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DISCRETE INTEGER MODEL OF COMPLEX TIME-DIVISION SIGNALS WITH A PERIODIC STRUCTURE

Abstract. Background. The determination of approaches to the analytical synthesis of ensembles of complex time-division signals without the use of a search-and-correction procedure is relevant for reducing the computational complexity of synthesis of complex time-division signals with a periodic structure. **Subject of the study:** a discrete integer model of complex time-division signals with a periodic structure and conditions for determining the cardinality of the set of discrete positions of the ensemble and achieving no more than one pulse overlap. **Objective:** development of a discrete integer model of complex time-division signals with a periodic structure and establishment of conditions based on this model for determining the cardinality of the set of discrete positions of the ensemble and achieving no more than one pulse overlap for each mutual discrete shift of signals. **Research tasks:** to develop a discrete integer model of complex time-division signals with a periodic structure; to determine the cardinality of the set of discrete positions of the ensemble; to determine the conditions for achieving no more than one pulse overlap for each mutual discrete shift of signals. **Methods.** The methods of discrete integer modeling and the apparatus of linear Diophantine equations were applied. **Results.** The mutual coprimality of the numbers of pulses in a pair of signals is a necessary and sufficient condition for the existence of no more than one integer solution of a linear Diophantine equation and the occurrence of no more than one pulse overlap for any mutual discrete shift. **Conclusions.** The minimum cardinality of the set of discrete positions for a pair of signals is determined as the least common multiple of the numbers of pulses in this pair, and for an ensemble as the maximum of such values obtained for all pairs of signals, which ensures the same duration of ensemble signals and constant integer pulse placement steps.

Keywords: complex time-division signals; periodic structure; discrete integer model; linear Diophantine equation; coprime numbers; set of discrete positions; pulse overlap.

1. Introduction

1.1 Research relevance

The development of ultra-wideband access systems for low-power applications follows several directions. Study [1] considers regulatory limitations, physical-layer specifications, and standardized impulse-radio communications. Equipment and standards compatibility across the physical layer, medium access control, and upper layers is investigated in [2]. The development of the physical layer is addressed in [3] through requirements for higher speed, lower power consumption, and fewer hardware resources. Transmitter and receiver architectures are considered in [4]. The characteristics and practical applications of ultra-wideband technology are analyzed in [5]. Study [6] focuses on advances in ranging, sensing, and data transfer in ultra-wideband systems. Study [7] analyzes the current use of IEEE 802.15.4 impulse-radio ultra-wideband technology and prospects for further adoption. Impulse-radio sensing and directions for its practical application are investigated in [8]. Study [9] considers the operating principles of ultra-wideband wireless systems, modulation schemes, and channel characteristics. Ultra-wideband radio-link quality is investigated in [10]. Distance- and frequency-dependent characterization and modeling of indoor ultra-wideband propagation channels under line-of-sight and non-line-of-sight conditions are investigated in [11]. A reconfigurable memristive pulse generator with pulse shaping for operation in the 3–5 GHz ultra-wideband range is proposed in [12]. A digitally configurable impulse-radio ultra-wideband transmitter based on a switching oscillator, with tunable pulse width and pulse position for high-data-rate and low-power operation, is proposed in [13]. IEEE 802.15.4a supplements IEEE 802.15.4 by adding alternative physical layers for low-rate wireless personal area networks [14].

IEEE 802.15.4z incorporates enhancements to ultra-wideband physical layers and ranging techniques [15]. IEEE 802.15.3f extends the millimeter-wave physical-layer specification to the 57–71 GHz band [16]. The reviewed results confirm that impulse transmission is a promising direction for further research.

Under conditions of a steady increase in the density of radio services and the number of users, a potential limitation of further development of ultra-wideband access systems based on the IEEE 802.15 family of standards is the use of a shared channel and probabilistic access mechanisms, when each end device can transmit data only if the channel is considered free, and in the case of overlapping transmissions, signals of different users become distorted and lost. The above limitation can be eliminated by transition to code-division multiple-access systems, which makes it possible to provide guaranteed simultaneous access for a certain number of users.

