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MODEL OF CHEESE-OSIPOV-LANCASTER AND ITS GENERATION

The article considers models of the type of the Lanchester type and indicates the priority of their creation by M.P. Osipov The analysis of application of these models to the research of social and economic systems is carried out. The generalization of the models of the Linchestan type by a system of equations is carried out and their classification, both in the multinational and in binary forms, is carried out. Equivalent classification models with precision to transformations are established.

Keywords: *lanchester models, priority, generalization, system, equivalence, phase portrait, conflicting pair.*

Introduction

Problem statement and analysis of recent researches and publications. It is known that as a result of the generalization of the accumulated experience and the natural evolution of science, a methodology for studying existing problems, both at micro and macro levels, is formed, based on a systematic approach. The use of the principle of systemicity includes, along with the quantitative and informative analysis of the investigated processes using the method of mathematical modeling, which is currently one of the most urgent areas in scientific research and allows: to explicitly describe the mechanisms of the functioning of processes and implement their prediction; replace the direct analysis of the main properties of phenomena by analyzing the properties and characteristics of mathematical models. The only difference in comparison with mathematical theory is that the consequences of axioms should not contradict empirical facts. Interesting is the philosophical question that needs a separate study on the reasons for the successful application of abstract concepts and theories to the description of the real world.

A powerful tool for scientific research is the deterministic mathematical model of objects, the carrier of a certain set of formal relations between parameters (external conditions and stable characteristics) and variables (the main characteristics, the analysis of which changes in values determines the main purpose of the modeling).

Scientists have made many attempts to simulate the military-economic antagonism of states with the help of ordinary differential equations [1 – 4]. Traditionally, the priority in creating a mathematical model of global armed confrontation was attributed to English mathematician F.V. Lanchester - a far-sighted genius, who during the First World War offered a mathematical model of air combat, during which the probable losses of aircraft opposing sides are proportional to the number of possible meetings. The simplest linear model of Lancaster has the form (7) (the formulas are in Table 1, 2), where $x = x(t)$, $y = y(t)$. Model (7) is used for battles in which the impact is carried out for plane purposes. In the course of scientific research it became clear that the creator of the very first and most comprehensive models was Mikhail Pavlovich Osipov. The personality of Osipov is known to domestic mathematicians by the article "The Influence of the Number of Battle Parties on Their Loss" published in the Journal of the Military Collection in 1915, where the author published the Lanchester models for the year before most of Lancaster. Recently, in English literature, there has been a tendency for the transition from the phrase "model of Lancaster" to "Osipov-Lanchester model". The underlying cause was the article by R. Helmbold, published under the title "Osipov - Russian Lanchester" [5]. Now there is no question of the priority of a Russian scientist in publishing the first model of global armed confrontation. Identifying the personality of Mikhail Pavlovich Osipov was helped by the American J.Kipp, having published an article [6] in 2004.

At the beginning of the First World War, Osipov speaks of ways to achieve victory. For the first time in the history of wars, its victory is determined not by successful actions at the front, but by the one which the party does not exhaust its resources for a long time, first of all human. For the case of ground combat, when the enemy's losses occur as a result of direct contact at the anterior edge, Osipov proposed a model (10), where x and y are the number of two opposing armies, and the coefficients A and B show the effectiveness of the use of weapons.

We note that model (10) is more complete than the system of equations of Lanchester (7), since it takes into account the possibility of interaction of inhomogeneous forces. For the Osipov model, the following statement holds true: if necessary, to hold in r times stronger opponent, need to be armed with r^2 times more effective weapon than him.

The purpose of the work is to generalize the models of the Lanchester type by a single system of differential equations, and to classify them in the multinational and binary forms. Establishing equivalent classification models with accuracy up to transformations.

