = 11/20, k_2 = 1/2 , k_3 = 1/20.

Discrete values of the second curved surface are formed according to the initial data $z_{A1} = 4$, $z_{A2} = 4$, $z_{A3} = 4$, $z_{A4} = 4$, $P_{ij} = -6$.

The system of equations (7) is

$$\begin{cases} 4z_{10} - 4z_{00} - 6 = 0 \\ 4 + z_{00} - 4z_{10} + 2z_{11} - 6 = 0 \\ 8 + 2z_{10} - 4\,z_{11} - 6 = 0 \end{cases} .$$

Solution of this system gives the result: $z_{00} = -9/4$, $z_{10} = -3/4$, $z_{11} = 5/8$.

The discrete values of the second surface application formed by the static-geometric method and by the specified value of the external shape-forming load P_{ij} = –6 distributed uniformly among the nodes of the discrete frame are presented in Table 3.

Table 3 – Applicative points values of the second surface with the load value of P_{ij} = –6

	i = –2	i = –1	i = 0	i = 1	i = 2
j = 2	8	5	4	5	8
j = 1	5	5/8	-3/4	5/8	5
j = 0	4	-3/4	-9/4	-3/4	4
j = –1	5	5/8	-3/4	5/8	5
j = –2	8	5	4	5	8

It can be computed the superimposition coefficients the values of the same three specified points of the reference contour $A_1(2;\ 0;\ 4)$, $A_2(0;\ 2;\ 4)$, $A_3(-2;\ 0;\ 4)$, and also the central node $A_4(0;\ 0;\ -2{,}25)$ to determine the coordinates of point $A_0(1;\ 0;\ -0{,}75)$.

According to the above initial data, the solution of the equations system (8) gives the values of superimposition coefficients: $k_1 = 37/100$, $k_2 = 0$, $k_3 = -13/100$.

To determine the coordinates of point $A_0(1;\ 1;\ 0{,}625)$, the superimposition coefficients is the values: $k_1 = 23/100$, $k_2 = 0{,}5$, $k_3 = -27/100$.

The value of the external shaping load and the nodal points application of the sought curved surfaces discrete frames as superimpositions of two discrete curved surfaces frames pre-formed by the static-geometric method (Fig. 3) is determined by the formulas:

$$P_{ij} = k_1 P_{ij}^1 + k_2 P_{ij}^2 \ ,$$

$$z_{ij} = k_1 z_{ij}^1 + k_2 z_{ij}^2 \ ,$$

where P_{ij}^1 – magnitude of the uniformly distributed external shape-forming load applied to the nodes of the modeled first surface, and P_{ij}^2 – that of the second one;

z_{ij}^1 – applicate of the ij-node of the first surface, and z_{ij}^2 – that of the second surface.

Considering the uniform step of the load size changing from 1 to 6: $6 - 1 = 5$; $1/5 = 0{,}2$ obtains $k_1 = k_2 = 0{,}2;\ 0{,}4;\ 0{,}6;\ 0{,}8$.

For example, the load value $P_{ij}^{A_4}$ and application of nodal points $z_{ij}^{A_4}$ of the discrete surface frames is determined by the formulas:

$$P_{ij}^{A_4=2{,}25} = 0{,}2\cdot(-6)+0{,}8\cdot(-1) = -1{,}2+(-0{,}8) = -2 \ ,$$

$$P_{ij}^{A_4=1{,}25} = 0{,}6\cdot(-6)+0{,}4\cdot(-1) = -3{,}6+(-0{,}4) = -4 \ ,$$

$$P_{ij}^{A_4=0} = 0{,}6\cdot(-6)+0{,}4\cdot(-1) = -3{,}6+(-0{,}4) = -4 \ ,$$

$$P_{ij}^{A_4=-1{,}25} = 0{,}8\cdot(-6)+0{,}2\cdot(-1) = -4{,}8+(-0{,}2) = -5 \ ,$$

$$z_{Pij=-2}^{A_4} = 0{,}2\cdot(-2{,}25)+0{,}8\cdot 3{,}375 = -0{,}45+2{,}7 = 2{,}25 \ ,$$

$$z_{Pij=-3}^{A_4} = 0{,}4\cdot(-2{,}25)+0{,}6\cdot 3{,}375 = -0{,}90+2{,}025 = 1{,}125 \ ,$$

$$z_{Pij=-4}^{A_4} = 0{,}6\cdot(-2{,}25)+0{,}4\cdot 3{,}375 = -1{,}35+1{,}35 = 0 \ ,$$

$$z_{Pij=-5}^{A_4} = 0{,}8\cdot(-2{,}25)+0{,}2\cdot 3{,}375 = -1{,}80+0{,}675 = -1{,}125 \ .$$

The recurrent dependence and nodal points application value of the surface curves discrete models is presented in Table 4.

