The article presents a mathematical description of an induction motor with further modeling in the MATLAB program. In the model, the method of direct torque control (DTC) is implemented. The basic idea of control is that at each step of the calculation the optimum state of the inverter voltage is determined, which causes a change in both the moment and the stator flux coupling in the necessary direction. The obtained transient processes.

**Keywords:** direct torque control, Flux linkage vector, induction motor model, MATLAB, DTC.

### Introduction

The induction motor in the fixed coordinate system x-y is described by a known system of equations [1]:

\[
\begin{align*}
\dot{\Psi}_{1x-y} &= R_1 \dot{I}_{1x-y} + \frac{d}{dt} \Psi_{1x-y}, \\
\dot{\Psi}_{2x-y} &= R_2 \dot{I}_{2x-y} + \frac{d}{dt} \Psi_{2x-y} - j \omega_0 \Psi_{2x-y}, \\
\dot{I}_{1x-y} &= L_1 \dot{I}_{1x-y} + L_2 \dot{I}_{2x-y}, \\
\dot{I}_{2x-y} &= L_3 \dot{I}_{1x-y} + L_4 \dot{I}_{2x-y}, \\
\omega &= \int \frac{M_d - M_C}{J} dt,
\end{align*}
\]

where:
- \(R_1\) – active resistance of the stator winding phase;
- \(R_2\) – active resistance of the rotor winding phase reduced to the stator;
- \(M_d\) – static load moment;
- \(\Psi_{1x-y}\) – vector flux linkage of stator;
- \(\Psi_{2x-y}\) – vector flux linkage of rotor;
- \(J\) – moment of inertia, reduced to the motor;
- \(U_{1x-y}\) – stator voltage vector;
- \(U_{2x-y}\) – rotor voltage vector; \(\omega\) – nominal angular velocity;
- \(\omega_0\) – rotor angular velocity;
- \(L_1\) – total inductance of the phase of the stator winding;
- \(L_2\) – total inductance of the phase of the rotor winding;
- \(L_m\) – magnetizing circuit inductance.

### Main part

This mathematical description for the MATLAB package can be represented in the following matrix form. Equation of stator and rotor stresses:

\[
\dot{\Psi}_3 = R_3 \dot{I}_3 + \frac{d}{dt} \Psi_3 - p_n C \cdot \Psi_3,
\]

where the matrix-columns of voltages, currents and flux linkages have the form:

\[
U_3 = \begin{bmatrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \end{bmatrix}, \quad \Psi_3 = \begin{bmatrix} \Psi_{1x} \\ \Psi_{1y} \\ \Psi_{2x} \\ \Psi_{2y} \end{bmatrix}, \quad I_3 = \begin{bmatrix} I_{1x} \\ I_{1y} \\ I_{2x} \\ I_{2y} \end{bmatrix}.
\]

Resistance Matrix \(R_3\) and the matrix \(C\), turning vector of flux linkage on 90°, have the form:

\[
R_3 = \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_1 & 0 & 0 \\ 0 & 0 & R_2 & 0 \\ 0 & 0 & 0 & R_2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.
\]

The linkage and torque of the motor are connected with current expressions:

\[
\Psi_3 = L_3 \cdot I_3, \quad M_d = (G \cdot \Psi_3) \cdot (F \cdot I_3).
\]

The matrix of inductances \(L_3\), as well as matrices \(F\) and \(G\) have the form:

\[
\begin{bmatrix} F \\ L_3 \\ G \end{bmatrix} = \begin{bmatrix} [-1 \ 0 \ 0 \ 0] & [L_1 \ 0 \ L_m \ 0] \\ [0 \ 1 \ 0 \ 0] & [0 \ L_1 \ 0 \ L_m] \\ [0 \ 0 \ 0 \ 0] & [L_m \ 0 \ L_2 \ 0] \\ [0 \ 0 \ 0 \ 0] & [0 \ L_m \ 0 \ L_2] \end{bmatrix}.
\]

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Based on the mathematical description, a computer model is developed in the MATLAB package, shown in Fig. 1.

