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MULTIPLICATIVE APPROXIMATION METHOD OF FUNCTIONAL DEPENDENCIES BY LINE SEGMENTS

Abstract. The article is devoted to the approximation problems of functional dependencies during conversions performed in intelligent measurement devices. Non-linearities are essential parts of most control processes and systems. When using a nonlinear transmitter with a conversion function in measurement information systems used in various fields, it is necessary to perform nonlinear functional conversion operations on numbers in microprocessors/microcontrollers during direct and indirect measurements. For this purpose, various approximation methods are used. The purpose of the approximation is to describe nonlinear functions in a simpler, more convenient way for utilization and calculations, with an insignificantly small loss of accuracy. Existent methods for linearization, although some of them are effective, can be burdensome for implementation in microprocessor-based systems. Here, one of the proposed methods for the approximation of nonlinear functional dependencies by line segments is proposed. In this method, the range of the argument changes in the function is divided into line segments, and the parts of the coordinate system bisector, remaining within the line segments of the function, is swapped to perform approximation. Having involved few simple mathematical operations, the proposed method can be implemented efficiently in microprocessors/microcontrollers to perform approximations in measurement systems.

Keywords: Linearization, Non-linear systems, approximation methods, multiplicative approximation method,

Introduction

In modern times, the development of measurement techniques is aimed at increasing their functional capabilities, improving metrological characteristics, and performing some intelligent functions that require mental activity.

Problem statement. When designing any measurement device, first of all, attention is paid to the requirements for its metrological characteristics.

Such metrological characteristics contain:

- accuracy,
- sensitivity,
- rigidity,
- obstacle resistance,
- dynamic range,
- reliability, etc.

In addition, in many cases, high requirements are expected for the linearity of the conversion characteristics (function) of measurement devices intended for use in information and measurement systems.

This is explained by the fact that the nonlinearity of the conversion characteristics of measurement devices is a cause of additional problems:

- presence of additional errors during the subsequent linear conversion of the measurement information;
- complexity of data processing algorithm;
- the complexity of presenting the input quantity value in its own measurement unit, etc.

On the other hand, since it is considered more relevant to present the measurement results in the unit of the input quantity, the measurement device must also perform the scaling operation.

In many practical cases, the conversion characteristics of measurement devices are nonlinear. There can be several reasons for this:

- nonlinearities due to the physical principles underlying the construction of measurement devices;
- non-linearity owing to imperfection in the design or technology of measurement device manufacturing process;
- nonlinearities depending on the nature of the measurement methods;
- nonlinearities arising from the aggregation of several of these reasons.

Nonlinearity of the transformation characteristic of a measurement device means its deviation from the linear characteristic of the real transformation characteristic.

In the direction of the effective organization of the measurement process and the implementation of some issues that need to be addressed by the program on the structure and programs developed by humans, as a result, led to the emergence of intelligent measurement methods and devices.

Purpose. The aim of the work is to formulate a method through which a linearization process can be implemented with high accuracy and less computational power to enable it suitable in microprocessor/ microcontroller applications.

Analysis of recent research and publications

Intelligent measurement devices are measuring devices in which hardware and software components operate together.

A large number of studies on the transformation process have been carried out in the past, including:

- near-optimal nonlinear regression [1],
- nonlinearly Preconditioned FETI Solver for Substructured Formulation [2],
- using piecewise linear functions [3-7].

Piecewise approximation techniques have a significant role in many fields of engineering and mathematics [8-10].

Linear programming techniques are also used in optimization problems

In general, the main methods of linearization of the conversion characteristics of measuring instruments are:

- - technological methods;
- - construction methods;
- - structural methods;
- - structural-algorithmic methods;
- - Algorithmic methods.

Technological methods include the preparation of individual elements and junctions of measuring instruments from special materials, stabilization of their mode of operation and conditions, etc. is carried out with technological limitations.

Construction methods are performed by making appropriate changes in the design of the measuring instrument or its constituent elements.

In this case, determining and implementing the optimal design is not an easy task. On the other hand, both methods require an individual approach to each measurement tool.

In the modern era, when microprocessors and microcontrollers are widespread and inexpensive, there are more opportunities for structural, structural-algorithmic and algorithmic methods.

When using structural methods, it is necessary to include additional functional blocks in the structure of the measuring instrument created to improve its metrological characteristics, as well as to linearize the conversion characteristics, and to organize the information conversion channel accordingly. Examples of structural methods are compensation methods.

In addition to the measures of the structural method in the structural-algorithmic method, certain control, calculation, etc. are also performed by the microprocessor computing devices included in the measuring instrument. algorithms are executed. This method includes additional equation methods and iteration methods.

