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MANAGEMENT IN HIGH-DIMENSIONAL MARKOV SYSTEMS

Abstract. A problem of analyzing Markov systems along with a large number of states has been considered. The conventional computational procedure for obtaining analytical ratios for calculating the distribution of system states is based on the use of a system of Kolmogorov differential equations. The system of linear algebraic equations being formed later can be easily solved numerically. However, the complexity of obtaining an analytical solution increases rapidly with the increase in the problem dimension. In this regard, the purpose is to develop an effective method for studying Markov systems, the computational procedure of which ensures the possibility of obtaining solutions for high-dimensional models. The method is based on the decomposition of states graph and system transitions. The obtained analytical expressions allow to set and solve the problem of rational resource distribution for changing the values of the system parameters to increase its efficiency. The method ensures the possibility of solving management problems in Markov systems along with a large number of possible states. An example of method application has been considered.

Keywords: Markov system, calculation of distribution of states, decomposition computational pattern, states probabilities management.

Introduction

The procedures of functioning of a significant part of modern complex technical, environmental, military and other systems can be described by a mathematical model of the same type in many important respects. Firstly, such systems can be in one of the many possible states at any specific time. Secondly, the system shifts from one possible state to another under the impact of one or more random events (requests, service requirements, etc). Thirdly, the service duration of each requirement is a continuous random variable, the distribution density of which is assumed to be well known (or it can be determined by the results of statistical tests). Fourthly, the system is manageable in the sense that it has a limited consumable resource that can be used to change certain given system parameters to increase its efficiency. The similarity of various systems under these distinctive features predetermines the possibility of using the same type of mathematical models to analyze them.

On the other hand, the observed differences in the functioning procedures for specific systems are primarily due to differences in their individual specifications. The most important role here is played by various mechanisms inherent in each system for the formation of random variables of the related parameters. Herewith, to the utmost, the nature of the mathematical description of the procedure of functioning of such systems is determined by the type of probability-theoretic models of the dynamics of system transitions from one possible state to another. The complexity level of solving the problems of analyzing such systems and managing them is determined by the type of dynamics models of these systems. Let's briefly analyze the traditional approaches.

Analyzing known results

The simplest model of system dynamics occurs unless all procedures within the system are Markov ones. In this event, a set of differential equations of A.N. Kolmogorov [1, 2] is used to analyze the system, obtained as follows [3]. For arbitrary pair of possible states of the system *(j, k)*, an indicator *R(j,k)* is entered, which shall be equal to *1* if a transition from the *j-th* state to the *k-th* one is possible in one step. Otherwise, *R(j,k)=0*. Now, for an arbitrary state *k* of the system, a set of Z_{k+} *of its states is entered,* from which a transition to state *k* is possible in one step, that is:

$$Z_k^+ = \{ j : R(j,k) = 1\},$$

and also, a set Z_k^- of such states in which a transition from state *k* is possible in one step, that is:

$$Z_k^- = \{ j : R(k,j) = 1\},$$

Next, $P_k(t)$ is entered, which is a function specifying the probability that the system at time period *t* shall be in the state *k*, $k \in Z$, *Z* is the set of possible states of the system, $Z = \{0;1;2;...;n\}$. The A.N. Kolmogorov's differential equations system in regard of the functions P_k, $k \in Z$, is as follows

$$\frac{dp_k(t)}{dt} = \sum_{j \in Z_k^+} \lambda_{jk} P_j(t) - P_k(t) \sum_{j \in Z_k^-} \lambda_{kj}, k \in Z. \quad (1)$$

