MATHEMATICAL METHODS, MODELS AND INFORMATION TECHNOLOGIES IN ECONOMY

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APPLICATION OF THE THEORY OF FUZZY SETS IN ASSESSING THE ECONOMIC EFFICIENCY AND RISK OF INVESTMENT PROJECTS UNDER CONDITIONS OF UNCERTAINTY

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Introduction. Extensive practice of real forecasting of an investment project shows the need for comprehensive consideration of various types of uncertainty in the assessment, planning and management of investment projects. The reality is that the influence of uncertainty factors on an investment project leads to the emergence of unforeseen situations that cause unexpected losses, damages, even in projects that were initially recognised as economically feasible for the enterprise, because negative scenarios of the development of events, even if unlikely, are not taken into account in the investment project. However, they may occur and disrupt the implementation of the investment project.

Analysis of recent research and publications. Making investment decisions, like any other type of management activity, is based on the use of various formalised and informal methods and criteria. In domestic and foreign practice, there are a number of methods that can be used to make calculations as a basis for decision-making in the area of investment projects. However, there is no universal method for all cases. The problems of applying the theory of fuzzy sets to the evaluation of the efficiency of investment processes are devoted to the work of such scientists as V.G. Chernov, O.V. Dorokhov, O.A. Dmitrieva, O.S. Zaitseva, M.O. Lysova, S.O. Kalmykov, Yu.I. Shokin, O.O. Nedosekin, O.V. Shchyrova and others. Among the foreign authors are R. Braley, J. Bailey, S. Hughes, U. Sharpa and others.

Objectives of the article. The consideration of information uncertainty and its effectiveness depend directly on the choice of mathematical apparatus determined by mathematical theory. The stage of justification and selection of a mathematical apparatus that provides an acceptable formalisation of uncertainty and an adequate solution to problems arising in the management of real investments is extremely important. Unreasonable and, as a consequence, incorrect choice of mathematical apparatus leads, first of all, to inadequacy of the created mathematical models, to obtaining incorrect results in the process of their application and, accordingly, to mistrust in the obtained results and to ignoring the conclusions based on them.

The main material of the study. The analysis of methods for quantitative assessment of the effectiveness of an investment project under conditions of uncertainty allows to conclude that existing methods either eliminate uncertainty from the investment project model, which is inadmissible since uncertainty is an integral characteristic of any forecast, or are unable to formally describe and take into account all possible diversity of types of uncertainty. The vast majority of methods formalise uncertainty only as probability distributions constructed on the basis of subjective expert judgements, which in a very large number of cases are clearly idealised. As a result, these methods identify uncertainty, regardless of its nature, with randomness and therefore do not allow for the full range of possible types of uncertainty affecting the investment project to be taken into account. The use of a probabilistic approach in investment analysis is hampered by reasons related to the lack of statistical information or the small (insufficient) sample size for some of the parameters of the investment project, which is due to the uniqueness of each investment project. In addition, the accuracy of the assessment of probabilities (objective and subjective) depends on a number of factors, ranging from the quality of statistical information to the quality of expert assessments, so the quality of the resulting assessment of the effectiveness and risk of an investment project depends too much on them, which has contributed to the growth of mistrust in the results obtained on the basis of forecast estimates and decisions. In this regard, top managers, bankers and financiers are of the opinion that the vast majority of forecasts are too idealised and far from practice. Many prefer to work based on experience and intuition. This is due, among other things, to the following main reasons:

– The specificity of the subject area of the study, as it lies at the intersection of modern applied mathematics, economics and psychology;

– the relative novelty and insufficient development of mathematical methods for analysing an investment project under conditions of uncertainty;

– low awareness of top managers of enterprises and financial specialists about new mathematical approaches to formalising and simultaneously processing heterogeneous information (deterministic, interval, linguistic, statistical) and about the possibilities of building specialised methods based on these approaches.

