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SYNTHESIS OF STRUCTURE OF THE ADDER BY MODULE

Abstract. The **subject** of the article is the study of the structure of low-bit binary adders for an arbitrary modulus of the residual class system (RCS). The **purpose** of this article is to develop an algorithm for synthesizing the structure of the adder of two residuals of numbers by an arbitrary value of the RCS module, by organizing inter-bit connections between the binary digits of the adder, the combination of which determines the structure of the adder modulo. **Tasks**: to investigate the possibility of performing the operation of addition of two residuals in RCS based on positional binary adders; to analyze the influence of additionally introduced interdigit connections into the positional binary adder on the value of the contents of this adder; to develop an algorithm for the synthesis of an adder by an arbitrary RCS module. **Research methods**: methods of analysis and synthesis of computer systems, number theory, coding theory in RCS. The following **results** were obtained. It is shown in the work that the introduction of additional interdigit connections in a positional binary adder allows changing the contents of this adder. The rules for the introduction of these additional links are formulated, on the basis of which an algorithm for the synthesis of an adder by an arbitrary RCS modulus is obtained. Specific examples of the synthesis of structures of binary adders for various values of the RCS modules are given. **Conclusions**. Thus, the paper proposes an algorithm for the synthesis of an adder by an arbitrary RCS module, which is based on the use of positional binary adders, by introducing additional inter-bit connections. The application of the considered algorithm expands the functionality of positional binary adders.

Keywords: number system, residual class system, positional binary adder, modular computation, computer system.

Introduction

The operation of adding of two numbers is the main operation, which is implemented by a computer system (CS), both in a positional binary notation (PN) and in a non-positional notation of residual classes (RCS). The adder of two numbers is the main part of operating device of CS in PN. Adders of two numbers by module m_i are also elements of CS along with positional adders [1-3]. In RCS, the modular addition operation $(a_i + b_i) \bmod m_i$ is implemented on base of usage of low-bit modulo m_i adders. One of the methods for implementation of the modular addition operation $(a_i + b_i) \bmod m_i$ is based on the usage of structures of positional low-bit binary adders [4-7]. This approach provides a wide range of options for implementation of the structure of such adders. This allows to fully use available practical experience in the design and selection of structures of binary adders. The article proposes an algorithm for synthesizing the structure of an adder of two remainders of numbers by an arbitrary RCS module.

Main part

The article discusses the synthesis of an adder of two residues of numbers by an arbitrary RCS module m_i. Synthesis of modulo adder is a procedure for constructing the structure of a non-positional adder from positional binary one-bit adders (BOA). In non-positional adder by an arbitrary module m_i, an addition circuit is used, which is implemented in the structure of adder by module $M = 2^n - 1$. This is achieved by organizing and using additional inter-bit connections $X_{\downarrow i \uparrow j}$, in the general case, between the j-th and the i-th BOA of the adder module M.

An arbitrary initial structure of a n-bit binary adder by module $M = 2^n - 1$ is shown in Fig. 1.

The task of adder by module m_i synthesis is to ensure the modular addition of two residues for a given modules by means of a adder by module $M = 2^n - 1$. In this article, this is achieved by introducing into the adder by module M additional links of the form $X_{\downarrow i \uparrow j}$, where the sign $X_{\downarrow i \uparrow j}$ denotes the connection between the output of the j-th BOA and the input of the i-th BOA. A diagram of the organization of such an additional connection between the output of the j-th BOA and the input of the i-th BOA is shown in Fig. 2.

The essence of constructing adders by module RCS is as follows. In the initial adder by module $M = 2^n - 1$, on base of certain rules, additional connections $X_{\downarrow i \uparrow j}$ are formed [1]. The usage of additional connections $X_{\downarrow i \uparrow j}$ in the adder by module $M = 2^n - 1$ allows to synthesize an adder for performing the operation of adding the residues of numbers by module m_i, since the introduction of additional connections $X_{\downarrow i \uparrow j}$ changes the weights of individual bits of the adder and reduces the module of the adder from the initial value M to the required modulus value m_i.

