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DME/DME AND VOR/DME POSITIONING ERRORS ESTIMATION

Currently, area navigation methods are used as alternative to Global Navigation Satellite System. The most popular alternative method of positioning is grounded on the usage of Distanse Measuring Equipment data in the algorithms of an aircraft Flight Management System. Estimation of aircraft position error is one of the most important tasks of navigation. An article considers the problem of positioning errors estimation by DME equipment. In accordance with the international requirements for the airborne equipment of civil aircraft, the problem is considered in terms of optimal DME pair usage. DME/DME and VOR/DME navigation accuracy for a pair of navigational aids is estimated.

Keywords: *DME, DME/DME, VOR/DME, accuracy, pair of navigational aids, airspace, positioning, alternative positioning methods, APTN.*

Introduction

Guidance or maintenance the certain position lines play a major role during the whole aircraft flight. It determine the movement of the aircraft on an exactly trajectory. This process is an integral requirement for navigational tasks and is applied at all stages of the aircraft flight. In accordance with international documents [1] strict requirements are set for the accuracy of maintaining the specified lines of position. An accuracy of maintaining the given parameters of motion is bound to be related to the deviation of the aircraft from the preplanned trajectory and directly affects the safety of aviation. Algorithms of alternative positioning techniques are used on board of a civil aircraft in case of problems of Global Navigation Satellite System (GNSS) functioning.

The most accurate alternative positioning algorithm is the positioning using a pair of DMEs [2]. This algorithm is used in the majority of modern Flight Management Systems (FMS). At the same time, it is important to choose the optimal pair of DMEs that will provide the highest accuracy of positioning. According to minimum equipment list, only two DME interrogators should be present at aircraft avionics. Therefore, positioning algorithms in FMS can operate simultaneously only with a one DME/DME or VOR/DME pair. Geometric factor associated with the geometry of navigational aids relative position is considered as the most significant factor of positioning accuracy decreasing. Another component of accuracy estimation is the precision of DME measurements, related to accuracy of determining the distances between aircraft and navigational aids.

Questions of positioning errors were considered in different scientific papers. In particular, Euiho Kim explored the use of new forms of DME navigation signals to improve measurement accuracy of DME [3]; S. Lo and P. Enge studied the components of distance deftermination error [4], S. Li, Y. Ni and N. Cai studied issues of DMEs location optimal choise [5]; I. Ostroumov investigated accuracy of positioning with all available DMEs in a certain part of airspace [6-8]. However, obtained results do not reflect the real capabilities of aircraft avionics but are directed towards to optimize DME ground network.

In accordance with mensioned above, the *main objective of the article is* to estimate the maximum positioning accuracy that can be obtained by choosing the optimal DME/DME or VOR/DME pair in flight management system for a certain aircraft location in some part of Ukrainian airspace.

Estimation of DME/DME error

Aircraft positioning using information from DME is based on the simultaneous use of two sets of equipment that determine distances to two different navigational aids. In the general case, it is assumed that results of measurements are obtained simultaneously, and sometimes delay of information from one of two navigational aids with longer distance can be neglected. In this case, an aircraft location is at the point of intersection of the aircraft position circular lines relative to navigational aid (fig. 1). The action of errors in determining distances Δ using DME leads to the fact that resulting aircraft location will differ from the true one. Thus, it is shown on fig. 1 that the results of measurements of distances R_A and R_B using DME A and DME B contain errors Δ_A and Δ_B respectively. The point of intersection of position lines A and B gives the aircraf location P_1, which differs from the true location P_2 by the error value Δ_P. Estimation of location determination errors is very important task of navigation systems. We perform an estimation of positioning error Δ_P using known errors of DME systems.

At long distances, the curvature of the position lines can be neglected and lines can be considered to be straight on fig. 1.

