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B.P. Boriak, A.M. Silvestrov

Poltava National Technical Yuri Kondratyuk University, Poltava

FILTERING AND FORECASTING SIGNALS ALGORITHM BASED ON EXPONENTIAL BROWN'S FILTER

In the article we reviewed exponential smoothing method, proposed by Robert Brown and field of its use in control systems and measurement systems. We proposed smoothing and forecasting signal algorithm, using nonius principle of Brown's filter structure increasing. This method gives an opportunity partly compensate disadvantages of exponential smoothing such as defects of introducing a lag relative to the input data. This algorithm might be implemented as program for digital information processing devices.

Keywords: *exponential smoothing, noise, forecast, original signal, low-pass filter, smoothing factor.*

Introduction

Exponential smoothing is data processing algorithm that is used for filtration and forecasting of time series data. Such processing information technique might be used as low-pass filter for signals that works using window function principle.

This method is widely used in the creation of statistics, signal processing, financial mathematics, smart transport, communications, automatic, astronomy, engineering and most other applied sciences that might describe processes as time series data.

One of the possible applications of this algorithm is its integration into the system of virtual and augmented reality [2], which nowadays are in current trends.

It should be noticed that exponential smoothing as usually can not guaranty strict accuracy during all time of data processing. This is because of the fact that even filtering systems with optimal parameters can be put under influence of the different noise with amplitudes that exceed values of determined noise amplitude.

Nowadays analysis and time series forecasting become more complicated because of non stationarity of different systems where it can be integrated. This problem can be solved by using adaptive smoothing and forecasting algorithm and filter structure improving.

Formulation of the problem

Brown's model assumes that the signal value at a certain time moment consists of two components. The first component is the product of the current signal value and weight coefficient α. The second component is product of the difference in the form (1- α) and the smoothed value of the signal in the previous time moment. The sum of these components can be represented as:

$$\hat{x}(k) = \alpha \cdot x(k) + (1-\alpha) \cdot \hat{x}(k-1), \quad (1)$$

where $\hat{x}(k)$ – smoothed signal value at time k; α – smoothing factor; x(k) – current signal value at step time k; (1 – α) – the second member of the infinite geometric progression series in model of Brown, that is weight coefficient of the filtered signal value second component; $\hat{x}(k-1)$ - filtered signal value at step time (k-1).

Let us consider the result of this algorithm on the example of linear function signal with determined amplitude noise. We see that the signal which consists of the linear function signal and imposed noise is fed to the system input. After the exponential smoothing process we observe that the filtered signal eliminates distortion caused by a noise superimposed on it. In the same time it lags for some time.

This process was modeled using the package of applied programs and programming language Matlab.

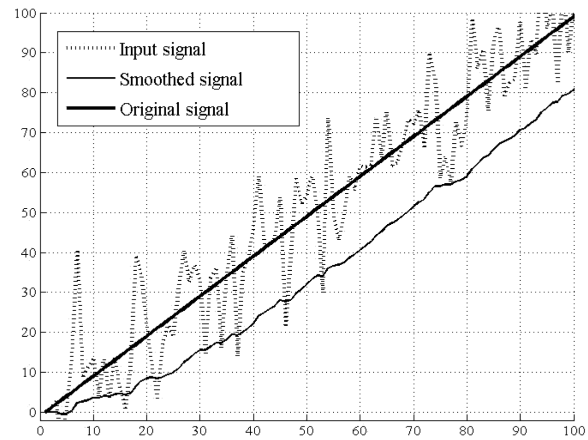

Fig. 1. Linear signal exponential smoothing process by Brown's filter

This negative effect can be ignored in a case when we need to analyze the signal after the researched process has finished and therefore there is no necessity to determine the accurate signal values, but it is possible to determine its form instead. The result of smoothing process gets worse if input original signal is nonlinear. Let us simulate exponential smoothing process when input signal is a quadratic function with imposed noise on it.

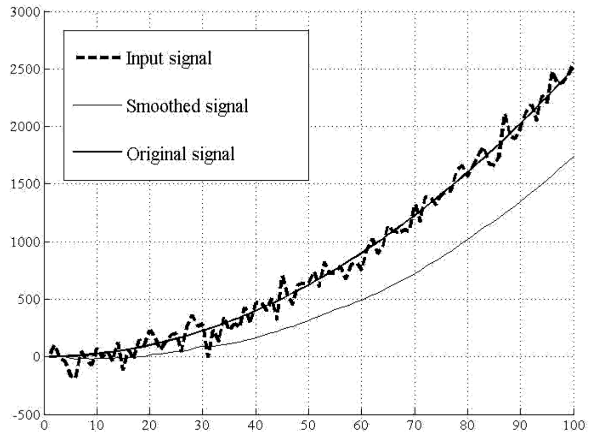

Fig. 2. Parabolic shape signal exponential smoothing process by Brown's filter

In this case we see that lag grows during the working time. As a result output data that we get after smoothing process does not give us information about real values of a signal but only can describe approximate shape of the signal. The difference between original signal and filtered signal grows that is why it does not give us opportunity to analyze even real shape of a signal and values of signal in a current time moment.