The extensive experience of domestic and foreign researchers convincingly shows that the probabilistic approach cannot be recognised as a reliable and adequate tool for solving semi-structured problems, which include problems of real investment management. In principle, any attempt to use statistical methods to solve such problems is nothing more than a reduction to well-structured (well-formalised) problems, and this kind of reduction significantly distorts the original formulation of the problem. The limitations and disadvantages of using "classical" formal methods in solving semi-structured problems are a consequence of the "principle of incompatibility" formulated by the founder of fuzzy set theory L.A. Zadeh [5]: "...the closer we come to solving real-world problems, the more obvious it is that as the complexity of the system increases, our ability to make accurate and confident conclusions about its behaviour decreases to a certain threshold, beyond which accuracy and confidence become almost mutually exclusive concepts."

Therefore, some domestic and foreign researchers are developing methods for assessing the effectiveness and risk of investment projects based on the TFZ apparatus. These methods use a probability distribution described by the membership function of a fuzzy number instead of a probability distribution.

Methods based on the theory of fuzzy sets refer to methods of evaluation and decision-making under conditions of uncertainty. Their application involves formalising the initial parameters and target performance indicators of an investment project (mainly NPV) in the form of a vector of interval values (fuzzy interval), falling into each interval of which is characterised by a certain degree of uncertainty. By performing arithmetic and other operations with such fuzzy intervals according to the rules of fuzzy mathematics, experts and the decision maker obtain the resulting fuzzy interval for the target indicator. Based on initial information, experience and intuition, experts can often quantitatively characterise the boundaries (intervals) of possible (acceptable) parameter values and the ranges of their most possible (preferred) values with a high degree of confidence.

Among the methods based on the theory of fuzzy sets, it is possible to include, as a special case, the old and widely known interval method. This method corresponds to situations where only the limits of the values of the analysed parameter within which it can vary are known quite precisely, but there is no quantitative or qualitative information about the possibilities or probabilities of realising its various values within a given interval. According to this method, the input variables of the investment project are specified in the form of intervals, the membership functions of which are classical characteristic functions of the set, so that it is then possible to directly apply the rules of fuzzy mathematics to obtain the resulting indicator of the effectiveness of the investment project in the form of an interval. In the interval method, the level (degree) of risk is proposed to be the amount of maximum loss per unit of uncertainty:

$$P = \frac{q_N - q_{\min}}{q_{\max} - q_{\min}} \text{ or } P = \frac{q_{\max} - q_N}{q_{\max} - q_{\min}},$$

where q_N is the required value of the parameter;

$q_{\min}$ – minimum value of the parameter;

q_{max} – maximum value of the parameter;

P – level (degree) of risk, or the ratio of the distance from the required value to its minimum (maximum) value to the interval between its maximum and minimum values.

The specific way in which the degree of risk is expressed depends on the performance measure used. For example, to assess the risk of an investment project using the NPV criterion, the first expression should be used, and the second expression should be used using the DPP criterion. This method of determining risk is fully consistent with the geometric definition of probability, but on the assumption that all events within a segment $[q_{min}; q_{max}]$ are equally likely. Obviously, such an assumption cannot be called reflective of reality.

If there is additional information about the values of a parameter within an interval, for example, if it is known that the value "a" is more possible than "b", the mathematical formalisation of uncertainties can be adequately implemented using a fuzzy interval approach. When using the mathematical apparatus of fuzzy set theory, experts need to formalise their ideas about the possible values of the estimated parameter of an investment project by specifying the characteristic function (membership function) of the set of values it can take. In this case, the experts are asked to indicate the set of values that they think the estimated value cannot take (for them, the characteristic function is equal to 0) and then to rank the set of possible values according to the degree of possibility (belonging to a given fuzzy set). After formalising the input parameters of the investment project, it is possible to calculate the probability distribution of the output parameter (investment project performance indicator) y by "α-level principle of generalisation" or "Zadeh's principle of generalisation":