Here λ_{jk} is the rate of transition of the system from state *j* to state *k*, the parameter of the distribution density of the random duration of the system's stay in state *j* prior to the transition to state *k*,

$$\phi_{jk}(t) = \lambda_{jk} e^{-\lambda_{jk} t}, t > 0.$$

To solve the differential equations system (1), one should use the Laplace transformation, which converts differential equations into algebraic ones. As it is known [3, 4], the Laplace transformation of the function *u(t)* is a function

$$L(u(t)) = \int_0^\infty u(t) e^{-st} dt = F(s). \quad (2)$$

Herewith the following important property of the Laplace transformation is used:

$$L(u'(t)) = \int_0^\infty u'(t) e^{-st} dt = u(t) e^{-st} \Big|_0^\infty +$$

$$+s\int_0^\infty u(t)e^{-st}dt = sL(u(t)) - u(0). \qquad (3)$$

By converting ratios (1) according to Laplace, we get

$$s\pi_k(s) - P_k(0) = \sum_{j\in Z_k^+} \lambda_{jk}\pi_j(s) - \pi_k(s)\sum_{j\in Z_k^-}\lambda_{kj}. \qquad (4)$$

Upon the reduction of such terms, the system of linear algebraic equations (4) shall be as follows

$$\begin{cases} b_{00}\,\pi_0(s) + b_{01}\,\pi_1(s) + \ldots + b_{0n}\,\pi_n(s) = C_0; \\ b_{10}\,\pi_0(s) + b_{11}\,\pi_1(s) + \ldots + b_{1n}\,\pi_n(s) = C_1; \\ \ldots \\ b_{n0}\,\pi_0(s) + b_{n1}\,\pi_1(s) + \ldots + b_{nn}\,\pi_n(s) = C_n. \end{cases} \qquad (5)$$

Following the solution of the equations system (5) according to Cramer's rule [5], we get

$$\pi_i(s) = \frac{D_i(s)}{D(s)}, \quad i = 0,1,2,\ldots,n,$$

where $D_i(s)$, $D(s)$ are the determinants of the related matrices.

Next, the inverse Laplace transformation is performed, while providing the required set of functions $P_k(t)$, $k = 0,1,\ldots,n$.

In terms of practice, the maximum variables of the obtained functions are of the greatest interest, i.e., variables equal to

$$\lim_{t\to\infty} P_k(t) = P_k, k = 0,1,\ldots,n,$$

Herewith the differential equations (1) are simplified to as follows:

$$\sum_{j\in Z_k^+} \lambda_{jk}P_j - P_k\sum_{j\in Z_k^-}\lambda_{kj}, k = 0,1,2,\ldots,n. \qquad (6)$$

The solution of this linear algebraic equations system, being easily obtained for any set of rate variables (λ_{jk}), determines the required distribution of system states. (P_k), $k=0,1,\ldots,n$. It is enough to analyze a specific system, thus in the predominant number of known works, the problem solution is limited thereto [6, 7]. However, the problems of managing the system resource to increase its efficiency the analytical expressions are required that explicitly reflect the dependences of the probabilities of system states on the values of related parameters. The equations system (6) allows to obtain the required analytical ratios unless the system order is minor $(n\le 5)$. But the technical and computational complexity of solving this system "manually" along with an increase in the problem dimension rapidly becomes difficult to overcome [6, 7]. This circumstance manifests itself especially demonstratively unless the system under consideration is multi-threaded. Let's enter, for instance, a graph of states and transitions of the simplest triple-threaded two-phase system (Fig. 1). This system has 27 states. It is difficult to analyze such a system using conventional methods. Pursuant thereto, the purpose of the study is to develop a method for calculating management in a high-dimensional Markov system.