Presenting main material

The dynamics of social and economic systems is always difficult to simulate because of their unpredictability, and at the same time, these models are very valuable and useful, since they sometimes help to predict the future almost accurately and predict the effects of one or another action in advance. The most famous of these models was the model of English mathematician Lewis Fray Richardson proposed in 1918. This model was the first experience of using dynamic modeling in the field of international relations, which is amazing with its simplicity and certain truthfulness. Richardson used it to describe the arms race between Austria-Hungary and Germany on the one hand, and Russia and France on the other, during the pre-I World War (1909-1913 biennium). The general view of Richardson's model is as follows:

$$\begin{cases} \dot{x} = a_1 x + a_2 y + a; \\ \dot{y} = b_1 x + b_2 y + b, \end{cases} \quad (1)$$

where x and y are the levels of arms of the two countries, a_2 and b_1 – «coefficients of defense»; a_2 and b_2 – coefficients of cost of military effort (negative); a and b – the coefficients of "aggressiveness", express the degree of militarism (claims, if a>0, b>0) or the peacefulness of foreign policy (goodwill if a<0, b<0).

It is easy to see that the model (1) is a generalization of Osipov's model (10). It should also be noted that with (1) at $a = b = 0$ Morse-Kimbal model is a model of armed conflict with operational losses. According to Taylor, this model is acceptable in the following cases:

• there are significant non-military losses (illnesses, accidents, desertion and others, not caused by the direct influence of the enemy);

• there is a loss of combat, proportional to own strength (an offensive in the zone of damage by the factors of its own nuclear explosion that took place at the Totsky training artillery range of the Privolzhsky-Ural Military District);

• there is fire support (in the first approximation, it can be assumed to be proportional to the enemy's strength on the front edge).

Models of Osipov and Lancaster and their ideas have been repeatedly applied, as well as corrected. So, in his works, Tam Tam (Tam 1998), he modeled the Ardenian operation (December 15, 1944 - January 16, 1945), J. Engel (Engel 1954) - an operation on Ivadzhi, P. Morse and R. Kimbal (Morse , Kimball 1950) - the battle for the Atlantic, etc. Russian scientists are modeled Ice fight, Kulikov battle (Alekseev 1988; Temeszhnikov 1988). The exceptional adequacy of the results obtained on the basis of models, corresponding to the historical data, is noted.

Model (8) describes the military confrontation of partisan forces with regular parts. It was developed by Brecken (1959) during the Vietnam War and generalized by V. M. Zakharov and GM Kurchinitsky in 2014 Party x is represented by guerrilla forces, and the party in - regular troops, which carry losses only in the ancestral region through the containment of partisans (attack block posts, sabotage actions, etc.), because they do not have reliable information about their dislocation and can control certain areas by bombing, aviation and rocket-artillery bombardments.

Analyzing the confrontation between the two superpowers in the Cold War, Peterson formulated a counteraction model (3), which, moreover, can be used to describe blockade and siege operations, which excludes any combat collision. In model (3), the number of victims is determined by its own number. This may be a model of the Cold War, when more of its submarines are fighting alert, the more they die.

The Lotka-Volterra model has the form (11), where $x(t)$ is the number of victims, $y(t)$ - the number of predators at time t, is structurally unstable. One of the reasons for the structural instability of the model is that it is conservative, that is, it has the first integral. Despite the main disadvantage of the model, the system of equations (11) allows us to draw non-trivial conclusions, confirmed by numerous observations. In particular, on the basis of (11) Volterra's principle is formulated: if in the "predator-victim" system both species are eliminated uniformly and in proportion to the number of their individuals, the average number of victims increases and the average number of predators decreases.

The model (18) proposed by J. Taylor in 1999 (75 years after the model (11)), to describe the hostilities, when the losses go, both at the forefront and as a result of the fire influence of the enemy in certain areas. It is established that the system (18) is replaced by the form (11) and vice versa. In the most general form, the classic Linux models for the two objects under study can be described by a system of differential equations:

$$\begin{cases} \dot{x} = a_{11}x^2 + a_{12}xy + a_{22}y^2 + a_1 x + a_2 y + a; \\ \dot{y} = b_{11}x^2 + b_{12}xy + b_{22}x^2 + b_1 x + b_2 y + b, \end{cases} \quad (2)$$

where a_{11}, a_{22}, b_{11}, b_{22}, – coefficients that have nonzero values in the "predator-victim" model and are a characteristic of the intraspecific competition of nature; a_{12} and b_{12} – the speed of losses due to the influence of plane targets; a_1 and b_1 – speed of non-hazardous losses; a_2 and b_2 – loss from the enemy's influence on the anterior edge; a i b – reserves.

