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INCOHERENT DEMODULATION OF TWO SYNCHRONOUS MUTUALLY NON-ORTHOGONAL DIGITAL SIGNALS WITH FREQUENCY SHIFT KEYING

Abstract. The synthesis method of the incoherent demodulation procedure of two synchronous mutually non-orthogonal digital signals with frequency modulation is considered. In the absence of interference this procedure degenerates into the procedure of classical incoherent demodulation of a digital signal with frequency modulation. When the instantaneous power of one of the signals significantly exceeds the instantaneous power of the other, the interference immunity of the latter approaches the immunity of reception in a channel with additive white Gaussian noise without interference. This procedure can be used in the development of modem compensators that ensure repeated use of the frequency resource as well as in the development of promising interference-protected radio communication devices.

Keywords: radio communication, digital signal, incoherent demodulation, frequency modulation.

Introduction

In modern conditions, reception of radio signals is carried out in an a priori uncertain signal-interference situation, which is due to the limitation of the radio frequency resource and the increase in the number and power of structural radiations of various origins. Therefore, the problem of demodulation of signals under the influence of interference was and remains relevant and a large number of works are aimed at its solution [1, 2].

In this article it is proposed to use demodulators of receiving devices synthesized on the basis of mathematical models of compensation procedures [3–6] to improve the immunity of the reception of a useful signal which is observed against the background of such strong interference.

The purpose and main content of the article is to solve the problem of synthesis of a mathematical model of the procedure of incoherent demodulation of mutually non-orthogonal digital signals with frequency shift keying (FSK). To achieve the goal, we will finalize and use the methodology given in [2, 7].

The procedure of incoherent demodulation of two synchronous mutually non-orthogonal digital signal with FSK

Let's write down the values of the useful FSK signal corresponding $(m=2)$ to two possible values of its discrete parameter $r_1 = 0{,}1$. Suppose that the discrete parameter $r_1 = 1$ is modulated and transmitted at the frequency ω_1, and $r_1 = 0$ at ω_2 then the general form of the useful FSK signal will be written as follows:

$$s_1(r_1, \varphi_{1c}, \varphi_{2c}, t) = r_1\left[A_0 \cos(\omega_1 t + \varphi_{1c})\right] + \\ +(1 - r_1)\left[A_0 \cos(\omega_2 t + \varphi_{2c})\right],$$

where φ_{1c}, φ_{2c} – are the initial phases of the useful signal at frequencies ω_1 and ω_2 respectively which are random parameters due to the fluctuation of the spread time in the communication channel; A_0 – is the amplitude of the useful signal, which is constant in frequency.

In turn, the powerful and similar interference FSK also takes two values of a discrete parameter $r_2 = 0{,}1$. For our observation model let the discrete parameter $r_2 = 1$ be transmitted at frequency ω_1 and $r_2 = 0$ at ω_2. Also, will be writing down the expression for a similar FSK interference as follows:

$$s_2(r_2, \varphi_{13}, \varphi_{23}, t) = r_2 A_{21} \cos(\omega_1 t + \varphi_{13}) + \\ +(1 - r_2) A_{22} \cos(\omega_2 t + \varphi_{23}),$$

where φ_{13}, φ_{23} – initial phases of interference at frequencies ω_1 and ω_2; A_{21}, A_{22} – amplitude of interference at frequencies ω_1 and ω_2.

We will assume that the frequency positions and clock points of the signal and interference coincide and the modulation of the interference at each of the two frequency positions is carried out without a phase break. The last condition makes it possible to use coherent (quasi-coherent) interference processing, and we will process the useful signal incoherently (quadrature). We will also assume that additive interference in the form of additive white Gaussian noise acts in the communication channel.

On the Fig. 1 shows the useful signal in vector form $s_1(r_1, \varphi_{1c}, t)$ and interference signal $s_2(r_2, \varphi_{13}, t)$ which rotate in the positive direction with the same angular velocities ω_1, but with different total phase values relative to the real axis.

First of all, in order to present a useful signal $s_1(r_1, \varphi_{1c}, t)$ against the background of powerful and structurally similar interference $s_2(r_2, \varphi_{13}, t)$ it is necessary to obtain the value of the in-phase and quadrature components of the amplitude of the useful signal relative to the interference.

To obtain the in-phase component of the amplitude, we project the vector of the useful signal onto the length A_0 of the signal-interference vector, and to obtain the quadrature component, we draw the normal to the beginning of the vector of the obstacle and get the projection of the vector of the useful signal on it.