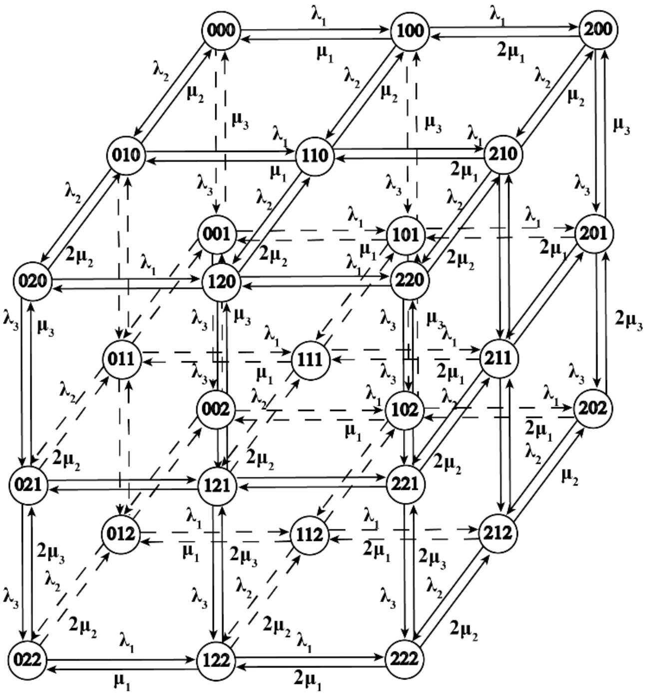

Fig. 1. Graph of states and transitions in a triple-thread two-phase Markov system

Development of a method for analytical calculation of system states probabilities

To obtain the required analytical expressions, one should use a technique based on the idea of decomposition of states [8]. The related computational procedure is implemented as follows. Initially, the entire set of possible system states Z is divided into non-overlapping subsets $(Z_1, Z_2, \ldots, Z_n)$, $\cup Z_k = Z$, $\cap Z_k = \varnothing$. The set of states included in a particular subset shall be called the related group state. At the first step of the procedure, a state transition matrix is formed for each group state, using which a system of linear algebraic equations is given with respect to the probabilities of states of this subset.

The solution of this equations system determines the conditional distribution of states of the allocated group state. At the second step, the probabilities of transitions from each group state to other states are calculated. For the allocated pair of group states, the probability of transition from the first state to the second one is equal to the sum of productions of the conditional probabilities of each state of the first subset multiplied by the probabilities of transition to incident states from the second subset. The aggregate of calculated probabilities of transitions between group states is used to find the distribution of group states. At the final step of the procedure, the system states probabilities are determined. Herewith the probability of a particular state from a certain group state is determined by the production of both the conditional probability of this state and the probability of the related group state.

Let's consider the simplest instance of this approach implementation. Let the graph of states and transitions for a double-channel Markov system along with two independent incoming threads is as shown in Fig. 2, where, λ_1 is the rate of the first incoming thread, μ_1 is the rate of the first thread requests servicing, λ_2 is the rate of the second incoming thread, μ_2 is the rate of the second thread requests servicing.

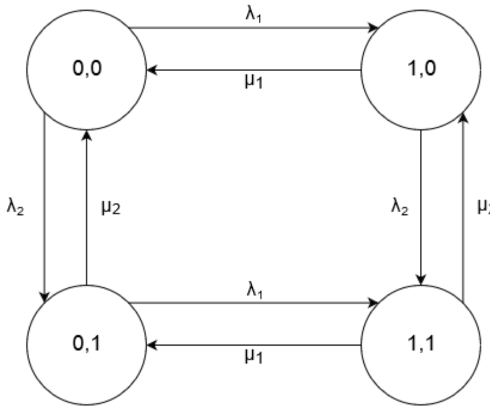

Fig. 2 Graph of states and transitions

Let's make a Kolmogorov equations system concerning the required probabilities

$$\begin{cases} \mu_1\pi_{10} + \mu_2\pi_{01} - \lambda_1\pi_{00} - \lambda_2\pi_{00} = 0, \\ \lambda_1\pi_{00} + \mu_2\pi_{11} - \mu_1\pi_{10} - \lambda_2\pi_{10} = 0, \\ \lambda_2\pi_{00} + \mu_1\pi_{11} - \lambda_1\pi_{01} + \mu_2\pi_{01} = 0, \\ \pi_{00} + \pi_{01} + \pi_{10} + \pi_{11} = 1, \end{cases}$$

or, upon ordering the addends in each of the equations, we get

$$(\lambda_1 + \lambda_2)\pi_{00} - \mu_2\pi_{01} - \mu_1\pi_{10} = 0,$$

$$\lambda_2\pi_{00} + (\mu_2 - \lambda_1)\pi_{01} + \mu_1\pi_{11} = 0,$$

$$\lambda_1\pi_{00} - (\mu_1 + \lambda_2)\pi_{10} + \mu_2\pi_{11} = 0,$$

$$\pi_{00} + \pi_{01} + \pi_{10} + \pi_{11} = 1.$$

Let's enter

$$H = \begin{pmatrix} \lambda_1 - \lambda_2 & -\mu_2 & -\mu_1 & 0 \\ \lambda_2 & \mu_2 - \lambda_1 & 0 & \mu_1 \\ \lambda_1 & 0 & -(\mu_1 + \lambda_2) & \mu_2 \\ 1 & 1 & 1 & 1 \end{pmatrix};$$