Block «Psi->I» solves the system of linear equations

$$\psi_3 = L_3 \cdot I_3,$$

realizing the operation

$$I_3 = L_3^{-1} \cdot \psi_3.$$

MATLAB Function in block has the form $$L_3 / u$$.

In block «Psi, I->M» on flux linkage $$\Psi$$ and current $$I_3$$ the electromagnetic moment is calculated in accordance with equation (3). Line «MATLAB Function» in block has the form:

$$(G*u(5:8))*(F*u(1:4))*p^{3/2} \quad \text{(Fig. 2)}.$$  

Further, the moment of resistance is subtracted from the electromagnetic moment $$M_e$$, and the resulting dynamic moment is integrated and divided by the moment of inertia $$J$$. The result is the rotational speed of the rotor. The transformation of the three-phase voltage into the two-phase required by the model is carried out using the block represented in Fig. 3.

As a result, based on the previously obtained subsystems, we obtain a general system which is shown in Fig. 4.

**Fig. 1.** Computer model of an induction motor

**Fig. 2.** Three-phase phase voltage conversion system (a-b-c) to (x-y)

**Fig. 3.** Two-phase phase voltage conversion system (x-y) to three-phase (A-B-C)
In the mid-80's, ABB proposed and later implemented the direct torque control (DTC) method [1]. The basic idea of control is that at each step of the calculation, the optimum state of the inverter voltage is determined, which causes a change in both the moment and the stator flux coupling in the required direction [10]. As a result, the separation of the torque and flow control channels of the induction motor is achieved.

The basis of the DTC system is the equation of the electromagnetic moment of an induction motor [1].

\[
M_R = \frac{3}{2} P_{II} K_1 \frac{L_1}{\sigma L_2} (\psi_1\psi_{2x} - \psi_{1x}\psi_{2y})
\]  

(5)

The projections of the flux link vector on the axis of the fixed coordinate system can be written in terms of the vector modules and the current angular values relative to the abscissa [1],

\[
\psi_{1x} = |\psi_1|\cos \theta_1; \psi_{1y} = |\psi_1|\sin \theta_1;
\]

\[
\psi_{2y} = |\psi_2|\cos \theta_2; \psi_{2y} = |\psi_2|\sin \theta_2
\]

Hence the expression for the moment is obtained in the form

\[
M_R = \frac{3}{2} P_{II} K_1 \frac{L_1}{\sigma L_2} |\psi_1||\psi_2|\sin \theta
\]

where \( \theta \) - the spatial angle between the vectors of the stator flux linkages \( \psi_1 \) and rotor \( \psi_2 \).

If the modules of vectors \( |\psi_1| = \psi_{1m} \) and \( |\psi_2| = \psi_{2m} \) keep constant, the magnitude of the torque can be controlled by changing the angle \( \theta \) [9].

The algorithm of the DTC system is constructed as follows. First, the vector of stator flux linkage \( \psi_1 \) and electromagnetic moment of motor are determined in some way. Then the vector and moment modules are compared with the set values, after which, using the comparators, called the relay regulators, the logical error signals \( d\psi \) and \( dM \) [9].

Based on these signals and knowing the position of the flux linkage vector in the plane of the reference vectors, one can choose a combination of the states of the inverter's keys, at which the base voltage vector is formed, minimizing the deviation from the set values. Thus, as a result of the operation of the system, the stator flux-stator module and the electromagnetic moment of the asynchronous motor will always be in the zone of the permissible deviation from the set value determined by the hysteresis value of the corresponding regulator. The choice of the basic vector of an error-minimizing control variable depending on the sector in which the vector is currently located \( \psi_r \), are produced using a switching table (Optimal Vector Selection Table) [9]. The switching table consists of the digitized signals of the flux mismatch (1.0) and the moment (1.0, -1) and information in which sector of the coordinate system associated with the stator is the stator flux vector of the induction motor. With the help of the switching table, the optimal voltage output vector is chosen which must be fed to the stator windings of the asynchronous motor in order to ensure a minimum deviation of the regulated quantities from the set values.