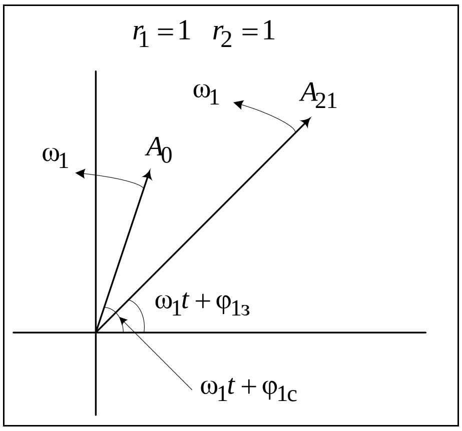

Fig. 1. Vector representation of useful signal and interference

Let us denote the angle between the vector of the useful signal and the interference $\Delta\varphi$ which in turn, will be equal to the difference of the complete phases of these vectors, namely $\Delta\varphi = \varphi_{1c} - \varphi_{13}$ (see Fig. 2).

The value of the in-phase A_1^s and quadrature components A_1^k of the amplitude of the useful signal $s_1(r_1, \varphi_{1c}, t)$ against the background of such interference $s_2(r_2, \varphi_{13}, t)$ is written as follows:

$$A_1^s = A_0 \cos(\varphi_{1c} - \varphi_{13});$$
$$A_1^k = A_0 \sin(\varphi_{1c} - \varphi_{13}).$$

Values A_2^s and A_2^k components of the useful signal $s_1(r_1, \varphi_{2c}, t)$ corresponding to the transmission of a discrete parameter $r_1 = 0$ against the background of such interference $s_2(r_2, \varphi_{23}, t)$, are obtained similarly

$$A_2^s = A_0 \cos(\varphi_{2c} - \varphi_{23}),$$
$$A_2^k = A_0 \sin(\varphi_{2c} - \varphi_{23}).$$

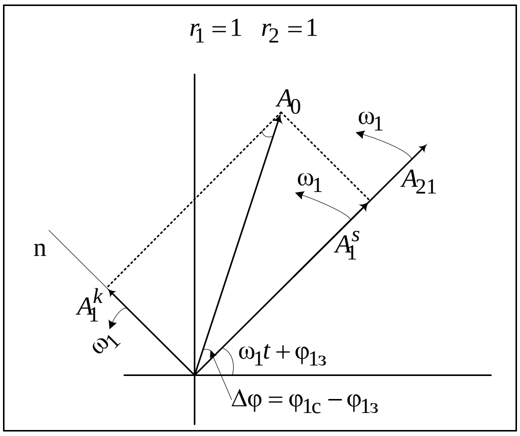

Fig. 2. Angular relations between signal and interference

In Fig. 3 signal interference is represented in vector form $s_2(r_2, \varphi_{13}, t)$, in-phase A_1^s and quadrature A_1^k components of the amplitude of the useful signal $s_1(r_1, \varphi_{1c}, t)$ rotate with the same angular velocity ω_1.

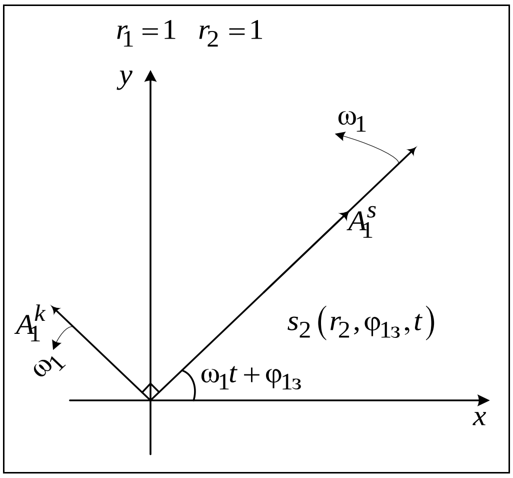

Fig. 3. Display of in-phase and quadrature components of the amplitude of the useful signal and interference

The value of the full phase relative to the axis x of the in-phase component A_1^s coincides with the value of the full phase $\omega_1 t + \varphi_{13}$ of the interference vector, the value of the full phase relative to the axis x of the quadrature component A_1^k - is greater $\pi/2$ than the full phase of the interference vector.

The use of the formula for summing trigonometric functions $\cos(\pi/2 + \alpha) = -\sin\alpha$, write down the full expression of the useful signal $s_1(r_1, \varphi_{1c}, t)$ that is observed against the background of a powerful and structurally similar interference $s_2(r_2, \varphi_{13}, t)$:

$$s_1(r_1, \varphi_{1c}, t) = r_1 \left[A_1^s \cos(\omega_1 t + \varphi_{13}) - A_1^k \sin(\omega_1 t + \varphi_{13}) \right] =$$
$$= r_1 \left[A_0 \cos(\varphi_{1c} - \varphi_{13}) \cos(\omega_1 t + \varphi_{13}) - \right.$$
$$\left. - A_0 \sin(\varphi_{1c} - \varphi_{13}) \sin(\omega_1 t + \varphi_{13}) \right]. \quad (1)$$

