Збірник наукових праць. Галузеве машинобудування, будівництво
Academic journal. Industrial Machine Building, Civil Engineering
http://journals.nupp.edu.ua/znp
https://doi.org/10.26906/znp.2021.56.2515

UDC 514.18

Topological importance of the segments of the technical system structure

Usenko Valerii[1]*, Vorontsov Oleg[2], Usenko Irina[3], Kodak Olga[4]

[1] National University «Yuri Kondratyuk Poltava Polytechnic» https://orcid.org/0000-0002-4937-6442
[2] National University «Yuri Kondratyuk Poltava Polytechnic» https://orcid.org/0000-0001-7339-9196
[3] National University «Yuri Kondratyuk Poltava Polytechnic» https://orcid.org/0000-0002-6217-4423
[4] National University «Yuri Kondratyuk Poltava Polytechnic» https://orcid.org/0000-0001-6138-6101
*Corresponding author E-mail: valery_usenko@ukr.net

The article discusses the structure of a redundant technical system in the form of a graph. It is indicated that the structure of a technical system is modeled by an undirected graph. The most important and least important elements of this structure from the point of view of topology have been identified and substantiated. The distribution of the number of spanning trees and cyclic subgraphs passing through each element of the structure of the technical system is shown. The relative distribution of linked subgraphs by structure sections is given. This representation, together with the representation of the total number of related subgraphs, conveys the participation of elements in the connectivity of the technical system. The topological significance of the structure sections of the technical system is shown, taking into account the restrictions that this structure is multipolar

Keywords: graph theory, system structure, structural reliability

Топологічна важливість ділянок структури технічної системи

Усенко В.Г.[1]*, Воронцов О.В.[2], Усенко І.С.[3], Кодак О.А.[4]

[1] Національний університет «Полтавська політехніка імені Юрія Кондратюка»
[2] Національний університет «Полтавська політехніка імені Юрія Кондратюка»
[3] Національний університет «Полтавська політехніка імені Юрія Кондратюка»
[4] Національний університет «Полтавська політехніка імені Юрія Кондратюка»
*Адреса для листування E-mail: valery_usenko@ukr.net

Розглянуто структуру резервованої технічної системи як граф, ребрами якого прийнято ділянки структури, а вершинами – вузли структури. Основні завдання дослідження надійності полягають у встановленні і обґрунтуванні вимог по надійності до системи і її складових частин, у виборі принципових напрямів проектного забезпечення надійності на етапах створення системи. Зазначено, що структура технічної системи моделюється неорієнтованим графом. Надійність структури мережі залежить від числа її резервованих та нерезервованих працездатних станів. Зі збільшенням останніх надійність структури збільшується і, відповідно, навпаки. Виявлено, що врахування ймовірності існування всіх працездатних станів дасть більш точну порівняльну оцінку надійності структури технічної системи. Для визначення числа циклічних зв'язних підграфів, що проходять через кожний елемент системи використовується алгоритм пошуку в глибину. Показано обчислення числа циклічних підграфів що проходять через дану ділянку. Визначено та проілюстровано циклічні підграфи структури технічної системи, що моделюють працездатні стани з резервом. Виявлено найважливіші ділянки для заданої системи за топологією, бо вони найбільш вразливі для структури. При виході останніх з ладу рівень працездатності структури буде значно меншим. І навпаки, з вилученням найменш важливих ділянок надійність структури знизиться в меншій мірі завдяки структурній надлишковості, що реалізована іншими ділянками. Приведено відносний розподіл зв'язних підграфів по ділянкам структури. Це представлення разом з поданням загального числа зв'язних підграфів передає участь ділянок у зв'язності структури технічної системи. Зазначено, що чим більшим є відносне число підграфів ділянки, тим важливішою є ділянка для зв'язності структури і навпаки. Показано топологічну важливість ділянок структури технічної системи за обмежень, що дана структура є багатополюсником, тобто кожний вузол має бути зв'язаним з кожним іншим вузлом структури.

Ключові слова: теорія графів, структура системи, структурна надійність

Introduction

To design and maintain complex systems, it is necessary to study the impact of their components on reliability [7-10]. Determination of the system structure components influence helps to solve the problems of optimal redundancy and reliability distribution among the elements [8]. This is necessary for monitoring, analyzing failures, restoring systems, and obtaining a generalized picture of the effect on the structural reliability of the failures process system of elements or their groups (components).