The choice of the optimal voltage vector is made from eight possible voltage base vectors [9]. DTC systems eliminated the shortcomings inherent in FOC systems: large amount of computations for forward and reverse transformation of a fixed and rotating coordinate system, the presence of a delay in the formation of an electromagnetic moment [9].

**Construction of a mathematical model**

**DTC system**

In Fig. 5 the system of direct torque control in the MATLAB package is shown.

In Fig. 5 function block numberkeys calculates the potentials at the output of all the arms of the converter.
according to the switches provided by the key. In the event that both keys are closed, an error message is displayed.

The function block calc_abc calculates the phase voltages at the output of the inverter, depending on which voltage potentials are present at the output of its arms. The input for this function is the result obtained in the previous one.

A program is developed in the language MATLAB of direct torque control on the rotor.

**Simulation results**

For the simulation, we used the parameters of the motor of type 4A80A4, which are given in Tabl. 1, 2.

In the simulation of direct torque control, a stepped reference to the flux linkage at the time $t=0s$ after, a stepped reference to the torque at the time $t=0.2s$ and the stepped moment of resistance to the motor shaft is equal 1. The resulting transient processes with respect to the motor variables are shown in Fig. 6-11.

<table>
<thead>
<tr>
<th>Motor parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>4A80A4</td>
</tr>
<tr>
<td>Efficiency, %</td>
</tr>
<tr>
<td>$\cos \phi$</td>
</tr>
<tr>
<td>$P_m$, kW</td>
</tr>
<tr>
<td>$S_m$, rel. units</td>
</tr>
<tr>
<td>$X_m$, rel. units</td>
</tr>
<tr>
<td>$R_1$, rel. units</td>
</tr>
<tr>
<td>$X_1$, rel. units</td>
</tr>
<tr>
<td>$R_2$, rel. units</td>
</tr>
<tr>
<td>$X_2$, rel. units</td>
</tr>
<tr>
<td>$J$, kg·m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters of the substitution scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{m1}$, H</td>
</tr>
<tr>
<td>$R_1$, Ohm</td>
</tr>
<tr>
<td>$L_{m2}$, H</td>
</tr>
<tr>
<td>$R_2$, Ohm</td>
</tr>
<tr>
<td>$L_{2m}$, H</td>
</tr>
</tbody>
</table>

Fig. 6. Current dependence ABC from time
Фіг. 7. Струмова залежність (a) статор XY; (b) ротор від часу

Фіг. 8. Залежність статорного напруги в координатах x-y від часу

Фіг. 9. Залежність статорного напруги в координатах A-B-C від часу
Conclusion

In this paper, a simpler mathematical model of an induction electric drive with direct torque control is developed. Modeling in the MATLAB package of a drive with an motor of the type 4A80A4 is made. The accuracy of torque and flux linkage obtained on the model is 0.041 and 0.02, respectively.

References


Рецензент: д-р техн. наук, проф. С.В. Козелков, Державний університет телекомунікацій, Київ.

ИССЛЕДОВАНИЕ СИСТЕМЫ ПРЯМОГО УПРАВЛЕНИЯ МОМЕНТОМ АСИНХРОННОГО ДВИГАТЕЛЯ

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В статье представлено математическое описание асинхронного двигателя с дальнейшим моделированием в программе MATLAB. В модели реализуется метод прямого управления моментом (DTC). Основная идея управления полягает в том, что на каждом шаге расчета определяется оптимальный порог напряжения инвертора, которое включается в момент, при котором индуктивная индуктивность статора в необходимом направлении. Представлены полученные переходные процессы.

Ключевые слова: прямое управление моментом, вектор потокосцепления, модель асинхронного двигателя, MATLAB, DTC.