A significant stage in the development of synthesis of ensembles of complex impulse signals was the approach proposed in [17], which provides synthesis of ensembles of periodic complex impulse signals with minimum energy interaction (hereinafter referred to as complex time-division signals with a periodic structure), for which no more than one pulse overlap occurs under arbitrary time shifts, making such ensembles suitable for use in asynchronous ultra-wideband systems. A characteristic feature of such ensembles is the constructive difference in signal energies caused by different numbers of equal-duration pulses in each signal [17, 18]. At the same time, synthesis of such signal ensembles is performed using a search-and-correction procedure [17], which leads to high computational complexity of the synthesis procedure.

Thus, the relevant research task is to determine approaches to analytical synthesis of ensembles of

complex time-division signals without using a search-and-correction procedure, which will significantly reduce the computational complexity of synthesis and provide possibilities for software-controlled adjustment of signal parameters in ultra-wideband code-division multiple-access systems. This enables software synthesis and reconfiguration of ensembles directly in transceiver devices by changing their parameters, creating prerequisites for adaptive modification of the operating mode of an ultra-wideband access system.

1.2 Literature review and problem definition

The approach proposed in [17] provides synthesis, by an exhaustive-search-and-correction procedure, of ensembles of periodic impulse signals with minimum energy interaction, for which no more than one pulse overlap occurs under arbitrary time shifts, making such ensembles suitable for asynchronous ultra-wideband systems. A characteristic feature of these ensembles is the inherent difference in signal energies caused by different numbers of equal-duration pulses in each signal, while their improved cross-correlation properties and preservation of these properties under arbitrary time shifts make ensembles of impulse signals with minimum energy interaction promising for use in ultra-wideband code-division multiple-access systems.

The construction of an ensemble of complex impulse signals with minimum energy interaction is performed in two stages [17]. First, the parameters of the impulse sequences that must satisfy the necessary synthesis condition are determined. Then, for the determined initial parameters, the sufficient condition of minimum similarity is checked. For the impulse sequences of the ensemble, the initial parameters are defined as

$$n_1 > n_2 > \dots > n_L,$$

$$n_i T_i = T, i = 1, \dots, L,$$

where n_i is the number of pulses in sequence i ; T_i is the pulse repetition period of sequence i ; T is the duration of the ensemble signals.

The above relations define the initial parameters of the impulse sequences but do not guarantee satisfaction of the sufficient condition of minimum similarity. Therefore, after the initial parameters have been determined, satisfaction of the sufficient condition of minimum similarity must be verified:

$$\max_{\tau} R_{ij}(\tau) \leq \frac{1}{\sqrt{n_i n_j}}, 1 \leq i < j \leq L,$$

where $R_{ij}(\tau)$ is the cross-correlation of impulse sequences i and j at time shift τ ; τ is the time shift between the sequences; L is the ensemble size.

The minimum-similarity condition for signals in the ensemble is achieved in [17] only by a separate exhaustive-search-and-correction procedure, which consists in sequentially checking, for each pair of impulse sequences and for each mutual discrete time shift, compliance with the constraint on the maximum cross-correlation value; if the condition is violated, the parameters of one of the sequences are corrected and the check is repeated until the sufficient condition is satisfied for all pairs of sequences in the ensemble.

Thus, the synthesis procedure for ensembles of complex signals with minimum energy interaction consists in forming an initial ensemble of periodic impulse sequences in which the sequence with the largest number of pulses is assigned the smallest initial period, whereas larger initial periods are selected for the remaining sequences so that all their pulses are fully located within the specified signal duration. Then, for each pair of sequences and for each discrete time shift, the admissible cross-correlation value equivalent to the condition of no more than one pulse overlap is calculated. If the condition is not satisfied, the parameters of one of the sequences are corrected, after which the verification is repeated iteratively. This synthesis procedure is based on exhaustive search and correction and has high computational complexity. Therefore, the known synthesis method does not provide a sufficient analytical basis for computationally efficient synthesis of complex-signal ensembles.