Review of the research sources and publications

In the studies carried out on the reliability of various technical systems [1-5], attention is focused on identifying the basic requirements for their reliability, reliability indicators were given, and methods for determining the number of spanning trees that can be associated with the limiting operational states of the topological structure were given. The number of spanning trees passing through each section of the structure is taken as an indirect indicator for comparing different structures in terms of reliability [2]. The literature provides a calculation of the number of spanning trees in graphs [1] and shows how to determine the number of spanning trees passing through individual sections of the structure [2]. However, for a more accurate assessment of each element's importance in the system structure, one should take into account all possible working conditions structure, including cyclic connected subgraphs of the structure [3].

Definition of unsolved aspects of the problem

To date, an assessment of the topological importance of the elements of the structure of a technical system as a characteristic of the elements' reliability distribution in the system structural reliability has not been derived.

Problem statement

The main tasks of the reliability study are to establish and substantiate the reliability requirements for the system and its components, in the choice of the principal directions and rational strategies for the design reliability assurance, in the elaboration of the reliability assurance issues at the subsequent system creation stages [4, 16, 17].

Basic material and results

The publication [8] reflects the ways of establishing the structural system elements' significance. There are limitations: there is no initial possibility of the parts reliability. A matrix of paths is used to find the significance of a system element as a value of its rank (R_i). Rank is the number of a particular element ties with others

$$R_i = \frac{A_i + A_i^2}{\sum_{i=1}^{m}\left(A_i + A_i^2\right)} \quad (1)$$

where A_i, A^2_i are the sums of the direct matrix paths rows with ones in the main diagonal and of the same matrix squared, respectively.

After studying the principle of system operation, a graph is drawn up that simulates the dominance of elements and a matrix of dominance. The value of the functional and structural rank of the element is

$$R_i = \frac{B_i + B_i^2}{\sum_{i=1}^{m}\left(B_i + B_i^2\right)} \quad (2)$$

where B_i, B_i^2 are the sums of the adjacency matrix rows and the same matrix squared.

The disadvantage of this method is that it does not set items with zero ranks. There is also no definition of the degree of mutual elements' influence.

In [13], the concept of the elements' weight in the system reliability was formulated. It does not depend on probability based on the concept of Boolean difference and the use of a logical-probabilistic method for studying the reliability of systems [10]. In [7], the reliability impact magnitude constituent components of the structure on the reliability of the system are described. This value is determined statistically.

Accordingly [11], the transition from the reliability parameters of individual elements to the reliability of their groups is insufficient without studying the importance of the component formed by the two elements. The influence of two elements on the system reliability is shown: temporary (conjunctive), total disjunctive, or separately (strictly disjunctive). The study describes finding the importance of two elements in the system reliability analysis. There is a prospect of generalization for a huge number of the system parts by the induction method.

Scientific sources [11, 14] highlight the property of weight for two elements. The weight of any logical function is determined by a probabilistic function with the condition of equal probability of truth and falsity of all arguments of the algebraic function of logic.

Most of the known methods for determining the magnitude of the impact of individual elements on the reliability of a system use the probability of initiating events. In [12], the structural significance of the element in a system with a monotonic structure in the form of a partial derivative is presented

$$B(i \setminus R) = \frac{\partial R_c}{\partial R_i} \quad (3)$$

where $R_c = f(R_1,...,R_m)$ is the system reliability depends on R_i.

If the value of R_i is unknown, the significance of the element x_i is expressed with a value of 0.5 for the reliability of all elements:

$$B(i) = \left.\frac{\partial R_c}{\partial R_i}\right|_{R_1 = ... R_m = 0,5} \quad (4)$$

In publications [9, 13], the partial derivative is the significance of the element:

$$\zeta_i = \frac{\partial R_c}{\partial R_i} = P\left\{\Delta_{x_i} y\left(x_m^{'}\right) = 1\right\} \quad (5)$$

where R_i is element reliability, R_c is system reliability.

Replacement in expression (4) of the variables x_i and x'_m by the value 0.5 for all i=1,...,m gives the dependence of the element's action:

$$g_{x_i} = P\left\{\Delta_{x_i} y\left(x'_m\right) = 1\right\}\Big|_{R_1 = \ldots R_m = 0,5} \tag{6}$$

In expressions (5) and (6), the influence of an element quantitatively coincides with the concept of significance.