$$\pi = \begin{pmatrix} \pi_{00} \\ \pi_{01} \\ \pi_{10} \\ \pi_{11} \end{pmatrix}; \; B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Then

$$H\pi = B, \text{ from which } \pi = H^{-1} B. \tag{7}$$

The obtained ratio allows to easily find a numerical solution, however, obtaining first-order definable ratios describing the analytical dependence of states probabilities on the transition rates even in this simple problem causes a rather cumbersome procedure. In this regard, one should use the technology of decomposition of the system structure.

Let's enter a set of lumped states E_0 and E_1:

$$E_0 = \{S_{00}, S_{10}\}, E_0 = \{S_{01}, S_{11}\}. \tag{8}$$

Herewith the conditional probability distributions of the system staying in the states of subsets E_0, E_1 are as follows

$$\hat{P}_0 = (\hat{P}_{00}, \hat{P}_{10}) = \left(\frac{\mu_1}{\mu_1 + \lambda_1}, \frac{\lambda_1}{\mu_1 + \lambda_1} \right), \tag{9}$$

$$\hat{P}_1 = (\hat{P}_{01}, \hat{P}_{11}) = \left(\frac{\mu_1}{\mu_1 + \lambda_1}, \frac{\lambda_1}{\mu_1 + \lambda_1} \right), \tag{10}$$

The probability of the system transition from the lumped state E_0 to the lumped state E_1 is equal to

$$W_{01} = \hat{P}_{00} \frac{\lambda_2}{\lambda_1 + \lambda_2} + \hat{P}_{10} \frac{\lambda_2}{\lambda_2 + \mu_1}. \tag{11}$$

In like manner, the probability of the system transition from the lumped state E_1 to the simplified state E_0 is equal to

$$W_{10} = \hat{P}_{01} \frac{\mu_2}{\lambda_1 + \mu_2} + \hat{P}_{11} \frac{\mu_2}{\mu_1 + \mu_2}. \tag{12}$$

Pursuant thereto, the stationary probabilities of the system staying in the lumped states E_0 and E_1 are equal to

$$Q = \{Q_0, Q_1\} = \left\{ \frac{W_{10}}{W_{01} + W_{10}}, \frac{W_{01}}{W_{01} + W_{10}} \right\}. \tag{13}$$

Then the stationary probabilities of the system staying in its possible states are determined by the following ratio

$$\begin{gathered} \pi_{00} = \hat{P}_{00} Q_0; \; \pi_{10} = \hat{P}_{10} Q_0; \\ \pi_{01} = \hat{P}_{01} Q_1; \; \pi_{11} = \hat{P}_{11} Q_1. \end{gathered} \tag{14}$$

Let's perform the required calculations. Let's substitute (9), (10) in (11), (12)

$$W_{01} = \frac{\mu_1}{\mu_1 + \lambda_1} \cdot \frac{\lambda_2}{\lambda_1 + \lambda_2} + \frac{\lambda_1}{\mu_1 + \lambda_1} \cdot \frac{\lambda_2}{\lambda_2 + \mu_1},$$

$$W_{10} = \frac{\mu_1}{\mu_1 + \lambda_1} \cdot \frac{\mu_2}{\lambda_1 + \mu_2} + \frac{\lambda_1}{\mu_1 + \lambda_1} \cdot \frac{\mu_2}{\mu_1 + \mu_2}.$$

Next

$$\pi_{00} = \hat{P}_{00} \frac{W_{10}}{W_{10} + W_{01}} = \hat{P}_{00} \frac{1}{1 + \dfrac{W_{01}}{W_{10}}}. \tag{15}$$