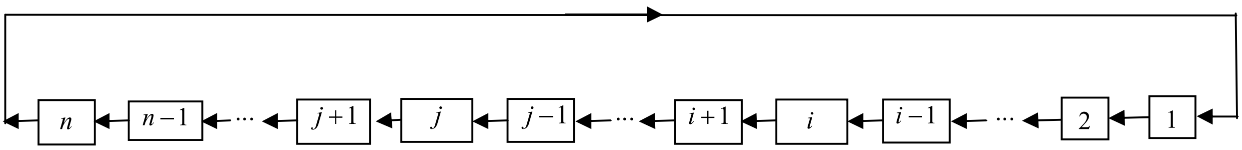

Fig. 1. The structure of a binary adder by module $M = 2^n - 1$

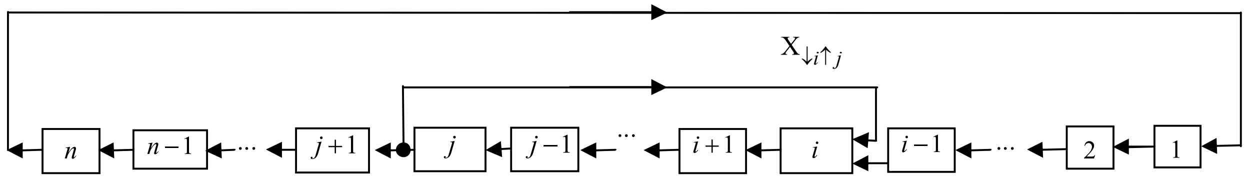

Fig. 2. Diagram of an adder by module $M = 2^n - 1$ with one additional connection $X_{\downarrow i\uparrow j}$

In the general case, the modulo adder synthesis algorithm consists of a sequence of performing the following operations.

1. Obtaining the structure of the adder by module $M = 2^n - 1$, where $n = [\log_2(m_i - 1)] + 1$.

2. Determination of the adder binary bits S_i for which equality $S_i = 0$ is true. The process of determining the condition $S_i = 0$ is based on the representation of the module in binary code.

3. Additional connection $X_{\downarrow i\uparrow j}$ begins with the most significant bit of the adder.

4. Additional connection $X_{\downarrow i\uparrow j}$ goes to the BOA input, for which $S_i = 0$.

Examples of synthesis of structures of adders by an arbitrary module

Two examples of synthesis of structures of adder are considered.

Example 1. $m_i = 53$. The stages of synthesis of an adder by module of RCS are as follows.

1. In accordance with the size of the module $m_i = 53$, the number n of BOA of adder by module $M = 2^n - 1$ is determined. For module $m_i = 53$ there is $n = [\log_2(m_i - 1)] + 1 = [\log_2(53 - 1)] + 1 = 6$. The structure of adder by module $M = 2^n - 1 = 63$ will be the following (Fig. 3).

Initial structure of adder by module $m_i = 53$ without additional connections $X_{\downarrow i\uparrow j}$ will be the same.

2. Module $m_i = 53$ in binary code $S_6\ S_5\ S_4\ S_3\ S_2\ S_1$ is 110101, which means $S_6 = 1$, $S_5 = 1$, $S_4 = 0$, $S_3 = 1$, $S_2 = 0$ and $S_1 = 1$. From the form of the module $m_i = 53$ which is represented in the binary code, $S_2 = S_4 = 0$ is determined.

3. Based on the obtained results, the structure of the adder by module $m_i = 53$ is represented in the following form.

In accordance with the synthesis method, two additional connections $X_{\downarrow 4\uparrow 6}$ and $X_{\downarrow 2\uparrow 6}$ are introduced into the adder by module $M = 2^6 - 1$. In order to check the correctness of the synthesis of the adder by module $m_i = 53$, the value of the RCS module $M = m_i$ for a given adder structure is determined. Based on the given structure of the adder (Fig. 4), a number of structures of individual parts of the adder by module $m_i = 53$ is composed.

