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ASSESSMENT OF THE IMPACT OF SPARSITY AND GEMAN-MCCLURE REGULARIZATION ON SIGNAL RECONSTRUCTION ACCURACY

Abstract. The article proposes a method for spectral reconstruction of signals using the Geman-McClure Function and Lagrange multipliers to optimize parameters. The method combines approaches of adaptive filtering and sparsity-based parameter regularization, incorporating Geman-McClure Function and Lagrange multipliers to ensure orthogonality and optimal parameter selection. The proposed approach efficiently identifies significant signal parameters, minimizes mutual interference among nonlinear components, reduces computational complexity, and improves reconstruction accuracy. Experimental modeling demonstrated that the developed method achieves a significant reduction in mean square error (MSE) by 10–15% and enhances the robustness of signal reconstruction by 10–15% across a signal-to-noise ratio (SNR) range from -10 to 10 dB, confirming its effectiveness for practical applications in cognitive telecommunication networks operating under severe noise conditions and nonlinear distortions.

Keywords: reconstruction, complex signals, Geman-McClure function, sparsity, regularization, nonlinearity, system, noise immunity, signal-to-noise ratio (SNR), Volterra series, optimization, signal parameters.

Abbreviations

GMF – Geman-McClure Function;
l_0-VLMS – l_0-Norm Variable Step-Size Least Mean Squares;
l_0-VNLMS – l_0-Norm Variable Step-Size Normalized Least Mean Squares;
MSE – Mean Squared Error.
SNR – Signal-to-Noise Ratio

Introduction

Statement of a scientific problem. In conditions of complex nonlinear communication environments with significant interference, effective reconstruction and precise analysis of signals become challenging due to inherent nonlinearities and distortions that significantly degrade signal quality. Traditional linear signal processing methods often fail to adequately address these nonlinear characteristics, leading to substantial inaccuracies in signal reconstruction, especially when high-order nonlinear interactions occur. The computational complexity associated with accurately modeling these nonlinear interactions using conventional methods increases exponentially with the order of the system, causing significant practical limitations.

Thus, the scientific problem addressed in this article is the development and justification of an optimized spectral reconstruction method using adaptive filtering, Volterra series modeling, and Lagrange multiplier-based optimization. This approach aims to effectively reduce computational complexity, ensure stability and robustness of signal processing in challenging radio conditions, and improve accuracy in modeling nonlinear interactions in cognitive telecommunication networks.

Research analysis. The analysis of modern methods for modeling nonlinear systems using Volterra series [1–15] confirms their effectiveness in spectral reconstruction of signals under complex interference conditions. Studies [1–3, 13] validate the advantages of Volterra kernel factorization and regularization in reducing computational complexity; however, they insufficiently address the impact of less significant parameters on reconstruction accuracy. Research [4, 6, 11, 15] examines the Geman-McClure Function (GMF), which effectively approximates the l_0-norm, ensuring sparsity of model parameters and reducing computational complexity. Moreover, the use of the Geman-McClure Function (GMF) in adaptive algorithm modifications, such as the l_0-VLMS and l_0-VNLMS methods [5, 9], significantly enhances reconstruction accuracy and stability. Publications [13, 14] highlight the benefits of factorization and regularization but overlook the complete evaluation of weaker signal components.

Overall, the analysis of the literature supports the integration of sparsity, regularization, and Lagrange multipliers to optimize model parameters, providing improved accuracy and robustness in Volterra-based spectral reconstruction.

The purpose of the work. The aim of the study is to develop a method for spectral reconstruction of signals in a complex interference-prone nonlinear environment by employing adaptive filtering based on Volterra series, utilizing the Geman-McClure function for regularization and Lagrange multipliers for parameter optimization. This approach ensures improved accuracy and robustness in signal reconstruction under conditions of nonlinear distortions and complex interference.

Presentation of the main material and substantiation of the obtained research results

In a complex interference-prone radio environment, modeling nonlinear systems using Volterra series imposes a significant computational burden, as it requires the calculation of a large number of parameters within the experimental model, especially when a substantial volume of input data is involved. Specifically, if the input signal $x(t)$ in a scientific and practical task is multi-component, it necessitates the analysis of intricate interactions occurring within the telecommunication system. In such cases, the computation of high-order Volterra series kernels $h_r(\tau_1, \tau_2, \dots, \tau_r)$ becomes increasingly complex, as the number of parameters grows exponentially with the order r [1].