But there is a difference: the weight g_{xi} is included in the logical model and the structural significance to the probabilistic one [10]. The reliability is

$$R_1 R_{c1}^{(i)} + Q_i R_{c0}^{(i)} = R_1 R_{c1}^{(i)} + (1 - R_i) R_{c0}^{(i)} \tag{7}$$

$$R_{c1}^{(i)} - R_{c0}^{(i)} = \zeta_i \tag{8}$$

where

$$R_{c1}^{(i)} = P(y_{c1}^{(i)}(x_m) = 1)$$

$$R_{c0}^{(i)} = P(y_{c0}^{(i)}(x_m) = 1)$$

Formula (8) is the basis of methods for calculating the significance of systems. Element significance is interpreted as the speed of the system's reliability changing. It detects elements, an increase in the reliability of which by an amount ΔR_i increases the reliability of the system by an amount

$$\Delta R_{\tilde{n}} = \zeta_i \Delta R_i \tag{9}$$

The significance ζ_i of an element depends not only on its place in the structure of the system but also on the reliability R_i of all other elements, except for itself. The probabilistic representation of significance helps to interpret the significance of an item as the system reliability if the item is in working order. The concept of an element's contribution was described in the study [10]. The contribution of an element to the system reliability is equal to:

$$B_{x_i} = R_i \frac{\partial R_c}{\partial R_i} = R_i \zeta_i \tag{10}$$

Formula (6) can be represented as

$$R_c - R_{c0}^{(i)} = R_i (R_{c1}^{(i)} - R_{c0}^{(i)}) = R_i \zeta_i \tag{11}$$

The contribution of the element B_{xi} is the probabilistic part of the reliability of the system R_c, which it receives with the restoration of the operability of the element R_i. Taking into account (9), the value of the contribution is presented as an increase in the reliability of the system by attracting an element to its structure, or by restoring its operability. Damage to an element of the structure of the system is the product of the probability of failure Q_i of the element by its significance

$$Ç_{x_i} = Q_i \frac{\partial R_c}{\partial R_i} = Q_i \zeta_i \tag{12}$$

The relationship between the contribution and loss of an element coincides with the ratio of its probabilities to reliability and failure [10]:

$$\frac{B_i}{Ç_i} = \frac{R_i}{Q_i} \tag{13}$$

Consider the structure of the redundant technical system (TS) as a graph A, edges of which we take parts of the structure x_i, i=1,...,n, and the vertices are the nodes of the structure v_j, j=1,...,m. The structure of TS is modeled by an undirected graph, the connections between nodes exist in two directions. The subgraph T_i of this graph, which has no cycles but has edges that connect all vertices of graph A, is a trunk tree. For the structure of the TS, such a tree simulates the operating state, which is the limit because it has no structural redundancy.

When the structure is inoperable, at least one node of the structure has no connection with other nodes and this corresponds to the removal of one or more edges of graph A that connect this vertex with others.

The reliability of the network structure depends on the number of its unreserved operational states. The number of remaining trees in graph A determines the number of these states [3]. As the number of trunks increases, the reliability of the structure increases, and, accordingly, as the number of trunks decreases, the reliability of the structure decreases, $P_i > P_j$, if $T_i > T_j$.

The number of carcass trees of a given graph is calculated from a symmetric binary matrix of adjacency of vertices, in which the elements of the main diagonal are numerical values of the degrees of the corresponding vertices [1]. Ever, in the structure of the TS, not only non-reserved but also redundant operational states are possible, which are modeled by connected cyclic subgraphs B_i of graph A of the structure of the TS. The sum probability of any related events number is reflected by the known formula [3]:

$$P\left(\sum_{i=1}^{n}(B_i)\right) = \sum_{i=1}^{n} P(B_i) - \sum_{i,j} P(B_i B_j) + \sum_{i,j,k} P(B_i B \ldots + (-1)^{n-1} P(B_1 B_2 \ldots \tag{14}$$

where B_i is a random event of the existence of a structure working state, which is modeled by the subgraph Q_i of graph A, n is the possible number of states, i = 1,…,n. The number of all possible subgraphs of graph A is 2^x, where x is the number of edges of graph A. If we take into account that always one of them is trivial, then the number of all possible subgraphs having edges

$$H_x = \sum_{j=1}^{x} \binom{x}{j} = 2^x - 1 \tag{15}$$

For the structure shown in Fig. 1 such subgraphs are $H_8 = 2^8 - 1 = 255$.