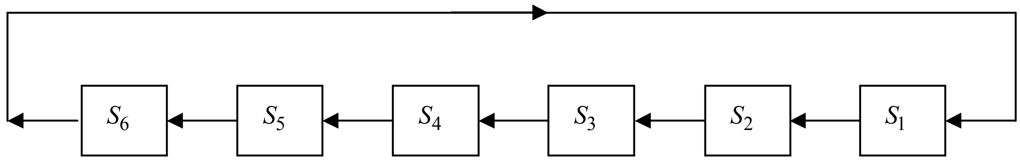

Fig. 3. Initial structure of adder by module $M = 2^6 - 1$

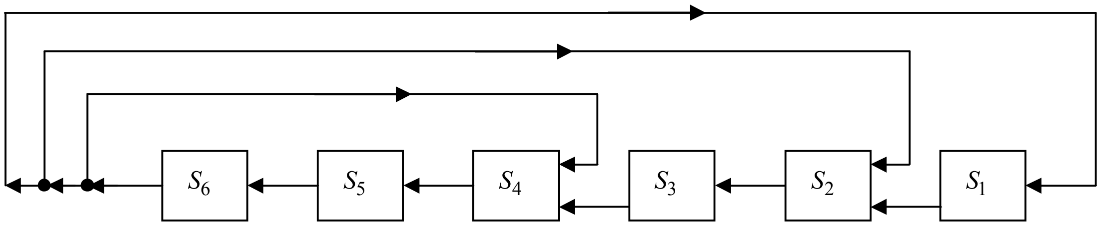

Fig. 4. Structure of adder by modulo $m_i = 53$

The first part of adder structure is shown on Fig. 5.

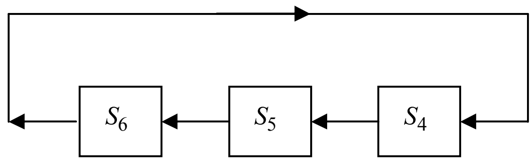

Fig. 5. First part of structure of adder by module m_i

For the first part of adder structure module M_1 will be the following $M_1 = \tau_3 \cdot \tau_5 \cdot \tau_4 - 1$. The second part of adder structure is shown on Fig. 6.

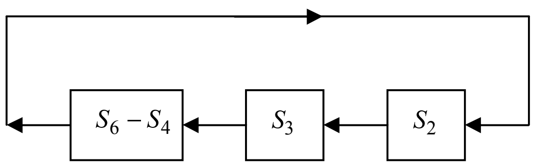

Fig. 6. Second part of structure of adder by module m_i

For this part of adder structure module M_2 :

$$M_2 = M_1 \cdot \tau_3 \cdot \tau_2 - 1 = (\tau_6 \cdot \tau_5 \cdot \tau_4 - 1) \cdot \tau_3 \cdot \tau_2 - 1 .$$

For adder by module the value of module $M = m_i$ of RCS will be determined as follows (fig. 4-6)

$$m_i = M_2 \cdot \tau_1 - 1 = \left[(\tau_6 \cdot \tau_5 \cdot \tau_4 - 1) \cdot \tau_3 \cdot \tau_2 - 1\right] \cdot \tau_1 - 1 =$$

$$= \left[(2^3 - 1) \cdot 2^2 - 1\right] \cdot 2 - 1 = 53.$$

Based on the performed calculations, there is the conclusion that the synthesis of the adder by module $m_i = 53$ (fig. 4) was carried out correctly.

Example 2. $m_i = 97$. The stages of synthesis of an adder by module of RCS are as follows.

1. In accordance with the size of the module $m_i = 97$, the number n of BOA of adder by module $M = 2^n - 1$ is determined. For module $m_i = 97$ there is

$$n = \left[\log_2 (m_i - 1)\right] + 1 = \left[\log_2 (97 - 1)\right] + 1 = 7 .$$

The structure of adder by module $M = 2^n - 1 = 2^7 - 1 = 127$ will be the following (Fig. 7).

2. Module $m_i = 97$ in binary code $S_7 S_6 \, S_5 \, S_4 \, S_3 \, S_2 \, S_1$ is 1100001, which means $S_7 = 1, S_6 = 1, S_5 = S_4 = S_3 = S_2 = 0, S_1 = 1$.

3. Based on the obtained results, the structure of the adder by module $m_i = 97$ is represented by Fig. 8. In accordance with the synthesis method, four additional connections $X_{\downarrow 5\uparrow 7}, X_{\downarrow 4\uparrow 7}, X_{\downarrow 3\uparrow 7}, X_{\downarrow 2\uparrow 7}$ .are introduced into the adder by module $M = 2^7 - 1$. In order to check the correctness of the synthesis of the adder by module $m_i = 97$, the value of the RCS module $M = m_i$ for a given adder structure is determined.