Since

$$W_{01} = \frac{\mu_1}{\mu_1 + \lambda_1} \cdot \frac{\lambda_2}{\lambda_1 + \lambda_2} + \frac{\lambda_1}{\mu_1 + \lambda_1} \cdot \frac{\lambda_2}{\lambda_2 + \mu_1} =$$

$$= \frac{\lambda_2}{\mu_1 + \lambda_1} \left(\frac{\mu_1}{\lambda_2 + \lambda_1} + \frac{\lambda_1}{\lambda_2 + \mu_1} \right) =$$

$$= \frac{\lambda_2 (\mu_1\lambda_2 + \mu_1^2 + \lambda_1\lambda_2)}{(\mu_1 + \lambda_1)(\lambda_1 + \lambda_2)(\lambda_2 + \mu_1)} =$$

$$\frac{\mu_1\lambda_2^2 + \lambda_2\mu_1^2 + \lambda_1\lambda_2^2 + \lambda_2\lambda_1^2}{(\mu_1 + \lambda_1)(\lambda_1 + \lambda_2)(\lambda_2 + \mu_1)},$$

$$W_{10}=\frac{\mu_1}{\mu_1+\lambda_1}\cdot\frac{\mu_2}{\lambda_1+\mu_2}+\frac{\lambda_1}{\mu_1+\lambda_1}\cdot\frac{\mu_2}{\mu_1+\mu_2}=$$

$$=\frac{\mu_2}{\mu_1+\lambda_1}\left(\frac{\mu_1}{\lambda_1+\mu_2}+\frac{\lambda_1}{\mu_1+\mu_2}\right)=$$

$$=\frac{\mu_2(\mu_1{}^2+\mu_1\mu_2+\lambda_1{}^2+\lambda_1\mu_2)}{(\mu_1+\lambda_1)(\mu_2+\lambda_1)(\mu_1+\mu_2)}=$$

$$=\frac{\mu_2\mu_1{}^2+\mu_1\mu_2{}^2+\mu_2\lambda_1{}^2+\lambda_1\mu_2{}^2}{(\mu_1+\lambda_1)(\mu_2+\lambda_1)(\mu_1+\mu_2)},$$

then

$$\frac{W_{01}}{W_{10}}=\frac{\begin{pmatrix}\mu_1\lambda_2{}^2+\lambda_2\mu_1{}^2+\\+\lambda_1\lambda_2{}^2+\lambda_2\lambda_1{}^2\end{pmatrix}\left(\mu_2+\lambda_1\right)\left(\mu_1+\mu_2\right)}{\begin{pmatrix}\mu_2\mu_1{}^2+\mu_1\mu_2{}^2+\\+\mu_2\lambda_1{}^2+\lambda_1\mu_2{}^2\end{pmatrix}(\lambda_1+\lambda_2)(\lambda_2+\mu_1)}=$$

$$=\frac{\mu_1\lambda_2(\lambda_2+\mu_1)+\lambda_1\lambda_2(\lambda_1+\lambda_2)}{(\lambda_2+\mu_1)(\lambda_1+\lambda_2)}\times$$

$$\times\frac{(\lambda_1+\mu_2)(\mu_1+\mu_2)}{\mu_1\mu_2(\mu_1+\mu_2)+\lambda_1\mu_2(\lambda_1+\mu_2)}= \qquad (16)$$

$$=\frac{\left(\mu_1\lambda_2/(\lambda_1+\lambda_2)+\lambda_1\lambda_2/(\lambda_2+\mu_1)\right)}{\left(\mu_1\mu_2/(\lambda_1+\mu_2)+\lambda_1\mu_2/(\mu_1+\mu_2)\right)}=$$

$$=\frac{\dfrac{(\mu_1/\lambda_1)\cdot\lambda_2}{1+\lambda_2/\lambda_1}+\dfrac{\lambda_2}{\lambda_2/\lambda_1+\mu_1/\lambda_1}}{\dfrac{(\mu_1/\lambda_1)\cdot\mu_2/\lambda_2}{1/\lambda_2+(\mu_2/\lambda_2)/\lambda_1}+\dfrac{\mu_2/\lambda_2}{(\mu_1/\lambda_1)\cdot(1/\lambda_2)+(\mu_2/\lambda_2)/\lambda_1}}.$$