It is obvious that taking into account the probability of the existence of all working conditions will give a more accurate comparative assessment of the TS structure reliability. If the number of cyclic subgraphs in the TS structure increases with the change of its internal connections, the TS reliability will increase, ie $P_i > P_j$, if $B_i > B_j$ and $Q_i > Q_j$.

Thus, the working states of the TS structure correspond to the cover trees and cyclic subgraphs of the structure graph. All vertices of graph A are connected by edges, and when one edge that is part of a loop is removed, the subgraph remains connected. When one edge is removed outside the cycle, the connection with one or more vertices of the graph is lost (the graph splits into two parts, which are components of connectivity) and this corresponds to the inoperable state of the TS structure.

It is possible to determine the number of cyclic connected subgraphs Q_i passing through each element of the system using the depth search algorithm [6]. The algorithm is based on a systematic search of the graph vertices in such an order that each edge x_i and each vertex of the graph v_i is considered only once.

The transition from the vertex v_j to v_{j+1} is the edge x_i that connects them. Using the depth search algorithm, you can view all possible subgraphs of the structure graph. The connected cyclic subgraph Q_i of graph A should contain a tree [1]. Therefore, the selection of a connected subgraph is performed by checking for the presence of a T_i tree.

The number of cyclic subgraphs Q_i passing through this section x_i is calculated by the formula

$$Q_i = S_n - S'_{n-1} \tag{16}$$

where S_n is the total number of connected subgraphs in the graph of structure A with a given section x_i; S'_{n-1} is the number of connected subgraphs without plot x_i.

Take the structure of the technical system (Fig. 1). T_8=32 different carcass trees pass through the whole structure; it has 32 different limit working states (Fig. 2) and Q_8=34 cyclic subgraphs (Fig. 3). If you remove segment x_1 from the structure (denote the segment x_1 through its ends, which are nodes 1-4). Graph A becomes subgraph A'. The number of remaining trees will be $T'_8 = det\ A'$= 24. Using the depth search algorithm, determine the number of cyclic subgraphs Q'_8=26. The number T and the number Q decreased by the number of subgraphs passing through section x_1, respectively.

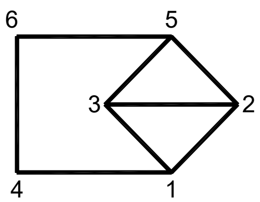

Figure 1 −The structure of the technical system

$T_{1\text{-}4}=T_i-T_j$=24 different carcass trees and $Q_{1\text{-}4}=Q_i-Q_j$=28 different cyclic subgraphs of the structure pass through sections $x_{1\text{-}4}$, $x_{4\text{-}6}$, $x_{5\text{-}6}$. These segments are the most important for this structure in terms of topology because they are the most vulnerable to the structure. The least important for this structure is the area $x_{2\text{-}3}$ because the smallest number of trees $T_{2\text{-}3}$ = 16 and $Q_{2\text{-}3}$ = 26 cyclic subgraphs pass through it. In the event of its failure (removal), the network structure will be more workable than in the removal of the above areas because it has a structural surplus, which is implemented in other segments.