To address this problem, previous experiments proposed the use of Volterra kernel factorization and regularization to reduce computational complexity [13]. However, this approach is insufficient for substantiating the impact of regularization and sparsity on signal reconstruction accuracy, as factorization primarily focuses on optimizing signal parameters while overlooking a detailed analysis of both significant and insignificant components within the studied model.

In the frequency domain, sparsity [6,15] allows retaining only those model parameters that have a significant impact on the accuracy of signal reconstruction $d(t)$. For instance, in the reconstruction of the signal spectrum $D(f)$ sparsity enables the preservation of only the most relevant frequency components $X(f_i)$, which correspond to fundamental harmonics or critical spectral variations. Components that do not significantly affect the signal (as determined by a predefined threshold) can be discarded; however, in some cases, such removal is infeasible without prior analysis of their actual influence.

The study [15] proposes incorporating sparsity and applying regularization based on a quantitative measure of sparsity, specifically the l_0 – norm, which enables the identification of significant model parameters while discarding those with negligible impact on the reconstruction outcome.

To ensure efficient practical implementation and reduce computational complexity, the authors propose the use of the Geman-McClure Function (GMF) [2, 4, 6, 15], which serves as an effective approximation of the l_0-norm. Originally introduced by Donald Geman and Steven McClure in the late 1980s, this mathematical function was designed for data processing as a robust method for mitigating the influence of outliers in general regression and optimization problems [6].

Essentially, this function belongs to the class of robust loss functions, which minimize the impact of extreme values on parameter estimates in experimental models, making it particularly effective in scenarios where data may be «contaminated». The authors specifically designed it for this purpose, as anomalies can significantly distort experimental evaluations when classical approaches, such as quadratic loss functions (l_2-norm), are used.

The key issue with the l_2- norm is its sensitivity to large outliers, as its quadratic nature disproportionately amplifies the influence of extreme values. In such cases, model results can become unstable, leading to reduced accuracy and efficiency in signal reconstruction [6,15]. In cognitive radio systems and nonlinear system modeling tasks, the use of the Geman-McClure Function (GMF) helps maintain a balance between model accuracy and robustness to interference. It reduces the impact of anomalies and enhances the distinction of the useful signal even in environments with low signal-to-noise ratio (SNR). Mathematically, the GMF is expressed as follows [15]:

$$G_\beta(w) = \sum_{i=1}^{N}\left(1 - 1/(1+\beta|w_i|)\right), \qquad (1)$$

where $w = [w_1, w_2, \dots, w_N]$– represents the model parameters; $\beta > 0$ – is a parameter that controls the smoothness and accuracy of the approximation to the l_0 – norm.

The GMF function is utilized in signal optimization tasks within cognitive radio systems due to the following advantages.

1. Improvement in the convergence of adaptive filtering algorithms: By incorporating GMF into algorithms such as l_0-VLMS and l_0-VNLMS [5,9], rapid convergence and model stability are achieved, even in complex radio environments.

2. Sparsity of parameters. GMF enables the selection of only the most significant parameters of the Volterra kernel while discarding insignificant frequency components or higher-order parameters [8].

3. Significant reduction in mean square error (MSE). Compared to traditional approaches, GMF substantially decreases MSE, enhancing the accuracy of signal reconstruction.

For a quantitative analysis of the impact of the parameter β on the dynamics of the Geman-McClure Function, calculations were performed for different β values. These calculations allow for the assessment of how smoothing and truncation of small coefficients change, as well as the determination of optimal values to minimize computational complexity without compromising accuracy. The corresponding results, illustrating the dependence of the GMF on β within the selected range of β = 5, 10, 20, 50, 100, are presented in Fig. 1 and Table 1.