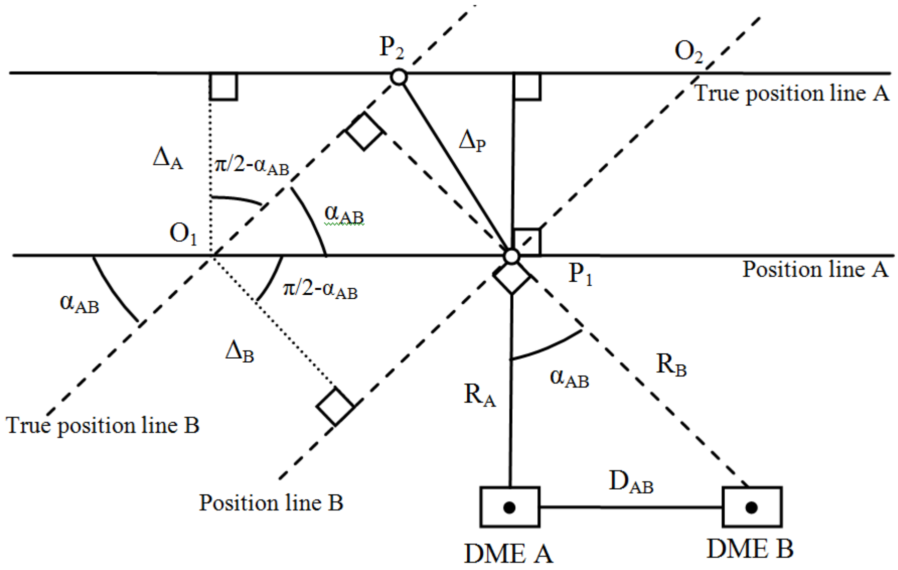

Fig. 1. Positioning error estimation

A case of aircraft coordinates determination using intersection of two lines of the location, formed from distance measurements (R_A and R_B) to DME A and DME B is shown on fig. 1. Point P_1 corresponds to determined aircraft location. The presence of errors (Δ_A and Δ_B) in the results of distance measurements allows to draw lines of the true location at the intersection of which P_2 (the point of the true aircraft position) is located.

The distance between points of location P_1 and P_2 is the radial error Δ_P, which determines the accuracy of aircraft coordinates.

To estimate an error value Δ_P, we use the angle α_{AB}, (angle between the directions on DME A and DME B), which can be determined by the basis of navigational aid pair:

$$\alpha_{AB} = \arccos\left(\frac{D_{AB}^2 - R_A^2 - R_B^2}{2R_A R_B}\right).$$

Since the lines of location are perpendicular to the radial distances of R_A and R_B, and the angles formed by two perpendicular angles are the same, then the angle of the line of position will be equal to the angle α_{AB}, formed by the directions on the DME. The error Δ_P is determined by the theorem of cosines from the triangle $O_1P_1P_2$:

$$\Delta_P = \sqrt{O_1P_1^2 + O_1P_2^2 - 2O_1P_1\,O_1P_2\cos(\alpha_{AB})},$$

where $$O_1P_1 = \frac{\Delta_B}{\cos\left(\frac{\pi}{2} - \alpha_{AB}\right)} = \frac{\Delta_B}{\sin(\alpha_{AB})},$$

$$O_1P_2 = \frac{\Delta_A}{\cos\left(\frac{\pi}{2} - \alpha_{AB}\right)} = \frac{\Delta_A}{\sin(\alpha_{AB})}.$$

Then,

$$\Delta_P = \sqrt{\left(\frac{\Delta_B}{\sin(\alpha_{AB})}\right)^2 + \left(\frac{\Delta_A}{\sin(\alpha_{AB})}\right)^2 - 2\frac{\Delta_B}{\sin(\alpha_{AB})}\frac{\Delta_A}{\sin(\alpha_{AB})}\cos(\alpha_{AB})};$$

$$\Delta_P = \sqrt{\frac{\Delta_B^2 + \Delta_A^2 - 2\Delta_A\Delta_B\cos(\alpha_{AB})}{\sin(\alpha_{AB})^2}}.$$

Since the errors Δ_A and Δ_B are random variables, then the radial error Δ_P will have a random nature.

Then

$$\sigma_P^2 = \frac{\sigma_B^2 + \sigma_A^2 - 2\rho\sigma_A\sigma_B\cos(\alpha_{AB})}{\sin(\alpha_{AB})^2},$$

where ρ is the coefficient of mutual correlation of measurement errors of the position line.

Since information about R_A and R_B is obtained from different DMEs, we will have independent measurement errors for the position lines, for which $\rho = 0$, then we will have:

$$\sigma_P^2 = \frac{\sigma_B^2 + \sigma_A^2}{\sin(\alpha_{AB})^2}. \quad (1)$$

Therefore, mean square error of determining the aircraft location depends on the mean square error of measuring the position lines and the angle between the directions to DME. The maximum accuracy of location will be observed when crossing of position lines will be perpendicular to each other.

Constant value of α_{AB} angle graphically corresponds to a circular line, the chord D_{AB} of which is the basis distance between DMEs. DMEs are located on a circular line at the points of intersection of the chord. Lines of the circles will be symmetrical on both sides of the chord. Lines of constant angles above the chord line for BRP (Boryspil) and IKV (Kyiv/Zhyliany) DMEs at FL20 within the area of their availability estimated by method [9] are represented on fig. 2.