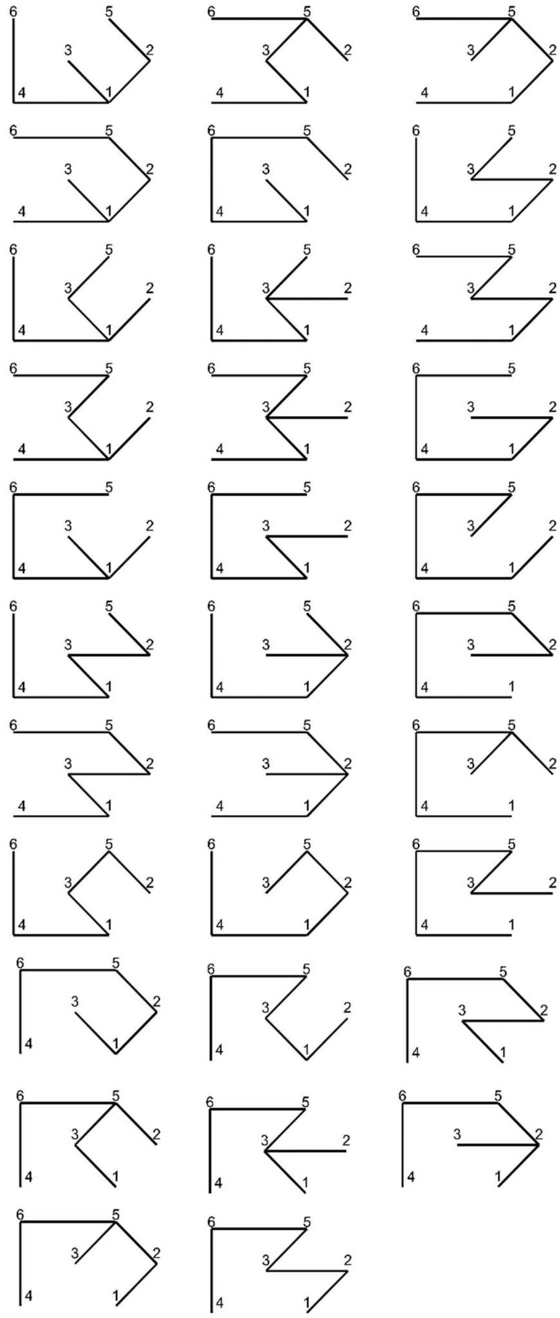

Figure 2 − All 32 carcass trees illustrate the ultimate operating state of a structure without reserve

In Fig. 4 shows the distribution of the trunk tree number passing through each section x_i of the TS structure. In fig. 5 – distribution of the number of cyclic subgraphs Q_i by structure sections. If the number of spanning trees for sections $x_{4\text{-}6}$ and $x_{2\text{-}3}$ is equal to $T_{4\text{-}6}$=24 and $T_{2\text{-}3}$=16, then taking into account the distribution of the numbers of cyclic subgraphs (Fig. 5), these values will change slightly: respectively, for section $x_{4\text{-}6}$ we have $Q_{4\text{-}6}$ = 52, and for section $x_{2\text{-}3}$ we obtain $Q_{3\text{-}5}$ = 42 (Fig. 6).

To study the distribution of reliability between elements of the system structure, we use the number of all connected subgraphs (carcass trees and cyclic subgraphs):

$$F_i = T_i + Q_i \qquad (17)$$

where F_i is the number of all connected subgraphs, T_i is the number of truncated trees, Q_i is the number of connected cyclic subgraphs.

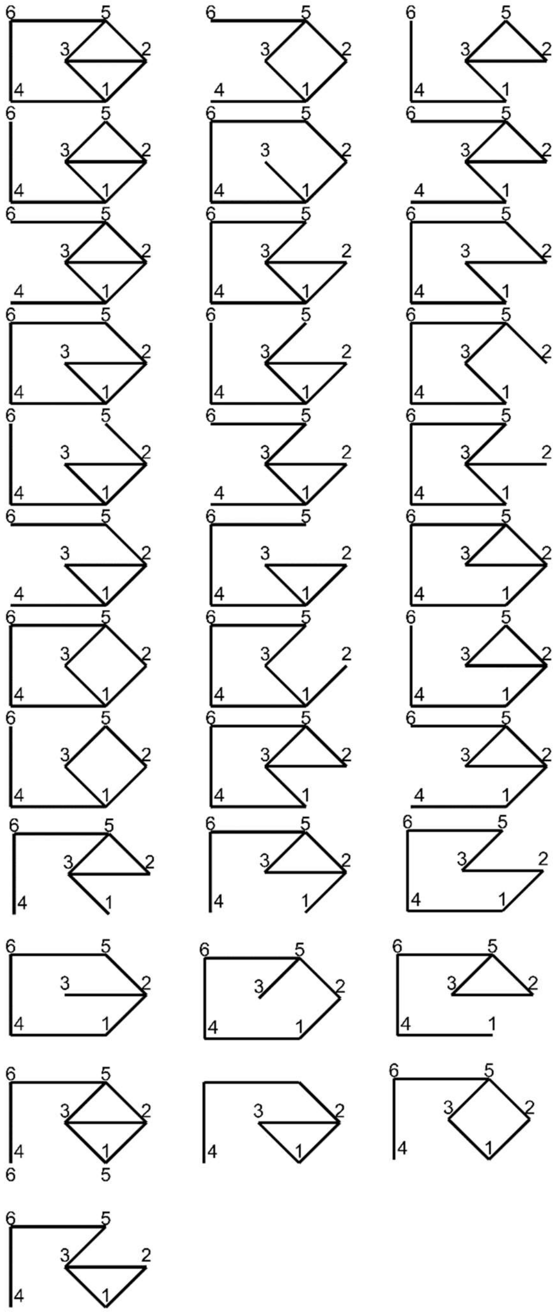

Figure 3 – All 34 cyclic subgraphs of the structure of the TS, simulating working conditions with a reserve

We will accept the distribution of connected subgraphs numbers on the TS structure sites is

$$C_i = \frac{F_i}{F} \qquad (18)$$

where F_i – the number of connected subgraphs passing through the section x_i; F - the number of connected subgraphs passing through the whole structure modeled by graph A.