Let's enter new variables: $x_1=\frac{\mu_1}{\lambda_1}; x_2=\frac{\mu_2}{\lambda_2}$. Then (15) considering (16) shall be as follows:

$$\pi_{00}=\frac{x_1}{(1+x_1)\left[1+\dfrac{\dfrac{x_1}{1+\frac{\lambda_2}{\lambda_1}}+\dfrac{1}{\frac{\lambda_2}{\lambda_1}+x_1}}{\dfrac{x_1x_2}{1+x_2\frac{\lambda_2}{\lambda_1}}+\dfrac{x_2}{x_1+x_2\frac{\lambda_2}{\lambda_1}}}\right]}= \qquad (17)$$

$$=\frac{x_1}{1+x_1}\cdot\frac{1}{1+A}.$$

Next, considering (14) we get:

$$\pi_{10}=\frac{\lambda_1}{\lambda_1+\mu_1}\cdot\frac{1}{A+1};$$

$$\pi_{01}=\frac{\mu_2}{\lambda_2+\mu_2}\cdot\frac{1}{1+A^{-1}}; \qquad (18)$$

$$\pi_{11}=\frac{\lambda_2}{\lambda_2+\mu_2}\cdot\frac{1}{1+A^{-1}}$$

Let's analyze the obtained ratios. Let

$\lambda_1=\lambda_2$, $\mu_1=\lambda_1$, $\mu_2=\lambda_2$.

Then in such a system that is completely symmetric in respect of all parameters, the probabilities of all states shall be the same and, as follows from (17), (18), equal to 0.25. Within normal functioning system, the request servicing rate is higher than that of the reception thereof, that is, $\mu_1>\lambda>_1$, $\mu_2>\lambda>_2$, i.e., $x_1>1>$, $x_2>1>$. Herewith the probability π_{00}, which determines the system efficiency, will be as higher, as greater the variables x_1 and x_2. The problem of optimizing a resource that can be used to increase the servicing rate, shall be provided with a restriction

$$x_1+x_2\le c,\quad x_1>1,\quad x_2>1 \qquad (19)$$

Now this resource allocation problem is formulated as follows: let's find a set (x_1, x_2) satisfying (19) and maximizing (17). The complexity of the analytical description of the objective function (17) restricts the possibility to solve the resulting mathematical programming problem using standard methods of both the first and second orders. In this context, one should use the downhill simplex method [7] to solve the problem. The objective function of the problem contains addends specifying penalties for violating restrictions (19):

$$F(x_1,x_2)=$$

$$=\pi_{00}(x_1,x_2)R_0\left[\max\{(x_1+x_2),c\}-c\right]^2+ \qquad (20)$$

$$+\sum_{i=1}^{2}R_i\left[1-\min\{1,x_i\}\right]^2.$$

Herewith the component

$$R_0\left[\max\{(x_1+x_2),c\}-c\right]^2$$

determines the penalty unless the variable (x_1+x_2) exceeds c. Components

$$R_i\left[1-\min\{1,x_i\}\right]^2$$

act in a similar way unless the values of variables x_1 or x_2 shall be less than one.

The coordinates of the vertices of the initial simplex *A*, *B*, *C* are given by the matrix

$$\gamma=\begin{pmatrix}1 & d_1+1 & d_2+1\\ 1 & d_2+1 & d_1+1\end{pmatrix};$$

$$d_1=\frac{1}{2\sqrt{2}}\left(\sqrt{3}+1\right)=0.96;\ d_2=\frac{1}{2\sqrt{2}}\left(\sqrt{3}-1\right)=0.26\ .$$

The coordinates of the vertices of this simplex were chosen so that the distance between any two vertices was equal to one. Indeed, let the vector $\begin{pmatrix}1\\1\end{pmatrix}$ sets the coordinates of vertex A of the simplex, the vector $\begin{pmatrix}1+\frac{1}{2\sqrt{2}}\left(\sqrt{3}+1\right)\\ 1+\frac{1}{2\sqrt{2}}\left(\sqrt{3}-1\right)\end{pmatrix}$ sets the coordinates of vertex B and the vector $\begin{pmatrix} 1+\frac{1}{2\sqrt{2}}\left(\sqrt{3}-1\right) \\ 1+\frac{1}{2\sqrt{2}}\left(\sqrt{3}+1\right) \end{pmatrix}$ sets the coordinates of vertex *C*.