Expression for the useful signal $s_1(r_1, \varphi_{2c}, t)$ that is observed against the background of a powerful interference $s_2(r_2, \varphi_{23}, t)$ is obtained in a similar way and has the following form:

$$s_1(r_1, \varphi_{2c}, t) = (1 - r_1) \left[A_2^s \cos(\omega_2 t + \varphi_{23}) - \right.$$
$$\left. - A_2^k \sin(\omega_2 t + \varphi_{23}) \right] = (1 - r_1) \left[A_0 \cos(\varphi_{2c} - \varphi_{23}) \times \right. \quad (2)$$
$$\left. \times \cos(\omega_2 t + \varphi_{23}) - A_0 \sin(\varphi_{2c} - \varphi_{23}) \sin(\omega_2 t + \varphi_{23}) \right].$$

The general model of monitoring the duration of the clock interval $T = t_k - t_{k-1}$ will be presented as:

$$y(t) = s_1(r_1, \varphi_{1c}, \varphi_{2c}, t) + s_2(r_2, \varphi_{13}, \varphi_{23}, t) + n(t) =$$
$$= r_1 \left[A_1^s \cos(\omega_1 t + \varphi_{13}) - A_1^k \sin(\omega_1 t + \varphi_{13}) \right] + \quad (3)$$
$$+ (1 - r_1) \left[A_2^s \cos(\omega_2 t + \varphi_{23}) - A_2^k \sin(\omega_2 t + \varphi_{23}) \right] +$$
$$+ r_2 A_{21} \cos(\omega_1 t + \varphi_{13}) + (1 - r_2) A_{22} \cos(\omega_2 t + \varphi_{23}) + n(t),$$

where $n(t)$ – additive white Gaussian noise.

We will also assume that the states of the discrete parameters r_1 and r_2 are equally likely and mutually independent and the initial phases $\varphi_{1,2,c,3}$ are uniformly distributed over the interval $[0,\ 2\pi]$.

In addition, with the already proposed rejection of the estimation A_0 of the amplitude of the useful signal, the equation is obvious $h_1^2 = h_2^2$. However, the assumption of abandoning the estimation of the amplitude of the useful signal and replacing it with its value $A_0 << A_{21}$, $A_0 << A_{22}$ does not allow to neglect the fact that in the general case $A_{21} \neq A_{22}$, because the case is possible when the difference in amplitudes $|A_{21} - A_{22}|$ commensurate with A_0.

In the future, for the sake of minimizing records and ease of understanding, we will assume that the useful signal is transmitted at a frequency ω_1 corresponding to the value of the discrete parameter $r_1 = 1$.

Received signal is considered as the sum of the useful signal and interference. For our case, the average power will be write as follows [8]:

$$P_{r_1,r_2,\varphi_{1c,3}} = \frac{1}{T}\int_{t_{k-1}}^{t_k}\left[s_1(r_1,\varphi_{1c},t)+s_2(r_2,\varphi_{13},t)\right]^2 \mathrm{d}t. \quad (4)$$

Using the formula of the likelihood function for a signal with a random initial phase [9, 10]

$$\Lambda_r\left[y(t);\ \varphi\right] = \exp\left\{-\frac{P_{r,\varphi}\cdot T}{N_0}\right\}\cdot\exp\left\{\frac{2T}{N_0}\cdot b_r\left[y(t);\ \varphi\right]\right\},$$

where $P_{r,\varphi}$ – the average power of the received signal $s(r,\varphi,t)$; $b_r\left[y(t),\ \varphi\right] = b_r = \frac{1}{T}\int_{t_{k-1}}^{t_k} y(t)\cdot s(r,\varphi,t)\mathrm{d}t$ – scalar product of the input observation $y(t)$ and $s(r,\varphi,t)$, has the following write of the conditional probability functional for the observation (3)

$$\Lambda_{r_1=1,r_2=1}\left[y(t);\ \varphi_{1c},\varphi_{13}\right] = \exp\left\{-\frac{P_{r_1,r_2,\varphi_{1c,3}}}{N_0}\right\}\times$$

$$\times\exp\left\{\frac{2}{N_0}\cdot b_{r_1}\left[y(t),\ \varphi_{1c}\right]\right\}\cdot\exp\left\{\frac{2T}{N_0}\cdot b_{r_2}\left[y(t),\ \varphi_{13}\right]\right\}, \quad (5)$$

where $b_{r_1}\left[y(t),\ \varphi_{1c}\right] = b_{r_1} = \int_{t_{k-1}}^{t_k} y(t)\cdot s_1(r_1,\varphi_{1c},t)\mathrm{d}t$,

$$b_{r_2}\left[y(t),\ \varphi_{13}\right] = b_{r_2} = \int_{t_{k-1}}^{t_k} y(t)\cdot s_2(r_2,\varphi_{13},t)\mathrm{d}t. \quad (6)$$