Table 4 – The value of the external shaping load and the nodal points ordinates of the sought surface curves discrete models

	$k_1 = 0{,}8$ $k_2 = 0{,}2$	$k_1 = 0{,}6$ $k_2 = 0{,}4$	$k_1 = 0{,}4$ $k_2 = 0{,}6$	$k_1 = 0{,}2$ $k_2 = 0{,}8$	
$P_{ij}^{A_{00}=3{,}375} = -1$	$P_{ij}^{A_{00}=2{,}25} = -2$	$P_{ij}^{A_{00}=1{,}125} = -3$	$P_{ij}^{A_{00}=0} = -4$	$P_{ij}^{A_{00}=-1{,}125} = -5$	$P_{ij}^{A_{00}=-2{,}25} = -6$
$P_{Pij=-1}^{A_{00}} = 3{,}375$	$P_{Pij=-2}^{A_{00}} = 2{,}25$	$P_{Pij=-3}^{A_{00}} = 1{,}125$	$P_{Pij=-4}^{A_{00}} = 0$	$P_{Pij=-5}^{A_{00}} = -1{,}125$	$P_{Pij=-6}^{A_{00}} = -2{,}25$
$P_{Pij=-1}^{A_{10}} = 3{,}625$	$P_{Pij=-2}^{A_{10}} = 2{,}75$	$P_{Pij=-3}^{A_{10}} = 1{,}875$	$P_{Pij=-4}^{A_{10}} = 1$	$P_{Pij=-5}^{A_{10}} = 0{,}125$	$P_{Pij=-6}^{A_{10}} = -0{,}75$
$P_{Pij=-1}^{A_{11}} = 4{,}0625$	$P_{Pij=-2}^{A_{11}} = 3{,}375$	$P_{Pij=-3}^{A_{11}} = 2{,}6875$	$P_{Pij=-4}^{A_{11}} = 2$	$P_{Pij=-5}^{A_{11}} = 1{,}3425$	$P_{Pij=-6}^{A_{11}} = 0{,}625$

The results of the computed superimposition coefficients of four specified points A_{20}, A_{02}, A_{-20}, A_{00} to determine the coordinates of unknown points A_{10} and A_{11} (considering the symmetry conditions) of the formed curved surfaces discrete models for various values of P_{ij} are presented in Table 5.

Table 5 – Values of superimposition coefficients

	A_{20}	A_{02}	A_{-20}	A_{00}	A_{10}	A_{11}
$P_{ij}=-1$						
z_{ij}	4	4	4	27/8	29/8	65/16
k_1					9/20	11/20
k_2					0	½
k_3					-1/20	1/20
$k_4= 1-k_1-k_2-k_3$					12/20	-2/20
$P_{ij}=-2$						
z_{ij}	4	4	4	9/4	11/4	27/8
k_1					11/28	9/28
k_2					0	½
k_3					-3/28	-5/28
$k_4= 1-k_1-k_2-k_3$					20/28	10/28
$P_{ij}=-3$						
z_{ij}	4	4	4	9/8	15/8	43/16
k_1					35/92	25/92
k_2					0	½
k_3					-11/92	-21/92
$k_4= 1-k_1-k_2-k_3$					68/92	42/92
$P_{ij}=-4$						
z_{ij}	4	4	4	0	1	2
k_1					3/8	2/8
k_2					0	½
k_3					-1/8	-2/8
$k_4=1-k_1-k_2-k_3$					6/8	4/8
$P_{ij}=-5$						
z_{ij}	4	4	4	-9/8	1/8	21/16
k_1					61/164	39/164
k_2					0	½
k_3					-21/164	-43/164
$k_4=1-k_1-k_2-k_3$					124/164	86/164
$P_{ij}=-6$						
z_{ij}	4	4	4	-9/4	-3/4	5/8
k_1					37/100	23/100
k_2					0	½
k_3					-13/100	-27/100
$k_4=1-k_1-k_2-k_3$					76/100	54/100

According to the results presented in Table 5, a certain regularity can be observed relating to the superimposition coefficient values of the specified three boundary and central nodes to determine the sought nodes coordinates that lies in the following.