Algorithmic methods do not involve the inclusion of any additional functional block or element in the structure of the measuring instrument and are based only on the processing of information from the measuring instrument by certain algorithms by a microprocessor computing device within the system [11-20].

Of the linearization methods of the conversion characteristics of measuring instruments, only algorithmic linearization methods meet the following modern requirements [19]:

- can be applied to linearization of various forms of nonlinear characteristics;
- the effectiveness of the method does not depend on the degree of deviation of the nonlinear characteristics of the measuring instrument from the linear characteristics;
- can be used for measuring a wide range of electrical quantities, especially non-electrical quantities;
- ensuring the accuracy of the given line with minimal costs;
- no need to use high-precision sample measurements of electrical and non-electrical quantities;
- there is no need to separate the measured quantities from their input in order to linearize the conversion characteristics of the measuring instruments;
- structure, principle of construction, construction, production technology, etc. of functional blocks operating in the system.

Solve the problem of linearization without interference, using only algorithms for processing measurement information in modern microprocessor computing devices.

Approximation methods of functional dependencies with corrective adjustments are based on the methodology of solving problems of linearization and correction of integral errors.

The range of the argument changes in the function is divided into line segments, and the parts of the coordinate system bisector, remaining within the line segments of the function, is swapped to perform approximation to $f(x)$.

Multiplicative Linearization

Depending on the mathematical operations used to perform the shifting, these methods are called:

- additive (A),
- multiplicative (M),
- combinatorial (K),
- additive-multiplicative (A-M)

approximation methods.

The article is devoted to the method of multiplicative approximation.

In the multiplicative approximation method of nonlinear functional dependencies, the function $f(x)$ is replaced by straight line segments approximating this function graph by rotating the ordinates of the linear

$$y = x$$

coordinate system bisector parts with the correcting coefficients.

Fig. 1 illustrates the i-th parts of the function $\varphi(x)$, which approximates the nonlinear function $f(x)$ by a multiplicative method.

Here A_1 is a multiplicative coefficient that rotates the i-th part of the bisector to the function f(x) and

$$\varepsilon_{i-1} = \varepsilon_i$$

are deviations of the functions $\varphi_i(x)$ and $f(x)$ on the boundaries of x_{i-1} and x_i.

The function $\varphi_i(x)$ that approximates the nonlinear function $f(x)$ in the i-th section $\varphi_i(x)$ is expressed with the following equation:

$$\varphi_i(x) = M_i \cdot x; \quad (1)$$

$$\text{if } f(x) > x \text{, then } M_i > 1;$$

$$\text{if } (x) < x \text{ then } M_i < 1.$$

The boundary values of these parts x_i are defined with the multiplicative coefficients M_i within these parts, and boundary deviation values ε_i having constant values and opposite signs.

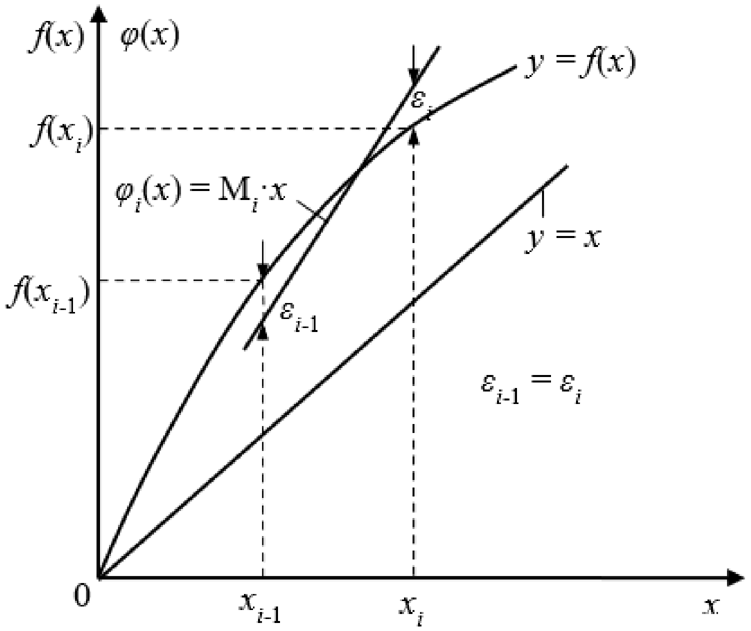

Fig. 1. Multiplicative approximation method by line segments

$$x_i = x_{i-1}[\, f(x_i) \pm \varepsilon\,]/[\, f(x_{i-1})] \mp \varepsilon; \quad (2)$$

$$M_i = [\, f(x_{i-1}) + f(x_i)] \;\; / \;\; (x_{i-1} + x_i); \quad (3)$$

$$\varepsilon_i = \pm 0.5 \cdot [M\Delta x_i - \Delta f(x_i)\,]; \quad (4)$$

where:

$$\Delta f(x_i) = f(x_i) - f(x_{i-1}); \Delta x_i = x_i - x_{i-1}; \quad (5)$$

$f(x_{i-1})$ and $f(x_i)$ – i-th part lower and upper boundaries $f(x_i) < 0$ (upward convexity),

It is obvious that the relative positioning of function $f(x)$ with the y=x, does not make changes in their corresponding expressions. Analysis of equation (3) shows

that if $\Delta f(x_i) = \Delta x_i$, then $\varepsilon_i \neq 0$.