Substitute (4) and (6) into (5)

$$\Lambda_{r_1=1,r_2=1}\left[y(t);\ \varphi_{1c,3}\right] = \exp\left[\frac{2}{N_0}\int_{t_{k-1}}^{t_k} y(t)\cdot s_1(r_1,\ \varphi_{1c},\ t)\mathrm{d}t + \right.$$

$$+\frac{2}{N_0}\int_{t_{k-1}}^{t_k} y(t)\cdot s_2(r_2,\varphi_{13},t)\mathrm{d}t - \frac{1}{N_0}\int_{t_{k-1}}^{t_k} s_1^2(r_1,\varphi_{1c},t)\mathrm{d}t -$$

$$\left. -\frac{1}{N_0}\int_{t_{k-1}}^{t_k} s_2^2(r_2,\varphi_{13},t)\mathrm{d}t - \frac{2}{N_0}\int_{t_{k-1}}^{t_k} s_1(r_1,\varphi_{1c},t)\cdot s_2(r_2,\varphi_{13},t)\mathrm{d}t\right]. \quad (7)$$

Substitute the expression for the useful signal (1) and the interference value $s_2(r_2,\varphi_{13},t)$ which corresponds to the transmission of the discrete parameter $r_2 = 1$ in (7)

$$\Lambda_{r_1=1,r_2=1}\left[y(t);\ \varphi_{1c};\ \varphi_{13}\right] =$$

$$=\exp\left\{\left[\left[\frac{2}{N_0}\int_{t_{k-1}}^{t_k} y(t)A_0\cos(\varphi_{1c}-\varphi_{13})\cos(\omega_1 t+\varphi_{13})\mathrm{d}t - \right.\right.\right.$$

$$\left. -\frac{2}{N_0}\int_{t_{k-1}}^{t_k} y(t)A_0\sin(\varphi_{1c}-\varphi_{13})\sin(\omega_1 t+\varphi_{13})\mathrm{d}t\right] +$$

$$+\left[\frac{2}{N_0}\int_{t_{k-1}}^{t_k} y(t)A_{21}\cos(\omega_2 t+\varphi_{13})\mathrm{d}t\right] -$$

$$-\left[\frac{1}{N_0}\int_{t_{k-1}}^{t_k} A_0^2\cos^2(\varphi_{1c}-\varphi_{13})\cos^2(\omega_1 t+\varphi_{13})\mathrm{d}t - \right.$$

$$\left. -\frac{1}{N_0}\int_{t_{k-1}}^{t_k} A_0^2\sin^2(\varphi_{1c}-\varphi_{13})\sin^2(\omega_1 t+\varphi_{13})\mathrm{d}t\right] -$$

$$-\frac{1}{N_0}\int_{t_{k-1}}^{t_k} A_{21}^2\cos^2(\omega_2 t+\varphi_{13})\mathrm{d}t -$$

$$-\frac{2}{N_0}\left[\int_{t_{k-1}}^{t_k} A_{21}A_0\cos(\varphi_{1c}-\varphi_{13})\cos^2(\omega_1 t+\varphi_{13})\mathrm{d}t - \right.$$

$$\left.\left. -\int_{t_{k-1}}^{t_k} A_{21}A_0\sin(\varphi_{1c}-\varphi_{13})\sin(\omega_1 t+\varphi_{13})\cos(\omega_1 t+\varphi_{13})\mathrm{d}t\right]\right\}. \quad (8)$$

Write the right-hand part of (8) as follows:

$$-\frac{2}{N_0}\int_{t_{k-1}}^{t_k} A_{21}A_0\cos(\varphi_{1c}-\varphi_{13})\cos^2(\omega_1 t+\varphi_{13})\mathrm{d}t,$$

$$\int_{t_{k-1}}^{t_k}\sin(\omega_1 t+\varphi_{13})\cos(\omega_1 t+\varphi_{13})\mathrm{d}t = 0.$$

Introduce the notation in (8) taking into account (5) and (6):

$$b_{r_1=1}^s = \frac{2}{N_0}\int_{t_{k-1}}^{t_k} y(t)A_0\cos(\varphi_{1c}-\varphi_{13})\cos(\omega_1 t+\varphi_{13})\mathrm{d}t =$$

$$= b_{r_1=1}^{s0}\cos(\varphi_{1c}-\varphi_{13});$$

$$b_{r_1=1}^k = \frac{2}{N_0}\int_{t_{k-1}}^{t_k} y(t)A_0\sin(\varphi_{1c}-\varphi_{13})\sin(\omega_1 t+\varphi_{13})\mathrm{d}t =$$

$$= b_{r_1=1}^{k0}\sin(\varphi_{1c}-\varphi_{13});$$

$$b_{r_2=1} = \frac{2}{N_0}\int_{t_{k-1}}^{t_k} y(t)A_{21}\cos(\omega_1 t+\varphi_{13})\mathrm{d}t;$$

$$h^2_{r_1=1}=\frac{1}{N_0}\int\limits_{t_{k-1}}^{t_k} A_0^2\cos^2\left(\varphi_{1c}-\varphi_{13}\right)\cos^2\left(\omega_1 t+\varphi_{13}\right)dt-$$

$$-\frac{1}{N_0}\int\limits_{t_{k-1}}^{t_k} A_0^2\sin^2\left(\varphi_{1c}-\varphi_{13}\right)\sin^2\left(\omega_1 t+\varphi_{13}\right)dt=$$