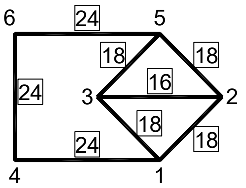

Figure 4 – Distribution of the number of carcass trees passing through sections of the structure

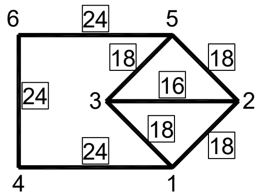

Figure 5 – Distribution of the number of cyclic subgraphs passing through sections of the structure

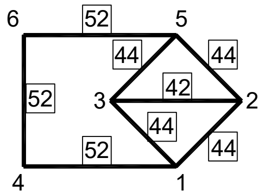

Figure 6 – Distribution of the numbers of all connected subgraphs by sections of the structure

In fig. 7 shows the relative distribution of connected subgraphs by sections of the structure.

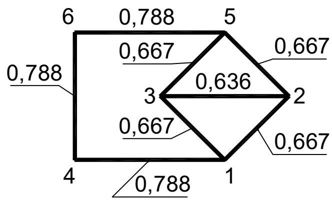

Figure 7 – Topological importance of TS structure

This representation, together with the representation of the total number of connected subgraphs, for example, for a given structure $F = 32 + 34 = 66$, indirectly conveys the sites' participation in the connectedness of the TS structure. It should be noted that the relative number of subgraphs C_i in segment x_i is a greater subgraph for the coherence of the structure and vice versa. $C_i > C_{i+1}$, where C_i is the topological importance of the site, C_{i+1} is the topological importance of the site x_{i+1}. In fig. 7 shows the topological importance of parts of the TS structure for the limited fact that this structure is multipolar, each university must be connected to each other node structure.

Conclusion

The participation of the structure segments in the distribution of the technical system reliability by elements can be estimated by the number of subgraphs: carcass trees and connected cyclic subgraphs of the technical system structure graph.

Assessment of the topological significance of a technical system structural element is a characteristic of the distribution of the elements' reliability in structural reliability of the system. Coverage trees and cyclic subgraphs of the structure graph correspond to the operating state of a technical system.

When less important segments fail, the network structure will be more efficient than removing critical segments, since it has structural redundancy implemented in other segments and vice versa.

More subgraphs of a segment than other segments indicate a higher level of structure importance and vice versa. The topological importance of the system structure elements is limited by the fact that this structure has many poles: input and output nodes. In this case, each node of the structure must be connected to each other.