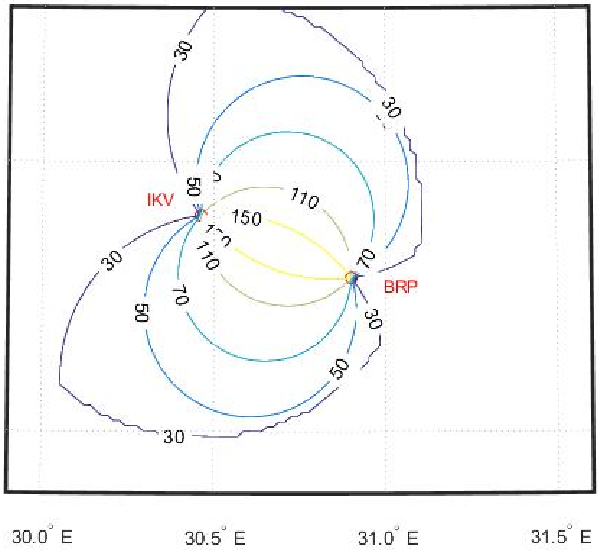

Fig. 2. Lines of the same angles between direction to IKV and BRP DMEs

Radius of circle is calculated as follow [10]:

$$R = \frac{d}{2\sin\left(\frac{\alpha_{AB}}{2}\right)}.$$

Coordinates of a circle center in the local coordinate system, center of which is located in DME A, and X axis is directed towards DME B, can be calculated by the following expression:

$$x_c = \frac{d}{2},\ y_c = \frac{d}{2}\operatorname{ctg}(\alpha_{AB}).$$

Since the denominator in (1) is a value with constant circular lines, and the effect of errors in the numerator is not very strong, the resulting lines of identical location determination errors will have the circular form (fig. 3). In addition, the angle and error values shown on fig. 2 and 3 correspondently are within DMEs operational area.

According to (1), the maximum positioning accuracy will occur for

$$\sin(\alpha_{AB}) = 1, \text{ with } \alpha_{AB} = 90^\circ.$$

For this circle, the distance D_{AB} to DME will be a diameter, and the center of circle is located at point with coordinates:

$$x_c = d/2,\ y_c = 0.$$

Operation area of DME/DME positioning is basicly limited by the level of double accuracy decreasing:

$$\sigma_P = \frac{\sqrt{\sigma_B^2 + \sigma_A^2}}{\sin(\alpha_{AB})};$$

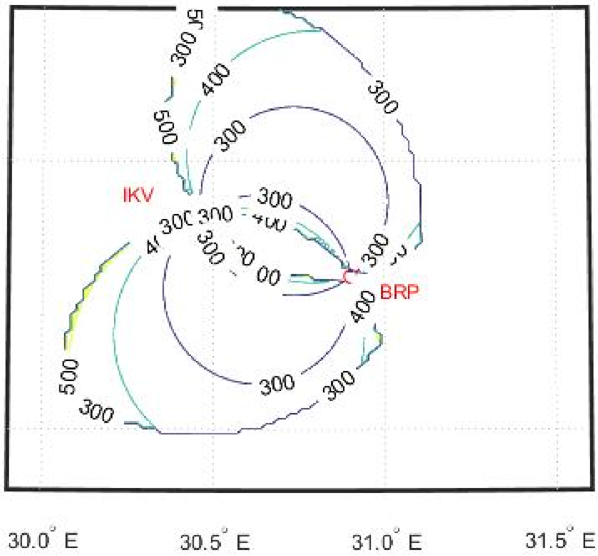

Fig. 3. Error of positioning by DME/DME

$$2\sqrt{\sigma_B^2 + \sigma_A^2} = \frac{\sqrt{\sigma_B^2 + \sigma_A^2}}{\sin(\alpha_{AB})};$$

$$\sin(\alpha_{AB}) = \frac{1}{2}.$$

Corespondingly, an area bounded by lines of fixed angles (30° and 150°) will correspond to a double accuracy decreasing. The values on fig. 2 and fig. 3 are represented within double accuracy decreasing area for BRP and IKV DMEs.