The first part of adder structure is shown on Fig. 9.

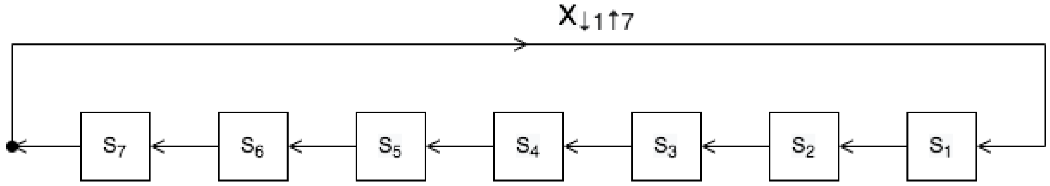

Fig. 7. Initial structure of adder by module $M = 2^7 - 1$

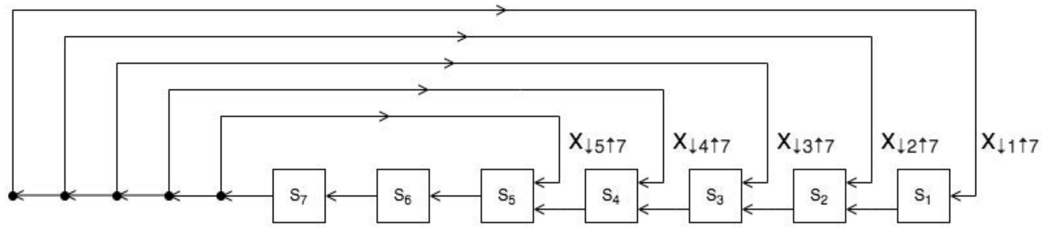

Fig. 8. Structure of adder by modulo $m_i = 97$

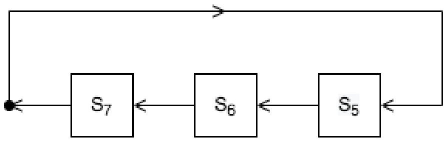

Fig. 9. First part of structure of adder by module m_i

For the first part of adder structure module M_1 will be the following $M_1 = \tau_7 \cdot \tau_6 \cdot \tau_5 - 1$. The second part of adder structure is shown on Fig. 10.

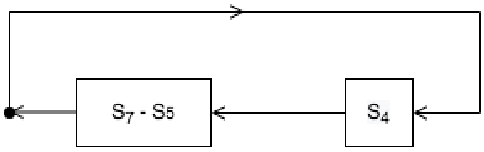

Fig. 10. Second part of structure of adder by module m_i

For the first part of adder structure module M_2 will be the following $M_2 = M_1 \cdot \tau_4 - 1$. The third part of adder structure is shown on Fig. 11. For the first part of adder structure module M_3 will be the following $M_3 = M_2 \cdot \tau_3 - 1$. The fourth part of adder structure is shown on Fig. 12.

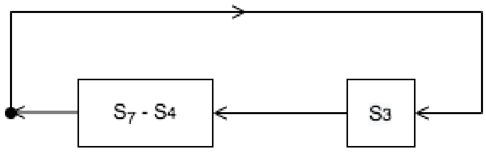

Fig. 11. Third part of structure of adder by module m_i

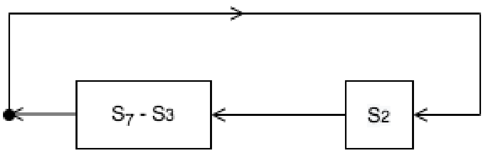

Fig. 12. Fourth part of structure of adder by module m_i

For the first part of adder structure module M_4 will be the following $M_4 = M_3 \cdot \tau_2 - 1$. The fifth part of adder structure is shown on Fig. 13.

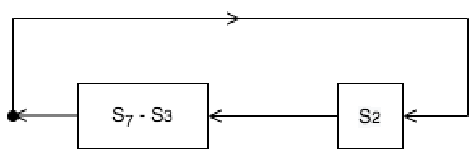

Fig. 13. Fifth part of structure of adder by module m_i

For the first part of adder structure module M_5 will be the following $M_5 = M_4 \cdot \tau_1 - 1$.