1.3 Research objective and tasks

The objective of the study is to develop a discrete integer model of complex time-division signals with a periodic structure and to establish, based on this model, the conditions for determining the cardinality of the set of discrete positions of the ensemble and achieving no more than one pulse overlap for each mutual discrete shift of signals. To achieve this objective, the following tasks were defined:

- to develop a discrete integer model of complex time-division signals with a periodic structure;
- to determine the cardinality of the set of discrete positions of the ensemble;
- to determine the conditions for achieving no more than one pulse overlap for each mutual discrete shift of signals.

The research results will enable the synthesis of ensembles of complex time-division signals based on the analytical determination of their parameters without the use of a search-and-correction procedure, which will reduce the computational complexity of synthesizing such signal ensembles.

2. Materials and methods

The object of the study is the process of formation and analysis of complex time-division signals with a periodic structure, in which the pulse start times are specified by discrete positions. The main hypothesis of the study: the application of a discrete integer model of complex time-division signals with a periodic structure provides analytical determination of the ensemble parameters without the use of a search-and-correction procedure.

The following conditions are adopted for the construction of the discrete integer model.

Condition 1. Complex time-division signals with a periodic structure have the same duration within the ensemble.

Condition 2. The pulses of the signals have the same duration.

Condition 3. The position of a pulse is determined by the moment of its start on the set of discrete positions of the signal.

Condition 4. Pulse overlap is determined by the coincidence of the numbers of discrete positions of their starts.

Condition 5. The periodic structure of signals is determined by constant integer pulse placement steps. The methods of discrete integer modeling and the apparatus of linear Diophantine equations are applied for the construction and analysis of the mathematical model.

The construction of the discrete model involves the transition from pulse start times to discrete position numbers, which makes it possible to determine the conditions of pulse overlap through the existence of integer solutions of a linear Diophantine equation.

3. Results of development and research of the mathematical model

3.1 Construction of the mathematical model of complex time-division signals with a periodic structure based on linear Diophantine equations

To construct and analyze the discrete integer model, we use the apparatus of linear Diophantine equations [19–21].

Let us assume that pulse positions are determined by their start instants, while pulse overlap is defined by coincidence of these instants. We introduce a discrete integer model of a complex time-division signal with a periodic structure, whose pulse positions are defined on a set of discrete integer quantities—signal positions. The elements of the set of signal positions correspond to all possible pulse start instants over the signal duration (hereinafter, the discrete set of signal positions). Pulse positions are the signal positions occupied by pulses of the corresponding sequence, and pulse overlap between two signals is determined by coincidence of the numbers of these positions. We pass from time to integer position numbers so that the overlap of two pulses is determined by coincidence of the position numbers of their start instants.

For each of the signals numbered i and j , considered as a signal pair, we define a time interval T_{ij} , identical for both signals of the pair, within which the start instants of their pulses of duration τ_u are determined as pulse positions on a set of N_{ij} discrete integer quantities – signal positions:

$$T_{ij} = N_{ij}\tau_u.$$

Each discrete signal position $m = 0, \dots, N_{ij}$ corresponds to the time instant $m\tau_u$; the value $m = N_{ij}$ corresponds to the right boundary of the interval T_{ij} .

The numbers of the discrete positions at the starts of the k th pulse of signal i and the l th pulse of signal j are denoted by $m_{i,k}$ and $m_{j,l}$, respectively. Since the discrete model takes into account only pulse start instants, pulses of duration τ_u occupy the half-open intervals:

$$\begin{aligned} &[m_{i,k}\tau_u, (m_{i,k} + 1)\tau_u); \\ &[m_{j,l}\tau_u, (m_{j,l} + 1)\tau_u). \end{aligned}$$

Because the pulse start instants are multiples of their duration τ_u , pulse overlap in time is possible only when the pulse start instants correspond to a signal position with the same number within the interval

$0, \dots, N_{ij} - 1$. Therefore, in the adopted discrete model, the pulse-overlap condition is defined as equality of the pulse-position numbers: $m_{i,k} = m_{j,l}$.

It should be noted that the end instants of the last pulses in signals i and j lie outside the interval T_{ij} . This is a limitation of the adopted discrete model but must be taken into account when implementing the signal-ensemble synthesis algorithm.