To summarize, the use of the exponential smoothing based on the Brown's model in determination of accurate values of the input signal is inappropriate. The algorithm of this process should be improved.

Nonius principle of filter structure increasing

During our research we used two filters connected in series that work according to the formula (1) – double exponential smoothing (Brown's DES 1). As a result we get well filtered signal $\hat{\hat{x}}(k)$ but with substantial lag relative to the input data.

In the following step we should subtract smoothed signal values $\hat{\hat{x}}(k)$ from input signal $x(k)$ (the sum of original signal and random noise). As a result we get lag values with noise disturbance. In order to get rid of distortion of signal $\varepsilon_1(k)$ we use exponential smoothing (Brown's DES 2) that has the same working principle with previous one. After we got filtered signal $\hat{\hat{\varepsilon}}_1(k)$ values we add it to smoothed signal $\hat{\hat{x}}(k)$ values from Brown's DES filter 1. In this way we can get rid of lag we had before getting signal $\hat{\hat{x}}_1(k)$. Let us exemplify the work of given algorithm using linear shape original signal with added noise.

If original signal is a signal with parabolic shape then we can observe the following tendency. The lag will stay and it will be soaring in determined diapason but it will not grow as it is shown on Fig. 2.

For compensation this lag we should expand the data processing algorithm structure similar to previous increasing.

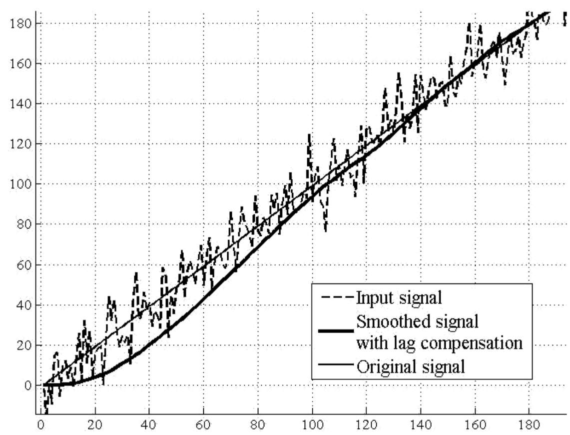

Fig. 3. Lag compensation process

The following data processing is based on determination of the second lag. This information can be got in a next way. We subtract smoothed signal $\hat{\hat{x}}_1(k)$ with compensated first lag from input disturbed signal $x(k)$. That is how we get second lag values $\varepsilon_2(k)$ with added noise. To get rid of disturbance we filter this signal with Brown's DES filter 3 (Brown's DES 3). In order to compensate second lag we need to add smoothed signal $\hat{\hat{x}}_1(k)$ with compensated first lag to smoothed second lag $\hat{\hat{\varepsilon}}_2(k)$.

As a result the signal smoothing process with compensation of the first and second lags is presented on the Fig. 4.

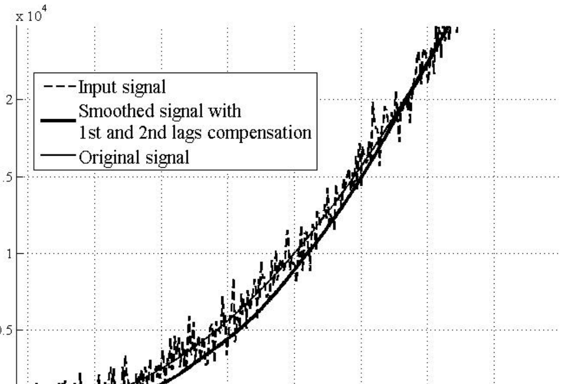

Fig. 4. The first and the second lags compensation process

The configure process takes some time due to the generation of the smoothing signal in the current time moment, which is determined basing on the previous observations. In addition, time that is needed for the filtered signal values to approximate to the original signal values depends on the smoothing factors which defines the smoothing value. With α decrease configure time is increased, however, in consequence we get a signal that is better smoothed.