The generalization of the image with a greater number of participants in the conflict:

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} a_{jj}^{i} x_j^2 + \sum_{j=1}^{n-1} a_{jj+1}^{i} x_j x_{j+1} + a_i, \quad i = \overline{1,n}.$$

Classical Linux models used to describe a military confrontation are derived from system (2) both in the form of a monomer (see Table 1) and in the form of a binary (see Table 2) with $a_{11} = a_{22} = b_{11} = b_{22} = 0$.

Table 1

Classification of the Linchestrian models in a multinational form

$a = b = 0$	$a_{12} = a_2 = 0$	$a_1 = a_2 = 0$	$a_{12} = a_1 = 0$
$b_{12} = b_1 = 0$	$\begin{cases} \dot{x} = a_1 x; \\ \dot{y} = b_2 y. \end{cases}$ (3) Peterson 1953	$\begin{cases} \dot{x} = a_{12} xy; \\ \dot{y} = b_2 y. \end{cases}$ (4)	$\begin{cases} \dot{x} = a_2 y; \\ \dot{y} = b_2 y. \end{cases}$ (5)
$b_1 = b_2 = 0$	$\begin{cases} \dot{x} = a_1 x; \\ \dot{y} = b_{12} xy. \end{cases}$ (6)	$\begin{cases} \dot{x} = a_{12} xy; \\ \dot{y} = b_{12} xy. \end{cases}$ (7) Lanchester 1916	$\begin{cases} \dot{x} = a_2 y; \\ \dot{y} = b_{12} xy. \end{cases}$ (8) Brecken 1959
$b_{12} = b_2 = 0$	$\begin{cases} \dot{x} = a_1 x; \\ \dot{y} = b_1 x. \end{cases}$ (9)	$\begin{cases} \dot{x} = a_{12} xy; \\ \dot{y} = b_1 x. \end{cases}$ (8) Brecken 1959	$\begin{cases} \dot{x} = a_2 y; \\ \dot{y} = b_1 x. \end{cases}$ (10) Osipov 1915

Table 2

Classification of the London model in a binary form

$a = b = 0$	$a_2 = 0$	$a_{12} = 0$	$a_1 = 0$
$b_1 = 0$	$\begin{cases} \dot{x} = a_{12} xy + a_1 x; \\ \dot{y} = b_{12} xy + b_2 y. \end{cases}$ (11) Lotka 1924, Volterra 1926	$\begin{cases} \dot{x} = a_1 x + a_2 y; \\ \dot{y} = b_{12} xy + b_2 y. \end{cases}$ (12)	$\begin{cases} \dot{x} = a_{12} xy + a_2 y; \\ \dot{y} = b_{12} xy + b_2 y. \end{cases}$ (13)
$b_{12} = 0$	$\begin{cases} \dot{x} = a_{12} xy + a_1 x; \\ \dot{y} = b_1 x + b_2 y. \end{cases}$ (14)	$\begin{cases} \dot{x} = a_1 x + a_2 y; \\ \dot{y} = b_1 x + b_2 y. \end{cases}$ Morse-Kimball 1950	$\begin{cases} \dot{x} = a_{12} xy + a_2 y; \\ \dot{y} = b_1 x + b_2 y. \end{cases}$ (15)
$b_2 = 0$	$\begin{cases} \dot{x} = a_{12} xy + a_1 x; \\ \dot{y} = b_{12} xy + b_1 x. \end{cases}$ (16)	$\begin{cases} \dot{x} = a_1 x + a_2 y; \\ \dot{y} = b_{12} xy + b_1 x. \end{cases}$ (17)	$\begin{cases} \dot{x} = a_{12} xy + a_2 x; \\ \dot{y} = b_{12} xy + b_1 y. \end{cases}$ (18) Taylor 1999

The systems (4), (6) and (5), (9) which are equivalent to the redefinition are not used as models, but their military content can be determined. Model (9), which is a reflection of a linear law, can be used, for example, to describe the confrontation between police forces and peaceful demonstrators. Both parties are losing solely due to the presence of arms in the party x and the perpetration of her violent actions.