The periodic structure of complex time-division signals i and j is defined through the constant integer pulse-placement steps Q_i and Q_j , which, for the numbers of pulses n_i and n_j on the set of N_{ij} discrete positions, are determined by the relations:

$$Q_i = \frac{N_{ij}}{n_i}, \quad Q_j = \frac{N_{ij}}{n_j},$$

which show that, on the set of N_{ij} discrete positions for the pair of signals i and j , the periodic structure of each signal spans an integer number of steps.

For signals i and j , denote the pulse numbers by k and l , respectively, and define the pulse-number ranges on the set of N_{ij} discrete positions as $k = 1, \dots, n_i$, $l = 1, \dots, n_j$.

Let the initial positions of the first pulses of signals i and j be the integers d_i and d_j , respectively, with $0 < d_i \leq Q_i$, $0 < d_j \leq Q_j$. Hence, the zero position $m = 0$, corresponding to time instant $t = 0$, is not the initial position of the first pulse of either signal. The discrete position numbers of pulses $m_{i,k}$ and $m_{j,l}$ can then be defined by the linear relations:

$$m_{i,k} = d_i + (k - 1)Q_i, \quad m_{j,l} = d_j + (l - 1)Q_j.$$

The mutual discrete shift between signals numbered i and j – C_{ij} is specified as an integer number of positions, with $0 \leq C_{ij} \leq N_{ij} - 1$.

The pulse-overlap condition defined by equality of their discrete position numbers $m_{i,k} = m_{j,l}$, for the mutual discrete shift C_{ij} and cyclic transmission, understood here as repetition of the periodic signal structure after completion of transmission, is written by introducing the transmission-cycle number r as an integer:

$$m_{i,k} = m_{j,l} + C_{ij} + rN_{ij},$$

where the mutual discrete shift C_{ij} defines the displacement of the signals in discrete positions, while the product of the transmission-cycle number r and the cardinality of the set of discrete positions N_{ij} is added to the right-hand side of the pulse-overlap condition to account for cyclic transmission. For further analysis within the time interval T_{ij} , it is sufficient to consider the transmission-cycle numbers $r = 0$ and $r = -1$: for $r = 0$, the term rN_{ij} is zero, whereas for $r = -1$, the term $-N_{ij}$ appears on the right-hand side of the pulse-overlap condition and accounts for positions of the preceding cycle relative to the time interval T_{ij} .

Substituting $m_{i,k} = d_i + (k - 1)Q_i$, $m_{j,l} = d_j + (l - 1)Q_j$ into Eq. $m_{i,k} = m_{j,l} + C_{ij} + rN_{ij}$ yields an equality relating pulse numbers k and l of signals i and j to the quantities d_i , d_j , C_{ij} , and rN_{ij} :

$$d_i + (k - 1)Q_i = d_j + (l - 1)Q_j + C_{ij} + rN_{ij}.$$

Rearranging the terms so that the terms depending on the pulse numbers remain on the left-hand side gives the linear equation:

$$Q_i(k - 1) - Q_j(l - 1) = C_{ij} + (d_j - d_i) + rN_{ij},$$

where $C_{ij} + (d_j - d_i) + rN_{ij}$ is the difference between pulse positions specified by the mutual shift C_{ij} , the difference between the initial positions $d_j - d_i$, and the shift by an integer number of signal positions rN_{ij} .

For convenience, in the expressions $m_{i,k} = d_i + (k - 1)Q_i$, $m_{j,l} = d_j + (l - 1)Q_j$, we replace the numbers of steps from the first pulses to pulses numbered k and l , equal to $(k - 1)$ and $(l - 1)$, respectively, with the intermediate integer variables $x = k - 1$, $k = 1, \dots, n_i$, $y = l - 1$, $l = 1, \dots, n_j$, $x = 0, \dots, n_i - 1$, $y = 0, \dots, n_j - 1$. Hence, the pulse-position-number relations are $m_{i,x+1} = d_i + xQ_i$, $m_{j,y+1} = d_j + yQ_j$. After the same substitution in the equality $Q_i(k - 1) - Q_j(l - 1) = C_{ij} + (d_j - d_i) + rN_{ij}$, we obtain the linear Diophantine equation:

$$Q_i x - Q_j y = C_{ij} + (d_j - d_i) + rN_{ij},$$

where x is the number of Q_i steps from the first pulse of signal i to pulse number $k = x + 1$, $x = 0, \dots, n_i - 1$; y is the number of Q_j steps from the first pulse of signal j to pulse number $l = y + 1$, $y = 0, \dots, n_j - 1$.