$$=h^2_{r_1=1,s}+h^2_{r_1=1,k};$$

$$h^2_{r_2=1}=\frac{1}{N_0}\int\limits_{t_{k-1}}^{t_k} A_{21}^2\cos^2\left(\omega_1 t+\varphi_{13}\right)dt;$$

$$R_{r_1=1,r_2=1}=\frac{1}{N_0}\int\limits_{t_{k-1}}^{t_k} A_{21}A_0\cos(\varphi_{1c}\text{-}\varphi_{13})\cos^2\left(\omega_1 t+\varphi_{13}\right)dt=$$

$$=R^0_{r_1=1,r_2=1}\cos(\varphi_{1c}\text{-}\varphi_{13}). \quad (9)$$

Taking into account notation (9), the conditional probability functional (8) has presented as follows:

$$\Lambda_{r_1=1,r_2=1}\left[y(t);\ \varphi_{1c},\varphi_{13}\right]=$$

$$=\exp\Big[\Big[b^{s0}_{r_1=1}\cos\left(\varphi_{1c}-\varphi_{13}\right)-b^{k0}_{r_1=1}\sin\left(\varphi_{1c}-\varphi_{13}\right)\Big]+b_{r_2=1}-$$

$$-h^2_{r_1=1}-h^2_{r_2=1}-2R^0_{r_1=1,r_2=1}\cos\left(\varphi_{1c}-\varphi_{13}\right)\Big]\Big]. \quad (10)$$

In case of quasi-coherent processing of interference and under the condition $h^2_{r_2=1}>>1$ the initial phase estimation error can be neglected $\varphi_{13}=0$. Average (10) over φ_{1c} the interval $[0,\ 2\pi]$ and get the unconditional probability functional:

$$\Lambda_{r_1=1,r_2=1}\left[y(t);\ \varphi_{1c}\right]=\frac{\exp\left(-h^2_{r_1=1}\right)}{2\pi}\exp\left(b_{r_2=1}-h^2_{r_2=1}\right)\times$$

$$\times\int\limits_0^{2\pi}\exp\Big[\left(b^{s0}_{r_1=1}\cos\varphi_{1c}-b^{k0}_{r_1=1}\sin\varphi_{1c}\right)-$$

$$-2R^0_{r_1=1,r_2=1}\cos\varphi_{1c}\Big]d\varphi_{1c}. \quad (11)$$

In the following, we will ignore the factor $\exp\left(-h^2_{r_1=1}\right)/2\pi$ which does not depend on the values of the discrete parameter r_1 and r_2. Taking into account the previously introduced notations (9) we write:

$$R^0_{r_1=1,r_2=1}=\frac{h^2_{r_2=1}A_0}{A_{21}}=\alpha_{21}h^2_{r_2=1}. \quad (12)$$

Introduce the notation that facilitates the integration procedure (11):

$$B_1=\sqrt{\left(b^{s0}_{r_1=1}\right)^2+\left(b^{k0}_{r_1=1}\right)^2};\quad \psi_1=\operatorname{arctg}\frac{b^{k0}_{r_1=1}}{b^{s0}_{r_1=1}}, \quad (13)$$

$$b^{s0}_{r_1=1}=B_1\cos\psi_1;\qquad b^{k0}_{r_1=1}=B_1\sin\psi_1. \quad (14)$$

Taking into account (12)–(14), (11) will be rewritten in the following form:

$$\Lambda_{r_1=1,r_2=1}\left[y(t);\varphi_{1c}\right]=\exp\left(b_{r_2=1}-h^2_{r_2=1}\right)\times$$

$$\times\int\limits_0^{2\pi}\exp\Big\{B_1\left(\cos\psi_1\cos\varphi_{1c}-\sin\psi_1\sin\varphi_{1c}\right)-$$

$$-2\alpha_{21}h^2_{r_2=1}\cos\varphi_{1c}\Big\}d\varphi_{1c}. \quad (15)$$

Values for unconditional probability functionals for the following options for transmitting discrete parameters: $r_1=1$ $r_2=0$; $r_1=0$ $r_2=1$; $r_1=0$ $r_2=0$, useful signal and interference obtained in a similar way:

$$\Lambda_{r_1=1,r_2=0}\left[y(t);\ \varphi_{1c}\right]=\exp\left(b_{r_2=0}-h^2_{r_2=0}\right)\times$$

$$\times\int\limits_0^{2\pi}\exp\left\{B_1\left(\cos\psi_1\cos\varphi_{1c}-\sin\psi_1\sin\varphi_{1c}\right)\right\}d\varphi_{1c};$$

$$\Lambda_{r_1=0,r_2=1}\left[y(t);\ \varphi_{2c}\right]=\exp\left(b_{r_2=1}-h^2_{r_2=1}\right)\times$$

$$\times\int\limits_0^{2\pi}\exp\left\{B_2\left(\cos\psi_2\cos\varphi_{2c}-\sin\psi_2\sin\varphi_{2c}\right)\right\}d\varphi_{2c};$$