$$\mu_{\tilde{Y}}\left(y^*\right) = \sup_{\substack{f\left(x_1^*, x_2^*, \ldots, x_n^*\right) = y^* \\ x_i^* \in supp\left(\tilde{X}_i\right),\ i=\overline{1,n}}} \left\{\min\left\{\mu_{\tilde{X}_1}\left(x_1^*\right), \mu_{\tilde{X}_2}\left(x_2^*\right), \ldots, \mu_{\tilde{X}_n}\left(x_n^*\right)\right\}\right\}$$

where $\mu_{\tilde{X}_i}\left(x_i^*\right)$ is the possibility that a fuzzy quantity $\tilde{X}_i$ will take the value x_i^*; $f\left(x_1^*, x_2^*, \ldots, x_n^*\right) = y^*$ – functional dependence of the output parameter of the investment project (NPV, PI, DPP, IRR, MIRR, etc.) on the input parameters.

Below are the main advantages of the fuzzy-interval approach to assessing the effectiveness and risk of investment projects compared to the above methods:

1. This approach allows for the formalisation and use of all available heterogeneous information (deterministic, interval, statistical, linguistic) in a single form, which increases the reliability and quality of strategic decisions.

2. Unlike the interval method, the fuzzy-interval method is similar to the Monte Carlo method, it generates a full range of possible scenarios for the development of the IP, not just lower and upper bounds, so that the investment decision is made not on the basis of two estimates of the investment project's efficiency, but for the entire set of estimates.

3. The fuzzy interval method allows to obtain the expected efficiency of an investment project both in the form of a point value and in the form of a set of interval values with its own distribution of possibilities, characterised by the membership function of the corresponding fuzzy number, which makes it possible to evaluate the integral measure of the possibility of obtaining negative results from the investment project, the degree of risk of the investment project.

4. The fuzzy interval method does not require an absolutely precise specification of membership functions, since, unlike probabilistic methods, the result obtained on the basis of the fuzzy interval method is characterised by low sensitivity (high stability) to changes in the type of membership functions of the original fuzzy numbers, which makes the use of this method more attractive in real conditions of poor quality of initial information.

5. Calculating estimates of investment project indicators based on the method of fuzzy intervals is effective in situations where the initial information is based on small statistical samples, i.e., in cases where it is impossible to obtain probabilistic estimates, which is always the case in the preliminary assessment of long-term investments and quite often in the subsequent prospective analysis, which is carried out in the absence of a sufficient information base.

6. The implementation of the fuzzy interval method on the basis of interval arithmetic provides ample opportunities for applying this method in investment analysis, due to the actual absence of competitive approaches to creating a reliable (in the sense of guarantee) and transportable (in the sense of inclusion) tool for solving numerical problems.

7. It is characterised by the ease of identification of expert knowledge.

The fuzzy interval approach also has advantages in solving the problem of creating an optimal portfolio of investment projects. In order to solve the problem of creating an optimal portfolio of investment projects, a large number of models for creating an optimal portfolio of investment projects have been developed, which differ from each other in the type of objective functions, characteristics of variables, mathematical methods used and the consideration of uncertainty. As a rule, to solve this problem, the apparatus of linear mathematical programming is used under conditions of certainty of the initial information: the problem is usually formulated as a problem of maximisation (or minimisation) of a given function on a given set of feasible alternatives described by a system of equalities or inequalities. For example,

$f(x) \to \max$, with restrictions $\phi_i(x) \le 0$, $i = 1,\ldots,m$, $x \in X$, where X – a given set of alternatives, $f : X \to R^1$ and $\phi : X \to R^1$ – specified functions.

As parameters of the target function $f(x)$ for the task of forming an optimal portfolio of an investment project, various integral indicators of the effectiveness of the investment project are used, however, despite certain advantages and disadvantages of each indicator, many researchers are inclined to believe that the most preferable is the use of NPV as parameters of the objective function, primarily because NPV has the property of additivity, which makes it possible to evaluate the profitability of the entire portfolio of an investment project as the sum of the returns of individual investment projects that form this portfolio. There are various options for formulating the task of forming an optimal investment project portfolio. Most often, the economic meaning of the objective function $f(x)$ is to maximise the economic effect from investment activities, and the meaning of restrictions $\phi_i(x) \le 0$, imposed on the set of feasible solutions to the problem, reflects the limited funds, taking into account the possibility of different budget constraints for each of the time periods of the project.