Herewith

$$R_{AB} = R_{AC} =$$

$$= \sqrt{\left(\frac{1}{2\sqrt{2}}\left(\sqrt{3}+1\right)\right)^2 + \left(\frac{1}{2\sqrt{2}}\left(\sqrt{3}-1\right)\right)^2} =$$

$$= \sqrt{\frac{1}{8}\left(4+2\sqrt{3}\right)+\frac{1}{8}\left(4-2\sqrt{3}\right)} = 1;$$

$$R_{BC} = 2\left[\left(\frac{1}{2\sqrt{2}}\left(\sqrt{3}+1\right)\right)+\left(\frac{1}{2\sqrt{2}}\left(\sqrt{3}-1\right)\right)\right]^2 =$$

$$= 2\left[\frac{1}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}\right]^2 = 1.$$

Resulting from the implementation of the downhill simplex method procedure the c=5 shall get a vector specifying the distribution (x_1, x_2) = (3.06; 1.94), while maximizing the system state probability whereby both channels are free.

Herewith, π_{00}=0.44, π_{10}=0.14, π_{01}=0.28, π_{11}=0.14. The problem has been solved.

The proposed decomposition technology of state aggregation allows to solve the problem of finding analytical ratios linking the numerical values of the parameters of high-dimensional Markov systems with its efficiency. These ratios provide an opportunity to formulate a criterion for the system efficiency and, thus, to set and solve the problems of system resource management. It should be also noted, that when solving a management problem in an ultra-high-dimensional Markov system, a hierarchically organized multi-stage decomposition can be used so that the problem dimension does not exceed the permissible value at each stage. An important area of further research in terms of practice is the dissemination of a decomposition approach to analyze multi-threaded systems with a priority set, for instance, using the pair-wise comparison method [9].

Summary

The known methods for solving the problem of Markov systems have been analyzed. It was confirmed that the conventional procedure based on solving a Kolmogorov equations system provides a numerical solution for problems of almost any dimension.

It was demonstrated that the computational complexity of obtaining analytical expressions for calculating the distribution of system states increases rapidly with an increase in the number of states and becomes compelling for systems of actual dimension.

A method for calculating the probabilities of states of a Markov system was proposed based on the decomposition of states graph, which reduced high dimension problems to a sequence of low-dimension ones.

The proposed approach allows to obtain analytical ratios that establish a relationship between the system probability distribution and a set of its initial parameters (system transition rates). The obtained ratios allow for system resources management to increase its efficiency.

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Управління у марківських системах високої розмірності

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Анотація. Розглянуто завдання аналізу марківських систем із великою кількістю станів. Традиційна обчислювальна процедура отримання аналітичних співвідношень для розрахунку розподілу ймовірностей станів системи заснована на використанні системи диференціальних рівнянь Колмогорова. Сформована в подальшому система лінійних алгебраїчних рівнянь легко вирішується чисельно. Однак складність отримання аналітичного рішення швидко зростає зі збільшенням розмірності задачі. У зв'язку з цим мета - розробка ефективного методу дослідження марківських систем, обчислювальна процедура якого забезпечує можливість отримання рішення для моделей високої розмірності. Метод заснований на декомпозиції графа станів та переходів системи. Одержувані аналітичні вирази дозволяють поставити і вирішити задачу раціонального розподілу ресурсу для зміни значень параметрів системи з підвищення її ефективності. Метод забезпечує можливість вирішення завдань управління у марківських системах з великою кількістю можливих станів. Розглянуто приклад застосування методу.

Ключові слова: Марківська система, розрахунок розподілу ймовірностей станів, декомпозиційна обчислювальна схема, управління ймовірностями станів.