The difference in the superimposition coefficient values of two specified nodes in the boundary contour k_1 and k_3 to determine the sought nodal points coordinates is a constant value and equals to 1/2 ($k_1 - k_3 = 1/2$) .

It can be proved this conclusion by writing the system of equations 7 for the above-determined curved surfaces discrete models in general terms:

$$\begin{cases} 4z_{10} - 4z_{00} = P_{ij} \\ 4 + z_{00} - 4z_{10} + 2z_{11} = P_{ij} \\ 8 + 2z_{10} - 4z_{11} = P_{ij} \end{cases} .$$

Solution of this system gives the result:

$$z_{00} = \frac{36 - 9P_{ij}}{8}, \quad z_{10} = \frac{36 - 7P_{ij}}{8}, \quad z_{11} = \frac{76 - 11P_{ij}}{16} .$$

Hence, the nodal point coordinates of the specified and modeled discrete curved surfaces is:

$$A_{00}(0;0;\frac{36}{8} - \frac{9}{8}P_{ij}), \quad A_{10}(1;0;\frac{36}{8} - \frac{7}{8}P_{ij}), \quad A_{11}(1;1;\frac{76}{16} - \frac{11}{16}P_{ij}) .$$

$$A_1(2;0;4), \quad A_2(0;2;4), \quad A_3(-2;0;4) .$$

Substituting coordinates of point A_{10} into the equation system (8), it is obtained:

$$\begin{cases} 1 - 0 = k_1(2 - 0) + k_2(0 - 0) + k_3(-2 - 0) \\ 0 - 0 = k_1(0 - 0) + k_2(2 - 0) + k_3(0 - 0) \\ \left(\frac{36}{8} - \frac{7}{8}P_{ij}\right) - \left(\frac{36}{8} - \frac{9}{8}P_{ij}\right) = k_1\left(4 - \left(\frac{36}{8} - \frac{9}{8}P_{ij}\right)\right) + k_2\left(4 - \left(\frac{36}{8} - \frac{9}{8}P_{ij}\right)\right) + \\ + k_3\left(4 - \left(\frac{36}{8} - \frac{9}{8}P_{ij}\right)\right) \end{cases}$$

Solution of this system gives the result:

$$k_1 = \frac{13P_{ij} - 4}{36P_{ij} - 16} \; ; \; k_2 = 0 \; ; \; k_3 = \frac{4 - 5P_{ij}}{36P_{ij} - 16}$$

Including: $k_1 - k_3 = 1/2$.

Substituting coordinates of point A_{11} into the equation system (8), it is obtained:

$$\begin{cases} 1 - 0 = k_1(2 - 0) + k_2(0 - 0) + k_3(-2 - 0) \\ 0 - 0 = k_1(0 - 0) + k_2(2 - 0) + k_3(0 - 0) \\ \left(\frac{76}{16} - \frac{11}{16}P_{ij}\right) - \left(\frac{36}{8} - \frac{9}{8}P_{ij}\right) = k_1\left(4 - \left(\frac{36}{8} - \frac{9}{8}P_{ij}\right)\right) + k_2\left(4 - \left(\frac{36}{8} - \frac{9}{8}P_{ij}\right)\right) + \\ + k_3\left(4 - \left(\frac{36}{8} - \frac{9}{8}P_{ij}\right)\right) \end{cases}$$

Solution of this system gives the result:

$$k_1 = \frac{7P_{ij} + 4}{36P_{ij} - 16} \; ; \; k_2 = \frac{1}{2} \; ; \; k_3 = \frac{12 - 11P_{ij}}{36P_{ij} - 16} .$$

Including: $k_1 - k_3 = 1/2$.

Conclusions. The method of constructing a grid based on superimpositions of pre-calculated two or more grids with the similar topology permits to easily determine the coordinates of an arbitrary node in a new grid according to the coordinates of the respective nodes in the known grids.

Four specified nodal point coordinates coefficient superimpositions values of discretely presented two-dimensional geometric images for determining the sought nodal point coordinates can be determined bas on known initial geometric images superimposition coefficients.

According to a single discrete model of the curved surface formed by the static-geometric method, it is possible to form, by means of the superimpositions method, any number of balanced discrete surface models with an arbitrary number of nodal points under the same boundary conditions and different values of the external shaping load without composing and solving cumbersome systems of linear equations, that permits to discretely modeling of various shapes surfaces and to solve the tasks of discrete spatial interpolation. Coordinates of any node in the modeled surfaces discrete frame is determined by the formulas (8).

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