The semi-logarithmic model (6) can be used, for example, to describe the actions of the occupation forces. In this case, the party y is completely disarmed (either by contract, or under the initial conditions when occupying the territory) and is losing proportionally to the number of contractions of the parties *x* and *y*. All losses on the part of *x* are related exclusively to the careless handling of the weapon.

The pair of systems (13) and (16) and the four (12), (14), (15) and (17) are equivalent because they are obtained from each other through transformations. The rest of the systems (12), (14), (15) and (17) are reduced by means of transformations into the system:

$$\begin{cases} \dot{u} = \alpha u + v + \beta; \\ \dot{v} = uv, \end{cases}$$

which is a partial case of a description of a mixed warfare model.

Consequently, from the foregoing follows the fact that from the binomial form of the Linchestrian models without a free member, the basis are: Lotki-Volterra, Morse-Kimbal, (12) and (13).

It should be noted that when conducting the classification of the system (2) by the structural elements of the right-hand side, the free members vanished. Nevertheless, they also find application in the models described in the literature. Thus, J. Engel used to describe the fighting at Iwadzim (one of the worst battles that lasted from February 16 to March 26, 1945, at the final phase of the Second World War in the Pacific Theater of War between the forces of the Japanese Empire and the United States for control of the island Ito) the following model:

$$\begin{cases} \dfrac{dx}{dt} = -Ay + M; \\ \dfrac{dy}{dt} = -Bx + K. \end{cases}$$

In this case, x is Americans, constantly increasing their presence on the island, and y are Japanese, whose reserves were obstructed from the outside by the American blockade. As a result of hostilities, the Japanese garrison, totally 20,000 people, was completely exterminated. The American loss amounted to about 20 thousand people, of which about one fifth died. Since the American wounded were immediately evacuated from the battlefield, therefore, they were credited with irretrievable losses. Conversely, Japanese wounded were not evacuated and fought to complete destruction.

Enzel also conducted an investigation into the capture of Crete. Based on the model, it was found that the combat effectiveness of the German troops was 6.5 times lower than the allies. And this despite the fact that the Germans participated in the elite parachute parts, and the Allies acted quite marshy in the battles ordinary land formation. Angezel's model is the simplest generalization of the Osipov equations system, which reduces to (10) by transforming the shift by dependent variables.

Consider the models in the three-part form, which are a partial case of generalization (2), for which $a_{11}^2 + a_{22}^2 + b_{11}^2 + b_{22}^2 \neq 0$. One of these models is the model of mundialism - any kind of interconnected communication. This is a system of Lotka-Volterra equations with a logistic correction:

$$\begin{cases} \dot{x} = \alpha x^2 + a_{12}xy + a_1 x; \\ \dot{y} = \alpha y^2 + b_{12}xy + b_2 y. \end{cases}$$

is obtained from (2) at

$$a_{11} = b_{22} = \alpha,\ a_{22} = b_{11} = a_2 = b_1 = a = b = 0.$$

Its behavior of solutions in the vicinity of the stationary point changes the form of coefficients.

It is known that the important feature of biological systems is switching from one mode of operation to another. Example:

- sleep and wakefulness are different types of metabolism. Switching occurs periodically and synchronized with geophysical rhythm;
- in most insects, one and the same organism can exist in the form of caterpillars, puppets, butterflies. The switching occurs sequentially according to the genetic program;
- tissue differentiation - the cells are obtained by division from one type of cell, but later each performs its functions.

On a phase plane, the trigger system in the simplest case corresponds to two or more stable stationary solutions separated by separatrices. All special points (stable and saddle) lie at the intersection of the main isoquins - isozolines of vertical and horizontal tangents. In fig. 1 shows a relatively simple phase portrait of a critical system that describes the phenomenon of competition between two identical species. The corresponding system of equations is a partial case of the system of the Lotka-Volterra equations with a logistic correction:

$$\begin{cases} \dfrac{dx}{dt} = x - xy - \delta x^2; \\ \dfrac{dy}{dt} = y - xy - \delta y^2, \end{cases}$$

at $a_{12} = b_{12} = -1,\ a_1 = b_2 = 1,\ \alpha = -\delta$.