It should be noted that after the substitutions $x = k - 1$, $y = l - 1$, the intermediate variables x and y become the direct unknowns of the linear Diophantine equation. Each pair of variables (x, y) determines the pulse numbers of signals i and j considered within the time interval T_{ij} . Fig. 1 shows an example of pulse placement for two signals on the set of discrete positions for $n_i = 3$, $d_i = 5$, $n_j = 5$, $d_j = 3$, $N_{ij} = 15$, $r = 0$, $T_{ij} = N_{ij}\tau_u = 15$, $m = 0, \dots, N_{ij} = 0, \dots, 15$, $C_{ij} = 0$.

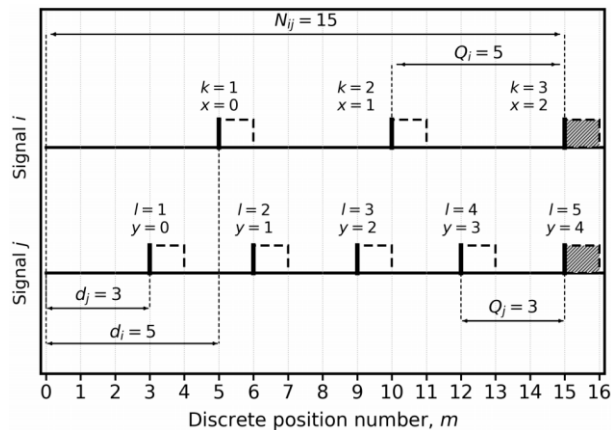


Fig. 1. Example of pulse placement for complex time-division signals with a periodic structure

The figure shows that pulse positions are determined by their start instants, whereas pulse overlap is determined by coincidence of these instants. Thus, in the constructed discrete integer model, the presence of pulse overlap between two complex time-division signals with a periodic structure is

determined by the existence of an integer solution of the linear Diophantine equation within the established ranges of the variables. The integer variables define the numbers of steps from the initial positions of the first pulses of the two signals, while the coefficients of the variables define the pulse-placement steps of the corresponding signals. Thus, the obtained model provides a basis for further determination of the number of integer solutions of the linear Diophantine equation depending on the mutual discrete shift and for establishing the condition under which the number of pulse overlaps between a pair of complex time-division signals with a periodic structure is minimal.

3.2 Determination of the cardinality of the set of discrete positions for the ensemble

Complex time-division signals with a periodic structure within an ensemble must have the same duration. In the introduced discrete integer model, each signal corresponds to a set of discrete positions at which pulses may be placed; therefore, equal duration of the ensemble signals corresponds to equal cardinality of this set.

For a pair of signals, integer pulse-placement steps are possible if the cardinality of the set of discrete positions is a common multiple of the numbers of pulses in both signals. The smallest possible interval corresponds to the least common multiple of the numbers of pulses in the pair. For the ensemble, all signal pairs must be taken into account; therefore, the cardinality of the set of discrete positions must be no smaller than the largest of the minimum values required for the individual pairs. Thus, for the ensemble as a whole, N must be selected so that the following inequality is satisfied:

$$N \geq \max_{1 \leq i < j \leq L} \text{LCM}(n_i, n_j),$$

which establishes the lower bound on the cardinality of the set of discrete positions for the ensemble: the time interval defined for all signals of the ensemble cannot be shorter than the largest among the minimum intervals required for the individual signal pairs. The equality

$$N = \max_{1 \leq i < j \leq L} \text{LCM}(n_i, n_j)$$

defines the minimum cardinality of the set of discrete positions, common to all signals of the ensemble, that provides equal duration of all ensemble signals, integer pulse-placement steps in the signals, and the maximum possible signal energy per unit time for fixed pulse duration and fixed numbers of pulses in the signals.