$$\Lambda_{r_1=0,r_2=0}\left[y(t);\ \varphi_{2c}\right]=\exp\left(b_{r_2=0}-h^2_{r_2=0}\right)\times$$

$$\times\int\limits_0^{2\pi}\exp\Big\{B_2\left(\cos\psi_2\cos\varphi_{2c}-\sin\psi_2\sin\varphi_{2c}\right)-$$

$$-2\alpha_{22}h^2_{r_2=0}\cos\varphi_{2c}\Big\}d\varphi_{2c}. \quad (16)$$

Let us write the unconditional likelihood functional in the general sense, taking into account (15) and (16)

$$\Lambda_{r_1,r_2}\left[y(t);\varphi_{1c},\varphi_{2c}\right]=$$

$$=\exp\left\{r_2\left(b_{r_2=1}-h^2_{r_2=1}\right)+(1-r_2)\left(b_{r_2=0}-h^2_{r_2=0}\right)\right\}\times$$

$$\times\int\limits_0^{2\pi}\int\limits_0^{2\pi}\exp\Big\{r_1B_1\left(\cos\psi_1\cos\varphi_{1c}-\sin\psi_1\sin\varphi_{1c}\right)+$$

$$+(1-r_1)B_2\left(\cos\psi_2\cos\varphi_{2c}-\sin\psi_2\sin\varphi_{2c}\right)-$$

$$-2r_1r_2\alpha_{21}h^2_{r_2=1}\cos\varphi_{1c}-2(1-r_1)(1-r_2)\times$$

$$\times\alpha_{22}h^2_{r_2=0}\cos\varphi_{2c}\Big\}d\varphi_{1c}d\varphi_{2c}.$$

For an equally probable discrete parameter of the useful signal the decision rule has the form:

$$r_1^*=\operatorname{rect}\Big[\Lambda_{r_1=1,r_2=0}\left[y(t);\varphi_{1c}\right]+\Lambda_{r_1=1,r_2=1}\left[y(t);\varphi_{1c}\right]-$$

$$-\Lambda_{r_1=0,r_2=1}\left[y(t);\varphi_{2c}\right]-\Lambda_{r_1=0,r_2=0}\left[y(t);\varphi_{2c}\right]\Big], \quad (17)$$

where $\operatorname{rect}(x\geq 0)=1$; $\operatorname{rect}(x<0)=0$ – decisive function.

Taking into account (15) and (16), the decision rule (17) for the equally probable discrete parameter of the useful signal:

$$r_1^*=\operatorname{rect}\Big[\exp\left(b_{r_2=0}-h^2_{r_2=0}\right)\times$$

$$\times\int\limits_0^{2\pi}\exp\left\{B_1\left(\cos\psi_1\cos\varphi_{1c}-\sin\psi_1\sin\varphi_{1c}\right)\right\}d\varphi_{1c}+$$

$$+\exp\left(b_{r_2=1}-h^2_{r_2=1}\right)\int\limits_0^{2\pi}\exp\Big\{B_1\left(\cos\psi_1\cos\varphi_{1c}-\sin\psi_1\sin\varphi_{1c}\right)-$$

$$-2\alpha_{21}h^2_{r_2=1}\cos\varphi_{1c}\Big\}d\varphi_{1c}-\exp\left(b_{r_2=1}-h^2_{r_2=1}\right)\times$$

$$\times\int\limits_0^{2\pi}\exp\left\{B_2\left(\cos\psi_2\cos\varphi_{2c}-\sin\psi_2\sin\varphi_{2c}\right)\right\}d\varphi_{2c}-$$

$$-\exp\left(b_{r_2=0}-h^2_{r_2=0}\right)\int\limits_0^{2\pi}\exp\left\{B_2\left(\cos\psi_2\cos\varphi_{2c}-\sin\psi_2\sin\varphi_{2c}\right)-\right.$$
$$\left.-2\alpha_{22}h^2_{r_2=0}\cos\varphi_{2c}\right\}d\varphi_{2c}\Big]. \qquad (18)$$

Equivalent and simplifying transformations of the procedure for incoherent demodulation of a useful signal with a FSK observed against the background of strong similar interference

Change the variable of integration in (18) to $\xi=\psi_{1,2}+\varphi_{1c,2c}$ after which we get from (18):

$$r_1^*=\mathrm{rect}\left\{\exp\left(b_{r_2=0}-h^2_{r_2=0}\right)\left[\int\limits_0^{2\pi}\exp\left(B_1\cos\xi\right)d\xi-\right.\right.$$
$$\left.-\int\limits_0^{2\pi}\exp\left[\left(b^{s0}_{r_1=0}-2\alpha_{22}h^2_{r_2=0}\right)\cos\varphi_{2c}-b^{k0}_{r_1=0}\sin\varphi_{2c}\right]d\varphi_{2c}\right]-$$
$$-\exp\left(b_{r_2=1}-h^2_{r_2=1}\right)\left[\int\limits_0^{2\pi}\exp\left(B_2\cos\xi\right)d\xi-\right.$$
$$\left.\left.-\int\limits_0^{2\pi}\exp\left[\left(b^{s0}_{r_1=1}-2\alpha_{21}h^2_{r_2=1}\right)\cos\varphi_{1c}-b^{k0}_{r_1=1}\sin\varphi_{1c}\right]d\varphi_{1c}\right]\right\}. \qquad (19)$$