Since strategic decisions, including those related to the formation of an optimal portfolio of investment projects, are aimed at the long-term perspective and, therefore, by their nature are associated with significant uncertainty and have a significant subjective component, the use of fuzzy mathematical programming to solve the problem of forming an optimal portfolio of investment projects has many advantages.

As an example, one can consider a situation in which the set of acceptable alternatives (investment projects) is a set of all possible ways of allocating resources that a decision maker is going to invest in order to form an optimal investment portfolio. Obviously, in this case, it is inappropriate to introduce in advance a clear boundary for the set of acceptable alternatives (e.g., clear restrictions on the size of the company's investment budget in a given period), since it may happen that resource allocations (investment projects) slightly outside this boundary (without restrictions) will have an effect that "outweighs" the lower desirability (e.g., in terms of investment costs) of these allocations for the decision maker. Thus, a fuzzy description turns out to be, in a sense, more realistic than an arbitrarily accepted clear description of the problem.

The forms of fuzzy description of initial information in decision-making problems can be different, hence the differences in the mathematical formulations of the corresponding fuzzy mathematical programming problems.

Consider, for example, a certain set $X = \{x\}$. Then the fuzzy subset *A* of *X* is called the set of ordered pairs:

$$A = \{x, \mu_A(x)\};\ x \in X;\ \mu_A : X \to [0,1].$$

Here the value $\mu_A(x)$ called membership function *x* to *A* and runs through the entire continuous set of values from 0 to 1. If $\mu_A(x)$ would take only two values 0 and 1, then the set *A* would be a normal subset *X* with indicator $\mu_A(x)$. Suppose that $X = \{x\}$ denotes a set of alternatives in making some decision. Then fuzzy go G from X it is possible to compare a fuzzy subset *G* of sets *X*. For example, if *X* is the set of real numbers, then the fuzzy goal expressed as: "*x* must be significantly greater than 7" can be represented by a fuzzy set with a membership function chosen (very subjectively) in the following form:

$$\mu_G(x) = \begin{cases} 0, & if \quad x < 7 \\ (1 + (x-7)^{-1})^{-1} & if \quad x \ge 7 \end{cases}$$

Similarly, one can define a fuzzy constraint *C* on *X* as a fuzzy subset of *X* sing the concepts of fuzzy goals and fuzzy constraints, it is possible to formulate the decision-making problem in a fuzzy formulation as a problem of finding the intersection of goals and constraints. In particular, for a given set of alternatives *X*, fuzzy solution *D* is defined as a fuzzy subset *X* for which $D = G \cap C$. Corresponding membership function for a fuzzy set *D* is expressed as follows: $\mu_D(x) = Min\{\mu_G(x), \mu_C(x)\}$.

In the general case of availability m goals $G_1,...,G_m$ and n restrictions $C_1,...,C_n$, the solution is a fuzzy set defined by the relation: $D = G_1 \cap G_2 ... \cap G_m \cap C_1 ... \cap C_n$.

Its membership function: $\mu_D(x) = Min\{\mu_{G_1}(x),...,\mu_{G_m}(x), \mu_{C_1}(x),...,\mu_{C_n}(x)\}$.

Finally, if a fuzzy solution D represented by its membership function $\mu_D(x)$, then the desired (clear) solution to the original problem D^G is a subset of D, which is defined as: $\mu_{D^G}(x) = \underset{x}{Max}\{\mu_D(x)\}$.

The value of x, that maximises $\mu_D(x)$ corresponds to the optimal solution. To illustrate, consider a simple example with two fuzzy goals G_1 and G_2, with one fuzzy constraint C_1, and with membership functions from Table 1.