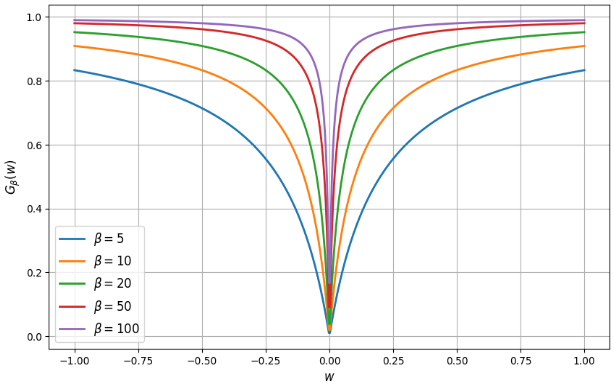

Fig. 1. Approximation of the l_0 – norm using the GMF

Table 1 – **Calculation of the Geman-McClure Function for Different β Values**

w_i	$\beta = 5$	$\beta = 10$	$\beta = 20$	$\beta = 50$	$\beta = 100$
-1,0	0,833	0,909	0,952	0,980	0,990
-0,5	0,714	0,833	0,909	0,952	0,980
-0,25	0,588	0,714	0,833	0,909	0,952
0,0	0,0	0,0	0,0	0,0	0,0
0,25	0,588	0,714	0,833	0,909	0,952
0,5	0,714	0,833	0,909	0,952	0, 980
1,0	0,833	0,909	0,952	0,980	0,990

The analysis of the calculations in Table 2 shows that as β increases, the GMF values for small w_i also increase, indicating enhanced smoothing and effective truncation of insignificant components. This improves the approximation of the l_0 – norm and reduces the mean squared error (MSE), which is a key factor in optimizing model parameters.

In Fig. 1, it is evident that as β increases, the GMF function approaches 1 more rapidly, confirming its ability to emphasize significant parameters while eliminating insignificant coefficients.

Thus, it can be concluded that the application of the Geman-McClure method enables a balance between the accuracy of l_0 - norm approximation and computational complexity. Specifically, for values of β>50 insignificant parameters can be excluded from the model, allowing for optimization focused on essential components. For β=100 the highest accuracy in approximating the l_0 - norm is observed, ensuring that only significant components are retained while reducing computational complexity.

Thus, strict regularization at high values of β may lead to the loss of significant parameters. To provide more flexible control over the truncation process in this method, an additional parameter γ is introduced, which determines the degree of dependence of the GMF on the absolute value of the weight coefficients. This allows for a more precise balance between model sparsity and accuracy, adapting to specific signal reconstruction conditions. Consequently, the classical GMF formula takes the following form:

$$G_\beta(w) = \sum_{i=1}^{N}\left(1 - 1/(1+\beta|w_i|^\gamma)\right), \qquad (2)$$

where γ – is a parameter that controls the smoothing rate of the influence of the parameter w_i on the Geman-McClure Function.

If γ = 1, GMF behaves as a classical approximation of the l_0- norm.

If γ > 1, the influence of small coefficients w_i is further smoothed, making the function even closer to the l_0- norm.

If γ < 1, the regularization effect decreases, allowing even small coefficients w_i to remain significant and not be discarded.

Thus, the choice of the parameter γ determines the balance between suppressing insignificant parameters and preserving weak signals. This is particularly important in telecommunication systems, where even low-amplitude components can carry useful information, and excessive smoothing may lead to the loss of essential signal characteristics (Table 2).

Table 2 – **GMF Dynamics for Different γ Values**

w_i	γ = 0.5, β = 20	γ = 1, β = 20	γ = 2, β = 20
-1, 00	0,952	0,952	0,952
-0, 50	0,934	0,909	0,833
-0, 25	0,909	0,833	0,556
0, 00	0,000	0,000	0,000
0, 25	0,909	0,833	0,556
0, 50	0,934	0,909	0,833
1, 00	0,952	0,952	0,952

The analysis of the values in Table 2 shows that for γ>1, the GMF values for small w_i decrease more rapidly, indicating effective truncation of weak components. In contrast, for γ<1, weak components are suppressed less aggressively, which is beneficial in cases where preserving even insignificant parameters is crucial for model stability.

Fig. 2 illustrates the dependence of the GMF function on the parameter γ at β=20.