Therefore, the following equation can be constructed:

$$\varepsilon_i = \pm 0.5 \;\cdot [\; M_i \;\cdot \Delta x_i - \Delta x_i\,) \,]= $$
$$= \pm 0.5 \;\cdot \Delta x_i \cdot (M_i - 1). \quad (6)$$

The main propery of this method is that, in the line segments with the multiplicative coefficient of $M_i \neq 1$, linear characteristics is approximated to $f(x)$.

However, in the line segments with the condition of $\Delta f(x_i) = M_i \cdot \Delta x_i$, the deviation could be zero, $\varepsilon_i = 0$. Therefore, the error of this method does not exist in line segments with $\Delta f(x_i) = \Delta x_i$, but can receive minimal values in line segments with $\Delta f(x_i) \neq \Delta x_i$.

Fig. 2 illustrates realization of multiplicative linearization method by line segments.

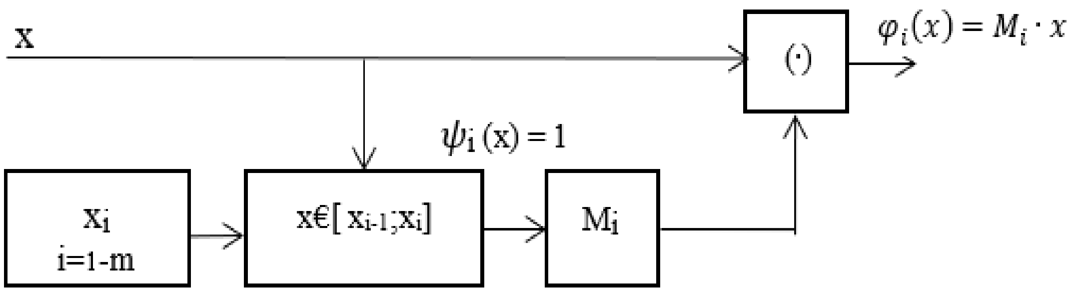

Fig. 2. Realization scheme of multiplicative method with line segments

Conclusions

A method through which a linearization process can be implemented with high accuracy and less computational power to enable it suitable in microprocessor/microcontroller applications is proposed. In the method, the boundary values within dynamic range of argument and constant multiplicative coefficent values within these line segments are stored in memory. The current argument boundary value x and its inclusion interval are defined to determine multiplicative coefficient, after which is multiplied by argument x. As a result, the value of $\varphi_i(x) = M_i \cdot x$ is calculated within line segments of argument.

To sum up, in the proposed method non-linear functional dependencies are just approximated using line segments few comparison and one multiplication operations.

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Метод мультиплікативної апроксимації функціональних залежностей відрізками лінії

Худавердієва Махаббат

Анотація. Стаття присвячена завданням апроксимації функціональних залежностей при перетвореннях, що виконуються в інтелектуальних вимірювальних пристроях. Нелінійності є невід'ємною частиною більшості процесів та систем управління. При використанні нелінійного перетворювача з функцією перетворення у вимірювальних інформаційних системах, що застосовуються в різних областях, необхідно виконувати операції нелінійного функціонального перетворення над числами мікропроцесорів/мікроконтролерів при прямих і непрямих вимірюваннях. Для цього використовуються різні методи апроксимації. Мета апроксимації полягає в тому, щоб описати нелінійні функції більш простим, зручним для використання та розрахунків способом із мізерно малою втратою точності. Існуючі методи лінеаризації, хоча деякі з них ефективні, можуть бути обтяжливими для реалізації в мікропроцесорних системах. Тут пропонується один із запропонованих методів апроксимації нелінійних функціональних залежностей відрізками прямих. У цьому методі діапазон зміни аргументу функції розбивається на відрізки прямих, а частини бісектриси системи координат, що залишилися в межах відрізків прямих функції, переставляють місцями для виконання апроксимації. Задіявши кілька простих математичних операцій, запропонований метод може бути ефективно реалізований у мікропроцесорах/мікроконтролерах для виконання апроксимацій у вимірювальних системах.

Ключові слова: лінеаризація, нелінійні системи, методи апроксимації, метод мультиплікативної апроксимації.