This system has four stationary solutions:

1) $x = y = 0$ – unstable knot;

2) $x = y = 1/(1+\delta)$ – saddle;

3) $x = 1/\delta,\ y = 0$ – stable knot;

4) $x = 0,\ y = 1/\delta$ – stable knot.

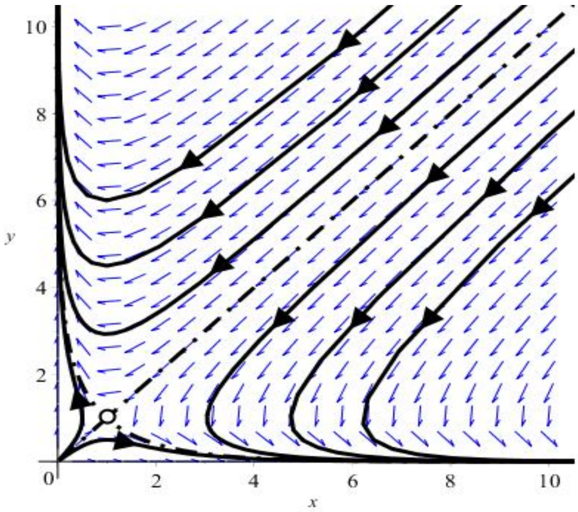

Fig. 1. A phase portrait of a critical system that describes the phenomenon of competition between two identical species

Suppose that pollution is in constant interaction with the surrounding environment, which has a purifying effect on pollution. We will also consider the system of "the environment - pollution" closed. Then the process of interaction with the environment can be described by the following system of equations:

$$\begin{cases} \dot{x} = a_{12}xy + a_1 x + a; \\ \dot{y} = b_{12}xy + b_{22}y^2 + b_2 y. \end{cases} \quad (19)$$

It is easy to see that the system of equations (19) is also a system of predator-victim, where the victim is contaminated (not a biological object), but as a predator, a biologically active environment.

Having replaced the variables in (19):

$$x = \frac{a_1}{b_{12}}u,\ y = \frac{a_1}{a_{12}}v,\ \tau = -a_1,$$

$$\alpha = -\frac{ab_{12}}{a_1^2},\ \beta = -\frac{b_2}{a_1},\ \gamma = \frac{b_{22}}{a_{12}}.$$

we obtain the simplest mathematical model of the interaction of pollution with the environment in the form of a system:

$$\begin{cases} \dot{u} = \alpha - u - uv; \\ \dot{v} = \beta v - uv - \gamma v^2. \end{cases} \quad (20)$$

In the system (20) the parameter α can be interpreted as the generalized power of the source of pollution; β – maximum permissible concentration of pollution (if $u > \beta$, $dv/dt < 0$, and nature is dying out); γ – characteristic of the ecosystem - the coefficient of intraspecific competition in nature.

The position of equilibrium of the system of equations (20), corresponding to the conditions $du/dt = 0$, $dv/dt = 0$, it is easy to find from the equation $\alpha - u - uv = 0$, $\beta v - uv - \gamma v^2 = 0$. Then

$$A_1(\alpha,0),\ A_2\left((\beta+\gamma+Q)/2,(\beta-\gamma-Q)/(2\gamma)\right),$$
$$A_3\left((\beta+\gamma-Q)/2,(\beta-\gamma+Q)/(2\gamma)\right),\ Q=\sqrt{(\beta+\gamma)^2-4a\gamma}.$$

The second and third equilibrium positions exist, if $(\beta+\gamma)^2-4a\gamma>0$. Using the standard linear analysis for the stability of these positions of equilibrium, it is easy to obtain a parametric portrait of a system of equations. Note that the hierarchical approach can also be used for Lantern models (see Table 1). At the lower level, the Monte Carlo method simulates the interaction of individual combat units, on the average, the interaction is described by the Markov models, and on the upper (aggregated, deterministic) level, the differential equations of the type of the Lanchester type are used. Above these models, by introducing in them controlled parameters (the distribution of forces and means in time - the introduction of reserves, etc.), you can add management tasks in terms of controlled dynamic systems of differential equations.