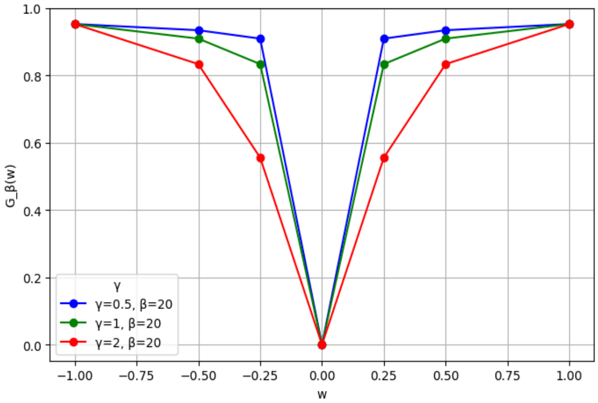

Fig. 2. GMF Function for Different γ Values at β = *20*

As shown in Fig. 2, the GMF function exhibits different behaviors:

γ = 0,5 – (blue curve) – the least suppression of small w_i values, with a gradual change in the function.

γ = 1 (green curve) – a moderate level of smoothing, corresponding to the classical GMF approach.

γ = 2 (червона крива) – (red curve) – the most aggressive truncation of weak components, with GMF values rapidly approaching 0 near $w_i \approx 0$.

The value of γ determines the level of regularization and the balance between truncating insignificant parameters and preserving essential model components. To quantitatively assess this impact, the Mean Squared Error (MSE) was calculated under GMF regularization, allowing for an evaluation of signal reconstruction accuracy depending on the choice of γ:

$$MSE(\beta) = \frac{1}{N}\sum_{i=1}^{N}\left(d_i - \hat{d}_i(\beta)\right)^2, \qquad (3)$$

where d_i and $\hat{d}_i(\beta)$ – respectively, the actual and reconstructed signal values after regularization with the parameter β.

As β increases, the error decreases since insignificant parameters are filtered out. However, excessive growth of β may lead to the loss of important information, resulting in an increase in MSE (Fig. 3, Table 3).

The analysis of the results in Table 3 confirms that the optimal value of β depends on the noise level in the environment.

For high noise levels, it is advisable to use $\beta \geq 50$, whereas for moderate noise levels, the optimal value is $\beta \approx 20$, as it ensures a balance between reconstruction accuracy and computational stability.

As seen in Fig. 3, the MSE value significantly decreases with an increase in β up to 50, after which it stabilizes.

This confirms that larger β values contribute to more accurate signal reconstruction; however, excessively high β values may lead to the loss of important signal components.

Nevertheless, optimizing β alone is insufficient to ensure stable signal recovery in a complex interference-prone environment.

To enhance reconstruction quality, both the selection of an optimal regularization level and the enforcement of parameter orthogonality within the model are crucial. Orthogonality helps reduce correlations between the Volterra series kernel components, thereby improving parameter estimation accuracy and robustness against interference.

Table 3 – **Dynamics of MSE Dependence on *β* at Different Noise Levels**

Testing conditions	β = 5	β = 10	β = 20	β = 50	β = 100
High noise level (SNR = -10 dB)	0,016	0,009	0,0053	0,0027	0,0016
Moderate noise level (SNR = 0 dB)	0,013	0,007	0,0042	0,0025	0,0012
Low noise level (SNR = 10 dB)	0,009	0,005	0,003	0,0015	0,0008

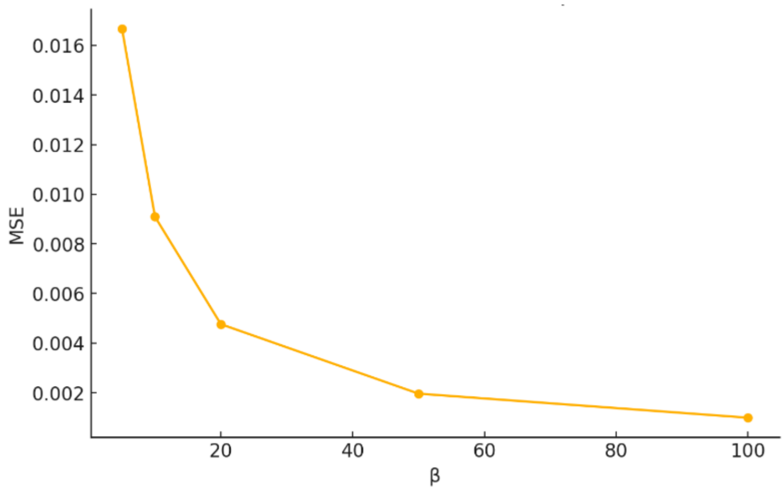

Fig. 3. Dependence of MSE on the parameter *β*

The use of the Geman-McClure function ensures a reduction in computational complexity while maintaining high signal reconstruction accuracy [2, 6, 15]. However, to guarantee computational stability and minimize interactions between the Volterra kernel parameters, their interdependencies must also be considered [3, 8, 13].