3.3 Determination of the condition for achieving no more than one pulse overlap under arbitrary mutual discrete shift

Based on the introduced discrete integer model, we determine the condition for achieving no more than one pulse overlap for each mutual discrete shift. For the asynchronous mode, cyclic signal transmission is taken into account, that is, repetition of the periodic signal structure after completion of transmission.

We determine the minimum cardinality of the set of discrete positions for a pair of signals that provides integer pulse-placement steps and the same time interval T_{ij} for both signals. We take into account that, for fixed pulse duration τ_u and fixed numbers of pulses n_i and n_j ,

decreasing the time interval T_{ij} increases the signal energy per unit time. Therefore, the cardinality of the set of discrete positions N_{ij} is determined from the condition of the smallest possible time interval T_{ij} .

For the cardinality of the set of discrete positions for the signal pair N_{ij} , the numbers of pulses n_i and n_j , and the pulse-placement steps Q_i and Q_j , we write the equality:

$$N_{ij} = Q_i n_i = Q_j n_j,$$

which implies that N_{ij} must be a common multiple of n_i and n_j .

Any common multiple of n_i and n_j can be expressed through their least common multiple $\text{LCM}(n_i, n_j)$ and a natural multiplicity factor q :

$$N_{ij} = q \cdot \text{LCM}(n_i, n_j).$$

If $q > 1$, the value N_{ij} exceeds the least common multiple by a factor of q , while the time interval T_{ij} exceeds the smallest possible interval by a factor of q . Therefore, the minimum value of N_{ij} is attained at $q = 1$:

$$N_{ij} = \text{LCM}(n_i, n_j).$$

The product of n_i and n_j equals the product of their least common multiple $\text{LCM}(n_i, n_j)$ and greatest common divisor $\text{GCD}(n_i, n_j)$:

$$n_i n_j = \text{LCM}(n_i, n_j) \cdot \text{GCD}(n_i, n_j),$$

hence the minimum cardinality of the set of discrete positions for the signal pair is

$$N_{ij} = \frac{n_i n_j}{\text{GCD}(n_i, n_j)},$$

which determines the shortest time interval:

$$T_{ij} = N_{ij} \tau_u.$$

It should be noted that increasing N_{ij} relative to $\text{LCM}(n_i, n_j)$ results only in additional repetitions of the periodic structure over a longer time interval.

We now generalize the results obtained for a signal pair to the ensemble, considering the ensemble as the set of all signal pairs. During synthesis of an ensemble of complex time-division signals with a periodic structure, we ensure compatibility of the periodic structures of individual signals and determine the cardinality of the set of discrete positions N , common to all signals of the ensemble. For this purpose, for each pair of signals i and j , $1 \leq i < j \leq L$, we select a value N that is a multiple of both the number of pulses n_i in signal i and the number of pulses n_j in signal j . For each signal pair, this choice ensures that the periodic structures of both signals are placed an integer number of times on the set of discrete positions. Thus, for the ensemble as a whole, N must be selected so that the inequality

$$N \geq \max_{1 \leq i < j \leq L} \text{LCM}(n_i, n_j),$$

which establishes the lower bound on the cardinality of the set of discrete positions for the ensemble: the time interval defined for all signals of the ensemble cannot be shorter than the largest among the minimum intervals required for the individual signal pairs. The equality

$$N = \max_{1 \leq i < j \leq L} \text{LCM}(n_i, n_j)$$

defines the minimum cardinality of the set of discrete positions, common to all signals of the ensemble, that provides equal duration of all ensemble signals, integer pulse-placement steps in the signals, and the maximum possible signal energy per unit time for fixed pulse duration and fixed numbers of pulses in the signals. The condition under which the number of pulse overlaps for a pair of signals does not exceed one for each value of the mutual discrete shift C_{ij} can be defined by assuming that, for a given mutual discrete shift C_{ij} , there are two distinct integer solutions of the linear Diophantine equation, each corresponding to one pulse overlap, with the sets of quantities for the first and second solutions being (x_1, y_1, r_1) and (x_2, y_2, r_2) , respectively. Subtracting the linear Diophantine equation for the set (x_2, y_2, r_2) from the linear Diophantine equation for the set (x_1, y_1, r_1) eliminates the mutual discrete shift C_{ij} and the difference between the positions $(d_j - d_i)$, yielding the relation:

$$Q_i(x_1 - x_2) - Q_j(y_1 - y_2) = (r_1 - r_2)N_{ij},$$

where variables x_1 and x_2 take values from 0 to $n_i - 1$, while variables y_1 and y_2 take values from 0 to $n_j - 1$.