Table 1

x	1	2	3	4	5
$\mu_{G_1}(x)$	0.1	0.0	0.4	0.3	1.0
$\mu_{G_2}(x)$	0.5	0.8	1.0	0.2	0.7
$\mu_{C_1}(x)$	0.2	0.3	0.5	1.0	0.1
$\mu_D(x)$	0.1	0.0	0.4	0.2	0.1

In this case, the optimal solution is $x = 3$, which maximises the membership function of a given fuzzy set.

Consider the problem of finding an optimal investment portfolio of n assets, the share of each asset x_k in the required portfolio is initially bounded from above and below. Assume also that there are m different scenarios in the financial market:

$$R_i(x) = \sum_{k=1}^{n} r_{ik} x_k \to \max \; ; \; i = 1,2,...,m \; ; \; \sum_{k=1}^{n} x_k = 1 \; ; \; X_k^{\min} \le x_k \le X_k^{\max} \; ; \; k = 1,2,...,n \; ;$$

Here r_{ik} denotes profitability k-th asset for i-th market scenario at the end of the investment period, and $R_i(x)$ – portfolio return for i-th scenario. The problem is a multi-criteria optimisation problem, which in general has a set of Pareto optimal solutions. Consider this problem in a fuzzy formulation. It is natural to interpret the uncertainty of a particular market scenario as the uncertainty of the portfolio's return in the event of its implementation. Suppose that $R_i^{\min}$ and $R_i^{\max}$ respectively, the minimum and maximum portfolio returns for i-th market scenario. Assuming that the investor's fuzzy goal is to "earn income $R_i(x)$ significantly greater than the value $R_i^{\min}$", the expression for the corresponding membership function (provided it is linear) is obtained:

$$\mu_i(R_i(x)) = \begin{cases} 0; & if \quad R_i(x) \le R_i^{\min}; \\ \dfrac{R_i(x) - R_i^{\min}}{R_i^{\max} - R_i^{\min}}; & if \quad R_i^{\min} \le R_i(x) \le R_i^{\max}; \\ 1; & if \quad R_i(x) > R_i^{\max}; \end{cases}$$

Then the problem will be reformulated as follows:

$$\underset{i=1,2,...m}{Min} \; \mu_i(R_i(x)) \to \max \; ;$$

$$\sum_{k=1}^{n} x_k = 1 \; ; \; X_k^{\min} \le x_k \le X_k^{\max} \; ; \; k = 1,2,...,n \; .$$

By introducing an auxiliary variable δ this problem reduces to the canonical linear programming problem:

$$\delta \to \max \; ; \; \mu_i(R_i(x)) \ge \delta \; ; \; i = 1,2,...,m \; ; \; \sum_{k=1}^{n} x_k = 1 \; ; \; X_k^{\min} \le x_k \le X_k^{\max} \; ; \; k = 1,2,...,n \; .$$

Conclusions. Thus, a comparative analysis of traditional methods of assessing the effectiveness of long-term investments, existing methods of forming an optimal portfolio of an investment project and the fuzzy interval method showed that the theory of fuzzy sets is one of the most effective mathematical theories aimed at formalising and processing uncertain information, largely integrating known approaches and methods. The theory of fuzzy sets once again confirms a truth widely known to researchers: the formal apparatus used, in its potential capabilities and accuracy, must be adequate to the semantics and correspond to the accuracy of the source data used. Therefore, methods of mathematical analysis are effectively used with accurate initial data. Mathematical statistics and probability theory use experimental data with strictly defined accuracy and reliability. The theory of fuzzy sets makes it possible to process the heterogeneous information characteristic of real investment analysis problems.

Making economically sound decisions always involves inaccurate cash flow modelling. In such cases, management relies on the knowledge of experts in the field of cash flow modelling. Cash flow forecasting based on triangular fuzzy sets makes it possible to recognise the rational use of expert opinion. It should also be remembered that the method discussed here is only one of several methods for dealing with uncertainty in investment analysis. Only by combining the results of several methods can a truly informed decision be made about the future of the project.