One of the effective approaches to maintaining a balance between signal reconstruction accuracy and parameter orthogonality is the application of Lagrange multipliers, which enables the consistent elimination of insignificant model components [7, 12, 14].

Mathematically, the orthogonality condition for a model with Volterra kernel components can be expressed as follows [3, 8, 13]:

$$\langle h_i, h_j \rangle = 0, \text{для всіх } \forall i \neq j, \quad (4)$$

where h_i, h_j – are the components of the Volterra kernel; $\langle .,. \rangle$ – is the inner product operator.

This condition implies that the model parameters in the frequency domain should be uncorrelated to ensure robustness against interference. Orthogonality also minimizes interactions between higher-order components, contributing to more accurate and stable signal reconstruction.

However, for practical implementation, effective parameter optimization methods must be applied to control the interdependence between model components and adaptively manage regularization. One such approach is the use of Lagrange multipliers, which allow for the formalization of constraints on parameter correlation and help achieve the required level of stability in signal reconstruction.

Conclusions and prospects for further research

The study presents a method for spectral reconstruction of signals in complex nonlinear noisy environments based on adaptive Volterra series models utilizing the Geman-McClure function (GMF) and Lagrange multipliers. The proposed approach allows effective consideration of parameter sparsity, reduces computational complexity, and minimizes mutual influence of nonlinear signal components. Experimental modeling has demonstrated that the application of GMF-based regularization significantly reduces computational complexity by effectively focusing on the most significant signal parameters, thereby decreasing the mean-square error (MSE) of signal reconstruction by up to 20,6%. Additionally, the proposed method improved signal robustness by 10–15% across varying signal-to-noise ratio (SNR) levels ranging from -10 dB to 10 dB.

Thus, the developed method provides efficient reconstruction of signals in complex nonlinear environments with significant distortions and noise conditions through an optimized combination of adaptive Volterra models, Geman-McClure regularization, and Lagrange multipliers.

Prospects for further research include extending the adaptive optimization approach to real-world multi-channel cognitive telecommunication networks, taking into account dynamically changing noise environments and nonlinear distortions, as well as developing advanced algorithms for automatic real-time parameter tuning of Volterra models.

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Оцінка впливу розрідженості та регуляризації на точність відновлення сигналу за допомогою функції Джемана-Маклюра

К. Г. Перець, О. М. Комар

Анотація. У статті запропоновано метод спектральної реконструкції сигналів з використанням функції Джемана-МакКлюра та множників Лагранжа для оптимізації параметрів. Метод поєднує підходи адаптивної фільтрації та регуляризації параметрів на основі розрідженості, включаючи функцію Джемана-МакКлюра та множники Лагранжа для забезпечення ортогональності та оптимального вибору параметрів. Запропонований підхід ефективно ідентифікує важливі параметри сигналу, мінімізує взаємні перешкоди між нелінійними компонентами, зменшує обчислювальну складність і покращує точність реконструкції. експериментальне моделювання продемонструвало, що розроблений метод досягає значного зменшення середньої квадратичної помилки (MSE) на 10–15% і підвищує надійність реконструкції сигналу на 10–15% у діапазоні відношення сигнал/шум (SNR) від -10 до 10 дБ, підтверджуючи його ефективність для практичного застосування в когнітивних телекомунікаційних мережах, що працюють в умовах сильного шуму та нелінійних спотворень.

Ключові слова: реконструкція, складні сигнали, функція Джемана-МакКлюра, розрідженість, регуляризація, нелінійність, система, завадостійкість, SNR, ряд Вольтерра, оптимізація, параметри сигналів.