For the case in which the numbers of pulses n_i and n_j are coprime, that is, $\text{GCD}(n_i, n_j) = 1$, it can be shown that the assumed two distinct integer solutions for the same mutual discrete shift C_{ij} have identical values of x , y , and r and therefore coincide. Hence, for each value of the mutual discrete shift C_{ij} , there can be no more than one integer solution of the linear Diophantine equation, and each integer solution corresponds to one pulse overlap. Thus, coprimality of the numbers of pulses n_i and n_j is a sufficient condition for the number of pulse overlaps for a signal pair not to exceed one for each value of the mutual discrete shift C_{ij} . When the greatest common divisor of the numbers of pulses n_i and n_j is greater than one, $\text{GCD}(n_i, n_j) > 1$, it can be shown that there exists a value of the mutual discrete shift C_{ij} for which the linear Diophantine equation has two distinct integer solutions that determine two separate pulse overlaps at different discrete positions $m = d_i$ and $m = d_i + N_{ij}/\text{GCD}(n_i, n_j)$ for the same value of the mutual discrete shift. Thus, if the greatest common divisor of the numbers of pulses in a signal pair is greater than one, there exists a mutual discrete shift for which at least two separate pulse overlaps occur at different positions.

Thus, taking into account the conclusion regarding the corresponding sufficient condition, coprimality of the numbers of pulses in a signal pair is a necessary and sufficient condition for limiting the number of pulse overlaps to one for each value of the mutual discrete shift.

4. Discussion of the results of the mathematical model investigation

To display all integer solutions of the linear Diophantine equation corresponding to pulse-overlap events for all values of the mutual discrete shift C_{ij} between signals i and j , and to confirm the obtained necessary and sufficient condition for limiting the number of pulse overlaps to one, we use the solution map

shown in Figure 2 for an example pair of complex time-division signals with a periodic structure having the parameters $n_i = 3$, $n_j = 5$, $N_{ij} = 15$, $d_i = 5$, $Q_i = 5$, $d_j = 3$, $Q_j = 3$. The numbers of pulses in the signals $n_i = 3$ and $n_j = 5$ satisfy the equalities $\text{GCD}(3,5) = 1$, $\text{LCM}(3,5) = 15$.

In Fig. 2, each row contains one marked cell; therefore, for each shown value of the mutual discrete shift C_{ij} , the linear Diophantine equation has one integer solution corresponding to one pulse-overlap event, and the absence of a second marked cell confirms the absence of a second integer solution for the same value of C_{ij} .

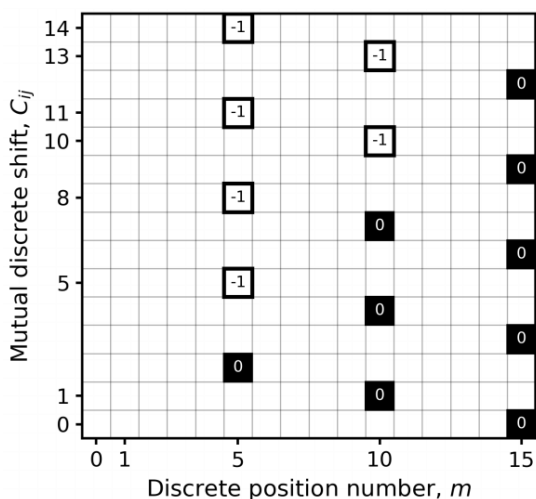


Fig. 2. Map of solutions of the linear Diophantine equation for a pair of signals with $n_i = 3$, $n_j = 5$, $N_{ij} = 15$, $d_i = 5$, $Q_i = 5$, $d_j = 3$, $Q_j = 3$. Black cells correspond to $r = 0$; white cells correspond to $r = -1$