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Olena Martynova, Ph.D. in Economics, Candidate of Economic Sciences, Docent, Associate Professor at the Department of Higher Mathematics and Economic and Mathematical Methods, Simon Kuznets Kharkiv National University of Economics. **Application of the theory of fuzzy sets in assessing the economic efficiency and risk of investment projects under conditions of uncertainty.**

This article describes an increasingly popular non-traditional approach to assessing the effectiveness of investment projects under conditions of uncertainty – the fuzzy set method. It is widely agreed that the key factor in analysing the effectiveness of investment projects is the analyst's ability to predict future values of key financial indicators. The fate of the project, and ultimately the well-being of both the investor and the analyst, depends on how accurately the analyst determines future cash flows, interest rates, company capabilities and flexibility. The paper is devoted to the topical issue of evaluating complex investment projects under conditions of risk and uncertainty. The main methods of risk accounting are considered and their main disadvantages are described in detail. As an alternative method, the author proposes to use the theory of fuzzy sets, which has recently become increasingly popular among specialists in various fields. The publication shows that the theory of fuzzy sets is one of the most effective mathematical theories aimed at processing uncertain information and largely integrates known approaches and methods. The author proposes a mathematical model for calculating the risks of investment projects based on fuzziness theory.

Key words: fuzzy set theory, economic efficiency assessment, risk, investment project, uncertainty.

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Мартинова Олена Вадимівна, кандидат економічних наук, доцент, доцент кафедри вищої математики та економіко-математичних методів, Харківський національний університет імені Семена Кузнеця. **Застосування теорії нечітких множин в оцінці економічної ефективності та ризику інвестиційних проектів в умовах невизначеності.**

Приймаючи традиційний підхід до оцінки проектів, можна стверджувати, що основним ворогом інвестиційного аналітика є невизначеність, властива його очікуванням щодо майбутніх значень більшості показників. У цій статті описаний набуває популярності нетрадиційний підхід до оцінки ефективності інвестиційних проектів в умовах невизначеності – метод нечітких множин. Багато хто погодиться, що ключове значення в аналізі ефективності інвестиційних проектів – уміння аналітика передбачати майбутні значення основних фінансових показників. Від того, наскільки точно він визначить майбутні грошові потоки, відсоткові ставки, можливості компанії та її гнучкість, залежить доля проекту, а, зрештою, – добробут як інвестора, так і самого аналітика. Стаття присвячена актуальній проблемі оцінки складних інвестиційних проектів в умовах ризику та невизначеності. Розглядаються основні методи обліку ризиків та докладно описуються їхні основні недоліки. Як альтернативний метод автором пропонується використання теорії нечітких множин, яка останнім часом стає все більш популярною серед фахівців різного профілю. У статті показано, що теорія нечітких множин є однією з найбільш ефективних математичних теорій, спрямованих на обробку невизначеної інформації та багато в чому інтегрує відомі підходи та методи. Також було запропоновано математичну модель до розрахунку величини ризиків інвестиційних проектів з урахуванням теорії нечіткості. Застосування теорії нечітких множин відкриває нові методи та можливості для вирішення завдань оцінювання проектів та формування оптимального портфеля проектів. По-перше, нечіткі множини дозволяють враховувати якісні характеристики проектів, перетворюючи їх у чисельний вигляд. По-друге, стосовно кількісних характеристик проекту, таких як NPV, теорія надає засоби для роботи з невизначеністю навіть у тих випадках, коли наявної інформації недостатньо, щоб робити статистичні висновки з необхідним рівнем достовірності. З іншого боку, розвинений багатий апарат переходу від нечітких оцінок до звичайним числам, що забезпечує можливість формування портфеля проектів з урахуванням їх нечітких оцінок шляхом ранжирування проектів чи рішення відповідної завдання математичного програмування. Гнучкість і потужність методів теорії нечітких множин дозволяють розглядати їх як перспективний та ефективний засіб для вирішення різних завдань управління проектами.

Ключові слова: теорія нечітких множин, оцінка економічної ефективності, ризик, інвестиційний проект, невизначеність.