The value of module m_i:

Fig. 9: $M_1 = \tau_7 \cdot \tau_6 \cdot \tau_5 - 1$;

Fig. 10: $M_2 = M_1 \cdot \tau_4 - 1 = (\tau_7 \cdot \tau_6 \cdot \tau_5 - 1) \cdot \tau_4 - 1$;

Fig. 11: $M_3 = M_2 \cdot \tau_3 - 1 = [(\tau_7 \cdot \tau_6 \cdot \tau_5 - 1)\tau_4 - 1]\tau_3 - 1$;

Fig. 12: $M_4 = M_3 \cdot \tau_2 - 1 = \{[(\tau_7 \cdot \tau_6 \cdot \tau_5 - 1) \cdot \tau_4 - 1] \cdot \tau_3 - 1\} \cdot \tau_2 - 1$;

Fig. 13: $M_5 = M_4 \cdot \tau_1 - 1 = -1 + \times (\{[(\tau_7 \cdot \tau_6 \cdot \tau_5 - 1) \cdot \tau_4 - 1] \cdot \tau_3 - 1\} \cdot \tau_2 - 1)$.

In this case, the result of the synthesis of the adder by module $m_i = 97$ (fig. 8) is correct.

The given examples of the synthesis of the structure of adders by module of RCS confirm the possibility of practical usage of the algorithm which is considered in the article.

Conclusions

The article considers an algorithm for synthesizing the structure of adders by module m_i of RCS. The algorithm for the synthesis of adders is based on the usage of existing adders by modules $M = 2^n - 1$, which are widely used in CS, operating both in the PN and in the RCS.

The article directly provides an algorithm for the synthesis of an adder by module m_i. The algorithm is implemented by introducing and using additional inter-bit connections $X_{\downarrow i \uparrow j}$. The article formulates the rules for introducing these additional connections. The usage of additional connections (based on the structure of adder by module $M = 2^n - 1$) allows to create an adder that implements the operation of adding two residues a_i and b_i of numbers. A set of k adders by module is an adder of two numbers $A = (a_1, a_2, ..., a_i, ..., a_k)$ and $B = (b_1, b_2, ..., b_i, ..., b_k)$ in RCS. Specific examples of the synthesis of adders by module for various values of the RCS modules m_i are given.

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Синтез структури суматора за модулем

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Анотація. **Предметом** статті є дослідження структури малоразрядних двійкових суматорів за довільним модулем системи залишкових класів (СЗК). **Метою** даної статті є розробка алгоритму синтезу структури суматора двох залишків чисел за довільним значенням модуля СЗК, шляхом організації міжрозрядних зв'язків між двійковими розрядами суматора, комбінація яких визначає структуру суматора за модулем. **Задачі**: дослідити можливість виконання операції додавання двох залишків у СЗК на базі позиційних двійкових суматорів; провести аналіз впливу додатково введених міжрозрядних зв'язків у позиційний двійковий суматор, на величину вмісту цього суматора; розробити алгоритм синтезу суматора за довільним модулем СЗК. **Методи** дослідження: методи аналізу і синтезу комп'ютерних систем, теорія чисел, теорія кодування у СЗК. **Отримані наступні результати**. В роботі показано, що введення додаткових міжрозрядних зв'язків у позиційний двійковий суматор, дозволяє змінити вміст даного суматора. Сформульовано правила введення цих додаткових зв'язків, на основі чого отримано алгоритм синтезу суматора за довільним модулем СЗК. Наведено конкретні приклади синтезу структур двійкових суматорів для різних значень модулів СЗК. **Висновки**. Таким чином, у роботі запропоновано алгоритм синтезу суматора за довільним модулем СЗК, який заснований на використанні позиційних двійкових суматорів, шляхом введення додаткових міжрозрядних зв'язків. Застосування розглянутого алгоритму розширює функціональні можливості позиційних двійкових суматорів.

Ключові слова: система числення, система залишкових класів, позиційний двійковий суматор, модульні обчислення, комп'ютерна система.