An important stage in the position accuracy calculation is the estimation of an area within which the probability of aircraft location will correspond to a certain *P* value. In case of independent measurements and the assumption that random errors in determining the position lines are determined by Gaussian distribution, then the distribution of location determination errors is described by double Gaussian distribution of a random variable:

$$\rho(\Delta_A, \Delta_B) = \frac{1}{2\pi\sigma_A\sigma_B} e^{-\frac{1}{2}\left(\frac{\Delta_A^2}{\sigma_A^2} + \frac{\Delta_B^2}{\sigma_B^2}\right)}.$$

The probability of aircraft location in a particular area will be constant within ellipse of errors that is described by equation [11]:

$$\frac{\Delta_A^2}{\sigma_A^2} + \frac{\Delta_B^2}{\sigma_B^2} = \kappa^2.$$

Probability of aircraft location within ellipse is as follows:

$$P = 1 - e^{-k^2/2} \quad \text{or} \quad k = \sqrt{-2\ln(1-P)}.$$

Also, an approach that estimates the area of a parallelogram formed by the lines of the true and measured position [11] can be used to evaluate the influence of the relative DME position:

$$S = \sigma_A \sigma_B \sin(\alpha_B).$$

Distance error of DME measurement is considered to be the sum of error occurring during the signal propagation in space (σ_{sis}) and an error introduced by the airborne interagetor σ_{air} [1, 12]:

$$\sigma_{DMEA,B}^2 = \sigma_{sis}^2 + \sigma_{air}^2.$$

The maximum permissible value of σ_{sis} is defined as 0.05 nm [1]. At that time, the maximum permissible value of σ_{air} is limited to 0.085 nm according to RTCA DO-189 [12], and, according to ICAO DOC-9613 [1], can be calculated as follows:

$$\sigma_{air} = \max\{0.085 \text{ nm}; 0{,}125\% \text{ R}\},$$

where R – measured distance.

Estimation of VOR/DME error

A method of position line determination by a combination of distance and angle measuring equipment is widely used in aircraft navigation systems. Location of DME and VOR at the same point allows to determine aircraft position using range and azimuthal crossing lines. The formula for positioning errors estimating is derived from triangles on fig. 4:

$$\sigma_P^2 = \sigma_d^2 + \left(2d\sin\left(\frac{\sigma_\alpha}{2}\right)\right)^2 = \sigma_d^2 + 2d^2\left(1-\cos\left(\sigma_\alpha\right)\right).$$

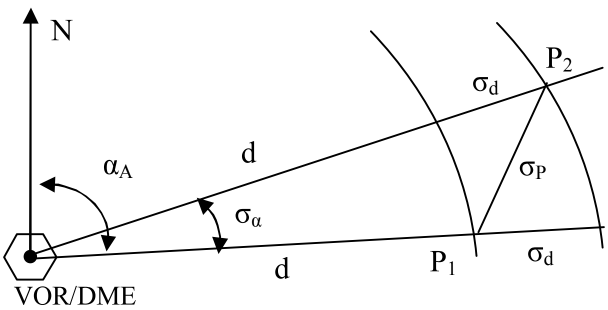

Fig. 4. Positioning error of VOR/DME method

Since the errors of azimuth determination basicly do not depend on the direction, the lines of identical errors have circular form with radii depending on the required accuracy:

$$R = \sqrt{\frac{\sigma_P^2 - \sigma_d^2}{2\left(1-\cos\left(\sigma_\alpha\right)\right)}}.$$

The results of positioning errors estimation for BRP VOR/DME are shown on fig. 5.

Selection of positioning method

During the flight, the choice of the appropriate positioning method and optimal navigational aids, that will provide the most accurate position line maintaining or determination of the coordinates of aircraft location, is important step in navigational algorithm. The value of mean-square deviation is used as an optimal criterion.

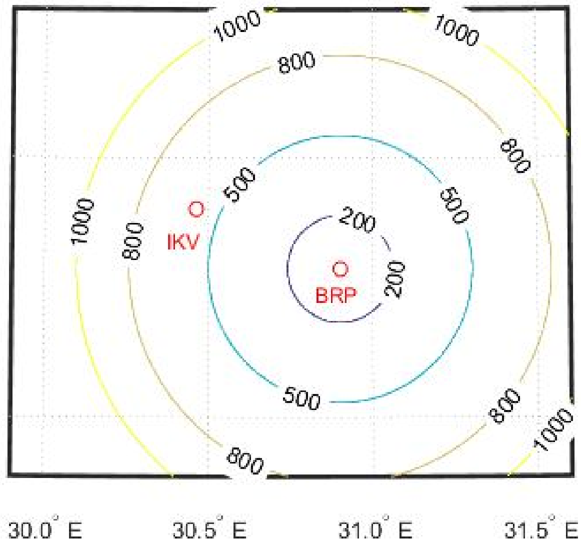

Fig. 5. Errors of BRP VOR/DME positioning

In general case, a pair of navigational aids is chosen to provide the least value of mean square deviation. This estimation is performed separately for each of the available positioning methods. A positioning method is selected grounding on the minimum error of positioning.