Thus, the presence of one marked cell in each row of the solution map establishes, for the pair of signals with $n_i = 3$ and $n_j = 5$, no more than one integer solution and no more than one pulse-overlap event for each value of the mutual discrete shift, which, for the example considered, corresponds to the necessary and sufficient condition established above for limiting the number of pulse overlaps to one. Therefore, coprimality of the numbers of pulses in a signal pair is a necessary and sufficient

condition for achieving no more than one pulse overlap under an arbitrary mutual discrete shift for complex time-division signals with a periodic structure, as confirmed by the analysis of the constructed map of solutions of the linear Diophantine equation for the signal pair. A further direction of this research may be the consideration of the continuous component of the mutual time shift of complex time-division signals with a periodic structure

5. Conclusions

1. Pulse overlap for a pair of complex time-division signals with a periodic structure at a given mutual discrete shift and transmission-cycle number is determined by the existence of an integer solution of the linear Diophantine equation.

2. The minimum cardinality of the set of discrete positions for a signal pair is determined as the least common multiple of the numbers of pulses in that pair, and for the ensemble as the largest of such values obtained for all signal pairs, which provides equal duration of the ensemble signals and constant integer pulse-placement steps.

3. Coprimality of the numbers of pulses in a signal pair is a necessary and sufficient condition under which, for any mutual discrete shift, there is no more than one integer solution of the linear Diophantine equation and no more than one pulse overlap occurs.

Conflict of interest

The authors declare that they have no conflict of interest regarding this research, including financial, personal, authorship-related, or any other type of conflict that could affect the research and its results presented in this article.

Funding

The research was conducted without financial support.

Data availability

The manuscript has no associated data.

Use of artificial intelligence tools

The authors confirm that artificial intelligence technologies were not used in the preparation of this work.

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Received (Надійшла) 16.06.2026

Accepted for publication (Прийнята до друку) 02.09.2026

Publication date (Дата публікації) 25.09.2026

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Дискретна цілочисельна модель складних часорозділювальних сигналів з періодичною структурою

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Анотація. Актуальність. Актуальним є визначення шляхів аналітичного синтезу ансамблів складних часорозділювальних сигналів без застосування перебірно-коригувальної процедури для зменшення обчислювальної складності синтезу складних часорозділювальних сигналів з періодичною структурою. **Предмет дослідження:** дискретна цілочисельна модель складних часорозділювальних сигналів з періодичною структурою та умови визначення потужності множини дискретних позицій ансамблю і досягнення не більше одного суміщення імпульсів. **Мета дослідження:** Метою дослідження є розроблення дискретної цілочисельної моделі складних часорозділювальних сигналів з періодичною структурою та встановлення на її основі умов визначення потужності множини дискретних позицій ансамблю і досягнення не більше одного суміщення імпульсів для кожного дискретного взаємного зсуву сигналів. **Завдання дослідження:** розробити дискретну цілочисельну модель складних часорозділювальних сигналів з періодичною структурою; визначити потужність множини дискретних позицій ансамблю; визначити умови досягнення не більше одного суміщення імпульсів для кожного дискретного взаємного зсуву сигналів. **Методи дослідження.** Застосовано методи дискретного цілочисельного моделювання та апарат лінійних діофантових рівнянь. **Результати дослідження.** Взаємна простота кількостей імпульсів у парі сигналів є необхідною та достатньою умовою існування не більше одного цілочисельного розв'язку лінійного діофантового рівняння та виникнення не більше одного суміщення імпульсів для будь-якого взаємного дискретного зсуву. **Висновки.** Визначено найменшу потужність множини дискретних позицій для пари сигналів як найменше спільне кратне кількостей імпульсів у цій парі, а для ансамблю – як найбільше з таких значень, одержаних для всіх пар сигналів, що забезпечує однакову тривалість сигналів ансамблю та цілочисельні сталі кроки розміщення імпульсів.

Ключові слова: складні часорозділювальні сигнали; періодична структура; дискретна цілочисельна модель; лінійне діофантове рівняння; взаємно прості числа; множина дискретних позицій; суміщення імпульсів.