References

1. Свами М., Тхуласираман К. (1984). *Графы, сети и алгоритмы*. Москва: Мир
2. Новохатний В.Г., Яворский Э.Б., Запорожец С.С. (1981). О топологических характеристиках анализа надежности трубопроводных сетей. *Деп. ВНИИИС,* 2302. Москва: ВНИИИС
3. Reinschke K., Ushakov I. (1987). *Application of Graph Theory for Reliability Analysis*. Berlin: Verlag Technik
4. Бобров В.И. (2004). *Надежность технических систем*. Москва: МГУП
5. Надежность и эффективность в технике: Справочник (1990). Под ред. В.И. Патрушева, А.И. Рембезы. Москва: Машиностроение
6. Кристофидес Н. (1978). *Теория графов*. *Алгоритмический подход*. Москва: Мир
7. Кузнецов Н. Ю. (1989). Об оценке влияния надежности различных элементов на надежность системы в целом. *Кибернетика*, 5, 110-119
8. Нечипоренко В.И. (1968). *Структурный анализ и методы построения надежных систем*. Москва: Сов. радио
9. Рябинин И.А. (2000). *Надежность и безопасность структурно-сложных систем*. С.-Петербург: Политехника
10. Рябинин И.А., Черкесов Г.Н. (1981). *Логико-вероятностные методы исследования надежности структурно-сложных систем.* Москва: Радио и связь
11. Рябинин И.А., Парфенов Ю.М. (1991). Определение характеристик важности совокупности элементов энергетической системы при исследовании ее безопасности. *Энергетика и транспорт,* 1, 44-57
12. Birnbraum Z.W. (1969). On the importance of different components in a multi-component system. Multivariate Analyses. *Academic Press*, 2, 581-592
13. Ryabinin I. A. (1976). *Reliability of Engineering Systems. Principles and Analysis*. Moscow: Mir
14. Ryabinin I. A. (1994). A Suggestion of a New Measure of System Components Importance by Means of a Boolean Difference. *Microelectronics and Reability*, 34(4), 603-613
15. Usenko V., Kodak O., Usenko I. (2020). Geometric reliability model of the five site redundant structure. *Engineering Review*, 40, 2, 10-15
http://doi.org/10.30765/er.40.2.02
16. Jin-Zhang Jia, Zhuang Li, Peng Jia, Zhi-guo Yang (2020). Reliability Analysis of Common Cause Failure Multistate System Based on CUGF. *Mathematical Problems in Engineering*, 2020, 1-14
https://doi.org/10.1155/2020/4608124
17. Cheng-Fu Huang, Ding-Hsiang Huang, Yi-Kuei Lin (2020). Network Reliability Evaluation for a Distributed Network with Edge Computing. *Computers & Industrial Engineering,* 147, 10649
https://doi.org/10.1016/j.cie.2020.106492

1. Swami M., Thulasiraman K. (1984). *Graphs, networks and algorithms.* Moscow: Mir
2. Novokhatny V.G., Yavorsky E.B., Zaporozhets S.S. (1981). On the topological characteristics of the analysis of the reliability of pipeline networks. *Dep. VNIIIS,* 2302. Moscow: VNIIIS
3. Reinschke K.,Ushakov I. (1987).*Application of Graph Theory for Reliability Analysis*. Berlin: Verlag Technik
4. Bobrov V.I. (2004). *Reliability of technical systems*. Moscow: MGUP
5. *Reliability and efficiency in technology: Handbook* (1990). Ed. V.I. Patrushev, A.I. Rembeza. (1990). Moscow: Mechanical Engineering
6. Christofides N. (1978). *Graph Theory*. *Algorithmic approach*. Moscow: Mir
7 Kuznetsov N. Yu. (1989). On the assessment of the reliability influence of various elements on the reliability system as a whole. *Cybernetics*, 5, 110-119
8. Nechiporenko V.I. (1968). *Structural analysis and methods for constructing reliable systems*. Moscow: Sov. radio
9. Ryabinin I.A. (2000). *Reliability and safety of structurally complex systems*. St. Petersburg: Polytechnic
10. Ryabinin I.A., Cherkesov G.N. (1981). *Logical-probabilistic methods for studying the structurally reliability of complex systems*. Moscow: Radio and communication
11. Ryabinin I.A., Parfenov Yu.M. (1991). Determination of the characteristics of the importance of a set of elements of the energy system in the study of its safety. *Energy and transport*, 1, 44-57
12. Birnbraum Z.W. (1969). On the importance of different components in a multi-component system. Multivariate Analyses. *Academic Press*, 2, 581-592
13. Ryabinin I. A. (1976). *Reliability of Engineering Systems. Principles and Analysis*. Moscow: Mir
14. Ryabinin I. A. (1994). A Suggestion of a New Measure of System Components Importance by Means of a Boolean Difference. *Microelectronics and Reability*, 34(4), 603-613
15. Usenko V., Kodak O., Usenko I. (2020). Geometric reliability model of the five site redundant structure. *Engineering Review*, 40, 2, 10-15
http://doi.org/10.30765/er.40.2.02
16. Jin-Zhang Jia, Zhuang Li, Peng Jia, Zhi-guo Yang (2020). Reliability Analysis of Common Cause Failure Multistate System Based on CUGF. *Mathematical Problems in Engineering*, 2020, 1-14
https://doi.org/10.1155/2020/4608124
17. Cheng-Fu Huang, Ding-Hsiang Huang, Yi-Kuei Lin (2020). Network Reliability Evaluation for a Distributed Network with Edge Computing. *Computers & Industrial Engineering,* 147, 10649
https://doi.org/10.1016/j.cie.2020.106492