We perform a comparison of DME/DME and VOR/DME positioning methods based on their accuracy. Let's compare values of DME/DME positioning errors on fig. 3 with errors of VOR/DME on fig. 5 and determine the optimal positioning method. In this case, the accuracy of VOR/DME is limited to 600m. Results of area estimation of an appropriate positioning method are shown on fig. 6.

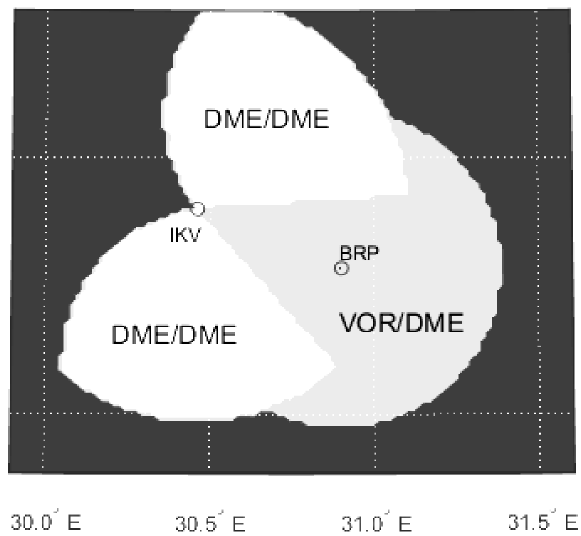

Fig. 6. Areas of an appropriate positioning method

Conclusion

An accuracy of maintaining an aircraft position line is one of the main characteristics of aviation safety. Given mathematical dependencies allow to estimate accuracy of positioning by a pair of DME/DME and VOR/DME navigational aids.

In addition, computer based simulation for BRP and IKV navigational aids was performed and areas of an appropriate positioning method was estimated in terms of accuracy.

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ПОХИБКА ПОЗИЦІОНУВАННЯ ЗА ПАРАМИ DME/DME ТА VOR/DME РАДІОМАЯКІВ

І.В. Остроумов

Сьогодні методи зональної навігації використовуються у якості альтернативних до глобальних супутникових систем позиціонування. Найбільш популярні альтернативні методи позиціонування у обчислювальній системі літаководіння гуртуються на використані дальномірних радіомаяків. Оцінювання похибок визначення координат місцеположення літака є одним з важливих завдань навігації. У статті розглянуто проблему оцінювання похибок позиціонування за далекомірним обладнанням DME. Відповідно міжнародним вимогам до складу бортового обладнання літака цивільної авіації проблема розглянута з точки зору застосування оптимальної пари DME. Виконано оцінювання максимальної точності навігації за DME/DME для повітряного простору України за умови використання оптимальної пари радіонавігаційних точок.

Ключові слова: *DME, DME/DME, VOR/DME, далекомірне обладнання, точність, пара радіонавігаційних точок, повітряний простір, Україна, позиціонування, альтернативні методи позиціонування, APTN.*

ОШИБКА ПОЗИЦИОНИРОВАНИЯ ПО ПАРАМ DME/DME И VOR/DME РАДИОМАЯКОВ

И.В. Остроумов

Сегодня методы зональной навигации используются в качестве альтернативных к глобальным спутниковым системам позиционирования. Наиболее популярные альтернативные методы позиционирования в вычислительной системе самолётовождения основываются на использованные дальномерных радиомаяков. Оценивание погрешностей определения координат местоположения самолёта является одной из важных задач навигации. В статье рассмотрена проблема оценки погрешностей позиционирования по дальномерному оборудованию DME. Согласно международным требованиям выдвигающемся к составу бортового оборудования самолёта гражданской авиации проблема рассмотрена с точки зрения применения оптимальной пары радиомаяков DME. Произведена оценка максимальной точности навигации по DME / DME для воздушного пространства Украины при условии использования оптимальной пары радионавигационных точек.

Ключевые слова: *DME, DME/DME, VOR/DME, дальномерное оборудование, точность, пара радионавигационных точек, воздушное пространство, Украина, позиционирование, альтернативные методы позиционирования, APTN.*