Introduce the notation similarly to (13), (14):

$$b^{s0}_{r_1=1,e}=b^{s0}_{r_1=1}-2\alpha_{21}h^2_{r_2=1};$$
$$B_{1e}=\sqrt{\left(b^{s0}_{r_1=1,e}\right)^2+\left(b^{k0}_{r_1=1}\right)^2};$$
$$b^{s0}_{r_1=0,e}=b^{s0}_{r_1=0}-2\alpha_{22}h^2_{r_2=0};$$
$$B_{2e}=\sqrt{\left(b^{s0}_{r_1=0,e}\right)^2+\left(b^{k0}_{r_1=0}\right)^2};$$
$$\eta_1=\mathrm{arctg}\frac{b^{k0}_{r_1=1}}{b^{s0}_{r_1=1,e}};\qquad \eta_2=\mathrm{arctg}\frac{b^{k0}_{r_1=0}}{b^{s0}_{r_1=0,e}},$$

whence follows

$$b^{s0}_{r_1=1,e}=B_{1e}\cos\eta_1;\qquad b^{k0}_{r_1=1}=B_{1e}\sin\eta_1;$$
$$b^{s0}_{r_1=0,e}=B_{2e}\cos\eta_2;\qquad b^{k0}_{r_1=0}=B_{2e}\sin\eta_2.$$

The decision rule (19) will have the following form:

$$r_1^*=\mathrm{rect}\Big[\exp\left(b_{r_2=0}-h^2_{r_2=0}\right)\left(\int\limits_0^{2\pi}\exp\left(B_1\cos\xi\right)d\xi-\right.$$
$$\left.-\int\limits_0^{2\pi}\exp\left\{B_{2e}\left(\cos\eta_2\cos\varphi_{2c}-\sin\eta_2\sin\varphi_{2c}\right)\right\}d\varphi_{2c}\right)-$$
$$-\exp\left(b_{r_2=1}-h^2_{r_2=1}\right)\left(\int\limits_0^{2\pi}\exp\left(B_2\cos\xi\right)d\xi-\right.$$
$$\left.-\int\limits_0^{2\pi}\exp\left\{B_{1e}\left(\cos\eta_1\cos\varphi_{1c}-\sin\eta_1\sin\varphi_{1c}\right)\right\}d\varphi_{1c}\right)\Big]. \qquad (20)$$

After replacing the integration variables in the second and fourth integrals the decision rule (20) into $\eta_{1,2}+\varphi_{1c,2c}$ is obtained [11]:

$$r_1^*=\mathrm{rect}\Big[\exp\left(b_{r_2=0}-h^2_{r_2=0}\right)\left[I_0\left(B_1\right)-I_0\left(B_{2e}\right)\right]+$$
$$+\exp\left(b_{r_2=1}-h^2_{r_2=1}\right)\left[I_0\left(B_{1e}\right)-I_0\left(B_2\right)\right]\Big], \qquad (21)$$

where $I_0(\cdots)$ – zero-order Bessel function.

We can see what

$$h^2_{r_2=1,r_2=0}>>1,\ h^2_{r_2=1,r_2=0}>>h^2_{r_1=1,r_1=0}$$
$$\exp\left(b_{r_2=1}-h^2_{r_2=1}\right)\Big|_{r_2=1}>>1;$$
$$\exp\left(b_{r_2=0}-h^2_{r_2=0}\right)\Big|_{r_2=1}\cong 0;$$
$$\exp\left(b_{r_2=1}-h^2_{r_2=1}\right)\Big|_{r_2=0}\cong 0;$$
$$\exp\left(b_{r_2=0}-h^2_{r_2=0}\right)\Big|_{r_2=0}>>1.$$

Then the decision rule (21) can be replaced by an asymptotically equivalent one:

$$r_1^*=\mathrm{rect}\Big[\left(b_{r_2=0}-h^2_{r_2=0}\right)\left[I_0\left(B_1\right)-I_0\left(B_{2e}\right)\right]\Big]+$$
$$+\mathrm{rect}\Big[\left(b_{r_2=1}-h^2_{r_2=1}\right)\left[I_0\left(B_{1e}\right)-I_0\left(B_2\right)\right]\Big]. \qquad (22)$$

The approximate decision-making procedure (22) r_1^* is two-stage where at the first stage a decision is made on which of the frequencies the interference is emitted $s_2\left(r_2,\varphi_{13},\varphi_{23},t\right)$.