Table 3

Hierarchical Model of Combat Action

Le-vel	Modified phenom-ena, processes	Modeling machine
5	Distribution of forces in space	The game of Colonel Blot-to and his modifications
4	Distribution of forces in time	Optimal control, repetitive games, etc.
3	Number dynamics	Equations of Lancaster and their modifications
2	"Local" interaction of divisions	Markov models
1	Interaction of indi-vidual combat units	Simulation simulation, Monte Carlo method

So, the Linux models can be used wherever there is a conflicting pair. Nobuo Taoka, founder of Tokyo Research Statistics Management Society, which in 1962 reorganized at different levels of the hierarchy of the London-based strategy in a model for capturing market share in business operations, is confirmed by this. As a result, Japan broke the American companies at that time in the competition for the share of the world market.

Conclusions

The article deals with the models of the Linchestrian type, which is a powerful tool for scientific research of deterministic mathematical models of objects, the carriers of a certain set of formal relations between parameters. The priority of their creation to MPs is highlighted. Osipov The analysis of application of these models to the research of social and economic systems is carried out.

The generalization of the models of the Linchestan type by a system of equations is carried out and their classification, both in the multinational and in binary forms, is carried out. Equivalent classification models with precision to transformations are established. For binomial forms of the Linchestrian models without a free member it is found that the basic ones are: Lotki-Volterra, Morse-Kimball.

An argumentative military interpretation of unused previously used models obtained as a result of generalization and classification has been given.

References

1. *Mityukov NV Definition of victims of wars through the lanchester models // Historical psychology and sociology of data by the user from the point of view of the resonance / N.V. Mityukov – 2009. – №2. – C. 122–140.*
2. *Saati TL Mathematical Models of Conflict Situations / per. from english / T. L. Saati - M. : Soviet Radio, 1977. – 455 c.*
3. *Lanchester F. W. Aircraft in Warfare: The Dawn of the Fourth Arm. London: Constable and Co, Ltd, 1916.*
4. *Osipov M. P. The Influence of the Numerical Strength of Engaged Forces in Their Casualties / transl. by R. L. Helmbold, A. S. Rehm. Naval Research Logistics 42, 1995. P. 435–490.*
5. *Helmbold R. L. Osipov: the «Russian Lanchester». European Journal of Operations Research: 1993, vol. 65, no2, P. 278–288.*
6. *Kipp J. W. Tracking down Russia's Lanchester. Journal of Slavic Military Studies 17: 2004, P. 257–269.*

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МОДЕЛЬ ЧЕЙЗА-ОСИПОВА-ЛАНЧЕСТЕРА ТА ЇЇ УЗАГАЛЬНЕННЯ

Ю.Г. Подошвєлєв, Н.В. Ічанська

У статті розглянуто моделі ланчестерського типу та зазначено пріоритет їх створення ученим М.П. Осиповим. Проведено аналіз застосувань даних моделей до дослідження соціально-економічних систем. Здійснено узагальнення моделей ланчестерського типу системою рівнянь та проведена їх класифікація, як в одночленній, так і в двочленній формах. Встановлено еквівалентні моделі класифікації з точністю до перетворень.

Ключові слова: *ланчестерські моделі, пріоритет, узагальнення, система, еквівалентність, фазовий портрет, конфліктуюча пара.*

МОДЕЛЬ ЧЕЙЗА-ОСИПОВА-ЛАНЧЕСТЕРА И ЕЕ ОБОБЩЕНИЯ

Ю.Г. Подошвелев, Н.В. Ичанская

В статье рассмотрены модели ланчестерского типа и указано приоритет их создания ученым М.П. Осиповым. Проведен анализ приложений данных моделей к исследованию социально-экономических систем. Осуществлено обобщение моделей ланчестерского типа системой уравнений и проведена их классификация, как в одночленной, так и в двучленной формах. Установлено эквивалентные модели классификации с точностью до преобразований.

Ключевые слова: *ланчестерские модели, приоритет, обобщение, система, эквивалентность, фазовый портрет, конфликтующая пара.*