According to Brown's method double exponential smoothing gives an opportunity to get the forecast with representation of $\hat{\hat{x}}(k)$ by Taylor series. For the m steps linear forecast it is sufficient to use first two elements of Taylor series:

$$\hat{\hat{x}}(k+m) = \hat{\hat{x}}(k) + m\cdot\Delta t\cdot\dot{\hat{\hat{x}}}(k), \quad (2)$$

where $\hat{\hat{x}}(k+m)$ – m steps forecast; m – steps amount of forecast; Δt – sample time; $\dot{\hat{\hat{x}}}(k)$ – derivative of double smoothed signal, can be presented as:

$$\dot{\hat{\hat{x}}}(k) = \left[\hat{\hat{x}}(k) - \hat{\hat{x}}(k-1)\right]\cdot\Delta t^{-1}, \quad (3)$$

Inserting formula (3) into formula (2) we will get:

$$\hat{\hat{x}}(k+m) = \hat{\hat{x}}(k) + m\cdot\left[\hat{\hat{x}}(k) - \hat{\hat{x}}(k-1)\right], \quad (4)$$

Due to the nonius principle of filter structure increasing as it is shown Fig. 4 this structure gives a possibility to get the forecast for m steps and compensate lags $\hat{\hat{x}}(k+m)$.

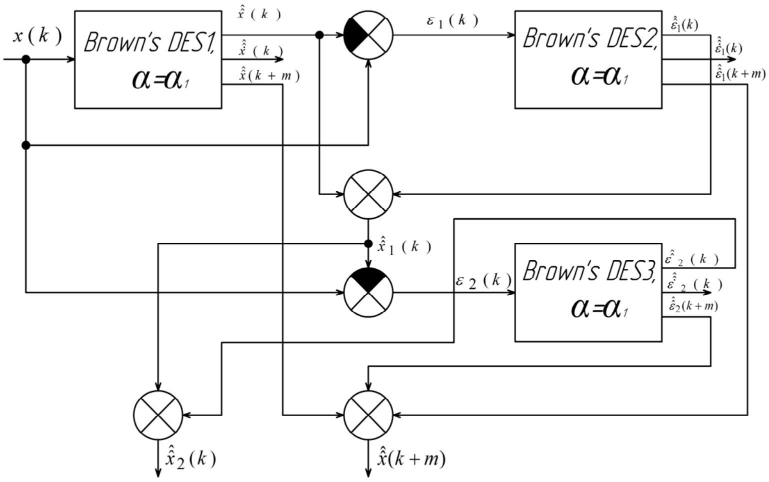

Fig. 5. Nonius principle of filter structure increasing

One of the algorithm advantages is a pretty high operation speed under conditions of its appliance to a system with small sample time. Another advantage is an algorithm flexibility that is accomplished due to the possibility to set up the connection between the smoothing factors of the different filters and due to the possibility of the independent functioning of the given elements with defined parameters.

Conclusions

As a result of the research the method of simple exponential filters connection was developed. Functioning of the filters is based on the Brown model. The filters give a possibility to get the smoothed value of the signal which is fed to the system input with noise in a current time moment. The given data processing algorithm also allows getting the forecast signal value.

One of the algorithm issues is a specific for the exponential smoothing configuring process that depends on the smoothing factor. The other issue is lack of the filter adaptation during the alteration of the noise amplitude.

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Рецензент: д-р техн. наук, проф. С.В. Козелков, Державний університет телекомунікацій, Київ.

АЛГОРИТМ ЗГЛАДЖУВАННЯ ТА ПРОГНОЗУВАННЯ СИГНАЛУ НА ОСНОВІ ЕКСПОНЕНЦІАЛЬНОГО ФІЛЬТРУ МОДЕЛІ БРАУНА

Б.Р. Боряк, А.М. Сільвестров

У статті було проведено огляд методу експоненціального згладжування, запропонованого Робертом Брауном та можливості його застосування у системах керування та вимірювання. Запропоновано алгоритм згладжування та прогнозування сигналу використовуючи ноніусний принцип нарощування структури експоненціального фільтра моделі Брауна. Даний метод дає можливість частково компенсувати негативні ефекти експоненціального згладжування, такі як похибка слідування першого і другого порядків. Даний алгоритм може бути реалізований у вигляді програми на цифрових пристроях обробки інформації.

Ключові слова: *експоненціальне згладжування, шум, прогноз, корисний сигнал, фільтр низьких частот, коефіцієнт фільтрації.*

АЛГОРИТМ СГЛАЖИВАНИЯ И ПРОГНОЗИРОВАНИЯ СИГНАЛА НА ОСНОВЕ ЭКСПОНЕНЦИАЛЬНОГО ФИЛЬТРА МОДЕЛИ БРАУНА

Б.Р. Боряк, А.М. Сильвестров

В статье было рассмотрен метод экспоненциального сглаживания, предложенный Робертом Брауном, и возможности его применения в системах управления и измерения. Предложено алгоритм сглаживания и прогнозирования сигнала, с использованием нониусного принципа наращивания структуры экспоненциального фильтра по модели Брауна. Этот метод дает возможность частично компенсировать негативные составляющие экспоненциального сглаживания, такие как погрешность следования первого и второго порядка. Данный алгоритм может быть реализован в виде программы на цифровых устройствах обработки информации.

Ключевые слова: *экспоненциальное сглаживание, шум, прогноз, полезный (искомый) сигнал, фильтр низких частот, коэффициент фильтрации.*