If the interference energy significantly exceeds the useful signal energy $s_1\left(r_1,\varphi_{1c},\varphi_{2c},t\right)$ then the value $\mathrm{rect}\left(b_{r_2=1,r_2=0}-h^2_{r_2=1,r_2=0}\right)$ in (22) due to the small influence of errors on the overall decision r_1^* should be replaced by one decision-making rule for coherent (quasi-coherent) reception of the FSK signal [9,10]:

$$r_2^*=\mathrm{rect}\left(b_{r_2=1}-b_{r_2=0}\right).$$

As a result (22) will become:

$$r_1^*=\mathrm{rect}\Big[\mathrm{rect}\left(b_{r_2=0}-b_{r_2=1}\right)\left(B_1-B_{2e}\right)+$$
$$+\mathrm{rect}\left(b_{r_2=1}-b_{r_2=0}\right)\left(B_{1e}-B_2\right)\Big], \qquad (23)$$

where it is taken into account that the function $I_0(x)$ is monotonic at $x>0$.

In the absence of interference $s_2\left(r_2,\varphi_{13},\varphi_{23},t\right)$ that is when $h^2_{r_2=1,r_2=0}=0$ the decision rule (21)–(23) degenerates into the classical rules of incoherent reception of the FSK signal.

We will evaluate the interference resistance of the received decision rule for the asymptotic case of an unlimited increase in the average interference power $s_2\left(r_2,\varphi_{13},\varphi_{23},t\right)$.

Assuming that the estimation errors of continuous $\left(A_{21},A_{22},\varphi_{1,23}\right)$ parameters and the discrete disturbance parameter will approach zero, we obtain the following expressions for $b^s_{r_1=1}$, $b^s_{r_1=0}$:

$$b^s_{r_1=1}\Big|_{r_2=1} = \frac{2}{N_0}\int_{t_{k-1}}^{t_k}\Big[\Big(A_1^s + A_{21}\Big)\cos\big(\omega_1 t + \varphi_{13}\big) + n(t)\Big]\times$$

$$\times A_1^s\cos\big(\omega_1 t + \varphi_{13}\big)\mathrm{d}t = 2h^2_{r_1=1,s} + 2\alpha_{21}h^2_{r_2=1}\cos\big(\varphi_{1c} - \varphi_{13}\big) + n_{\text{ш}1};$$

$$b^s_{r_1=0}\Big|_{r_2=0} = \frac{2}{N_0}\int_{t_{k-1}}^{t_k}\Big[\Big(A_2^s + A_{22}\Big)\cos\big(\omega_2 t + \varphi_{23}\big) + n(t)\Big]\times$$

$$A_2^s\cos\big(\omega_2 t + \varphi_{23}\big)\mathrm{d}t = 2h^2_{r_1=0,s} +$$

$$+2a_{22}h^2_{r_2=0}\cos\big(\varphi_{2c} - \varphi_{23}\big) + n_{\text{ш}2}\,. \qquad (24)$$

It can be seen from comparison (23) and (24) that under the above-mentioned assumptions (about the absence of errors in the estimation of interference parameters), the components in the correlation integrals $b^s_{r_1=1,r_2=0}$ generated by its presence are fully compensated.

The noise components $n_{\text{ш}1}$ and $n_{\text{ш}2}$ remain the same as for the classical case of incoherent reception of the FSK signal.

Thus, the potential interference immunity of the incoherent demodulation algorithm (21) of the FSK signal, provided that the average power of such FSK interference significantly exceeds the power of the useful signal and there are no errors in the estimation of its parameters, is the same as in its absence.

Conclusions

This procedure of incoherent demodulation of mutually non-orthogonal digital signals with frequency modulation has a number of advantages:

– provided that the average power of such interference significantly exceeds the power of the useful FSK signal and there are no errors in the estimation of the interference parameters, the potential (limit) immunity of the incoherent demodulation procedure (23) is the same as in the absence of interference;

– provided that the average power of such interference significantly exceeds the power of the useful FSK signal and there are no errors in the estimation of the interference parameters, the potential (limit) immunity of the incoherent demodulation procedure (23) is the same as in the absence of interference;

– this mathematical model of the procedure can be used in the implementation of frequency resource reuse programs and in the development of promising interference-protected radio communication tools.

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Некогерентний прийом двох синхронних взаємно заважаючих цифрових сигналів з частотною маніпуляцією

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Анотація. Розглянуто синтез процедури некогерентної демодуляції двох синхронних взаємно неортогональних цифрових сигналів з частотною маніпуляцією. За відсутності завади дана процедура вироджується в процедуру класичної некогерентної демодуляції цифрового сигналу з частотною маніпуляцією. Коли миттєва потужність одного з сигналів значно перевищує миттєву потужність іншого, завадостійкість останнього наближається до завадостійкості прийому в каналі з адитивним білим гаусовим шумом без завади. Ця процедура може бути використана при розробці модемних компенсаторів, що забезпечують повторне використання радіочастотного ресурсу, а також при розробці перспективних завадозахищених пристроїв радіозв'язку.

Ключові слова: радіозв'язок, цифровий сигнал, некогерентна демодуляція, частотна модуляція.